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MURAKKAB SHAKILDAGI ANIZOTROP YUPQA PLASTINANING TERMO-ELEKTRO-MAGNIT-ELASTIKLIK DEFORMATSIYALANISH JARAYONLARINI MATEMATIK MODELLASHTIRISH

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Annotatsiya. Ushbu maqolada murakkab shakldagi anizotrop yupqa plastinalarning termo-elektro-magnit-elastiklik deformatsiyalanish jarayonlarini matematik modellashtirish bo'yicha RFM usuli yoritiladi. Murakkab geometrik shakllar va material xususiyatlarining hisobga olinishiga imkon berib, termo-elektro-magnit ta'sirlarning plastinalardagi deformatsiyalariga ta'sirini baholashga yordam beradi. Model natijalari plastinalarning sanoat va ilmiy tadqiqotlardagi qo'llanilish ehtimolini oshirish maqsadida amaliy natijalar bilan taqqoslanadi.

Kalit so’zlar: Anizotrop, termo, elektro, magnit, elastiklik, matematik modellashtirish, deformatsiya.

1 Kirish

Hozirgi vaqtda texnika katta tezlik bilan rivojlanmoqda, ayniqsa elektrotexnika, kompyuter qurilmalari va h.k. Bular o‘z navbatida yangi innovatsion yondoshishlarni talab qilmoqda. Texnik qurilmalarni ishlab chiqarishda bazaviy elementlarining holatlarini elektromagnit va mexanik maydonlarning o‘zaro ta’siri natijasida o‘zgarishlarini tadqiq qilish muhim ahamiyat kasb etmoqda. Texnik qurilmalarning ko‘pchiligi geometrik shakllari murakkab ko‘rinishdagi plastina va qobiqlardan tashkil topgan bo‘ladi.

2 Matematik modellashtirish

Plastina x, y, z Dekart koordinatalar tizimida shunday joylashganki, plastinaning o‘rta tekisligi x, y tekisligi bilan ustma – ust tushadi. Elektromagnit maydonda joylashgan yupqa plastina uchun matematik modeli Gamilton-Ostrogradskiy variatsion tamoyili va Maksvell tenglamalari ko‘rinishidagi elektrodinamika tenglamalari asosida olinadi.

$$\int_t (\delta K - \delta \Pi + \delta A)dt = 0; \tag{1}$$

bu yerda: $\delta -$ variatsiya amali; $K -$kinetik energiya; $\Pi -$ potensial energiya; $A -$tashqi hajm va sirt kuchlari bajargan ish.

Plastinaning harakat tenglamalarini keltirib chiqarishda ko‘chishning o‘zgarish qonunlari sifatida quyidagi Kirxgof-Lyav gipotezasidan foydalaniladi. Bunda yupqa plastinaning Oz koordinata o‘qi bo‘ylab deformatsiyalanishi yo‘q deb faraz qilinadi va plastina o‘rta tekisligining ko‘chish proyeksiyalari quyidagicha ifodalanadi:

$$u_1 = u(x, y, t) - z\frac{\partial w}{\partial x},\ u_2 = v(x, y, t) - z\frac{\partial w}{\partial y},\ u_3 = w(x, y, t) \tag{2}$$

bu yerda: $u, v, w -$ko‘chishlar.

Gamilton-Ostrogradskiy variatsion tamoyili asosida yupqa murakkab konstruksiyaviy shakldagi magnitelastik plastinaning geometrik nochiziqli deformatsiyalanish holatiga Koshi munosabatlari, Guk qonuni va Maksvell elektromagnit tenzor ko‘rinishidan foydalanib, elektromagnit maydon ta’sirlari ko‘riladi. Natijada ko‘chishga nibatan boshlang‘ich va chegaraviy shartlarga ega xususiy hosilali differensial tenglamalar tizimi ko‘rinishidagi matematik model olinadi.

3 Variatsion kinetik energiyani aniqlash

Gamilton-Ostrogradskiy variatsion tamoyili asosida elektromagnit maydonda joylashgan yupqa murakkab konstruksiyaviy shakldagi anizotrop plastinalarning geometrik nochiziq deformatsiyalanish holatini matematik modelini ishlab chiqish uchun birinchi bosqichda variatsion kinetik energiyani hisoblashimiz kerak bo‘ladi.

Kinetik energiyaning o‘zgarishini hisoblashda quyidagi munosabatdan foydalanamiz .

$$\int_t \delta K dt = \frac{1}{2}\int_t\int_V \delta\rho\left[\left(\frac{\partial u_1}{\partial t}\right)^2+\left(\frac{\partial u_2}{\partial t}\right)^2+\left(\frac{\partial u_3}{\partial t}\right)^2\right]dVdt. \quad (3)$$

Endi kinetik energiyaning variyatsiyasini hisoblaymiz. Buning uchun kinetik energiyaning umumiy formulasiga variatsiya amalini ta’sir qildirib quyidagiga ega bo‘lamiz:

$$\int_t \delta K dt = \int_t\int_V\left(\rho\frac{\partial u_1}{\partial t}\delta\frac{\partial u_1}{\partial t}+\rho\frac{\partial u_2}{\partial t}\delta\frac{\partial u_2}{\partial t}+\rho\frac{\partial u_3}{\partial t}\delta\frac{\partial u_3}{\partial t}\right)dVdt. \quad (4)$$

bunda: ρ – qaralayotgan materialining zichligi; $u_1, u_2, u_3 -$ ko‘chish, $V-$hajm, $t-$vaqt.

Variatsiya ostidagi kinetik energiyaning (4) umumiy ko‘rinishi $t-$ vaqt bo‘yicha integrallanadi va (5) ga ega bo‘lamiz:

$$\int_t \delta K = \int_V\left(\rho\frac{\partial u_1}{\partial t}\delta u_1+\rho\frac{\partial u_2}{\partial t}\delta u_2+\rho\frac{\partial u_3}{\partial t}\delta u_3\right)dV\Bigg|_t -$$
$$-\int_t\int_V\left[\frac{\partial}{\partial t}\left(\rho\frac{\partial u_1}{\partial t}\right)\delta u_1+\frac{\partial}{\partial t}\left(\rho\frac{\partial u_2}{\partial t}\right)\delta u_2+\frac{\partial}{\partial t}\left(\rho\frac{\partial u_3}{\partial t}\right)\delta u_3\right]dVdt. \quad (5)$$

Bu yerdagi $u_1,\, u_2,\, u_3$ larning o‘rniga (2) formuladagi qiymatlarini keltirib qo‘yib variatsion kinetik energiyaning quyidagi ko‘rinishiga ega bo‘lamiz:

$$\int_t \delta K = \int_V\left\{\rho\left[\left(\frac{\partial u}{\partial t}-z\frac{\partial^2 w}{\partial t\partial x}\right)\delta u-\left(z\frac{\partial u}{\partial t}-z^2\frac{\partial^2 w}{\partial t\partial x}\right)\delta\frac{\partial w}{\partial x}+\left(\frac{\partial v}{\partial t}-z\frac{\partial^2 w}{\partial t\partial y}\right)\delta v-\right.\right.$$
$$\left.\left.-\left(z\frac{\partial v}{\partial t}-z^2\frac{\partial^2 w}{\partial t\partial y}\right)\delta\frac{\partial w}{\partial y}+\frac{\partial w}{\partial t}\delta w\right]\right\}dV\Bigg|_t-\int_t\int_V\left\{\rho\left[\left(\frac{\partial^2 u}{\partial t^2}-z\frac{\partial^3 w}{\partial t^2\partial x}\right)\delta u-\right.\right. \quad (6)$$
$$-\left(z\frac{\partial^2 u}{\partial t^2}-z^2\frac{\partial^3 w}{\partial t^2\partial x}\right)\delta\frac{\partial w}{\partial x}+\left(\frac{\partial^2 v}{\partial t^2}-z\frac{\partial^3 w}{\partial t^2\partial y}\right)\delta v-\left(z\frac{\partial^2 v}{\partial t^2}-z^2\frac{\partial^3 w}{\partial t^2\partial y}\right)\delta\frac{\partial w}{\partial y}+$$
$$\left.\left.+\frac{\partial^2 w}{\partial t^2}\delta w\right]\right\}dVdt.$$

Natijada variatsion kinetik energiya hosil bo‘ldi:

$$\int_t \delta K = \int_x\int_y\left\{\rho h\frac{\partial u}{\partial t}\delta u+\rho h\frac{\partial v}{\partial t}\delta v+\rho h\frac{\partial w}{\partial t}\delta w\right\}dydx\Bigg|_t+\int_y\rho\frac{h^3}{12}\frac{\partial^2 w}{\partial t\partial x}\delta w\Bigg|_x dy\Bigg|_t+ \quad (7)$$
$$+\int_x\rho\frac{h^3}{12}\frac{\partial^2 w}{\partial t\partial y}\delta w\Bigg|_y dx\Bigg|_t-\int_t\int_x\int_y\left\{\rho h\frac{\partial^2 u}{\partial t^2}\delta u+\rho h\frac{\partial^2 v}{\partial t^2}\delta v+\rho h\frac{\partial^2 w}{\partial t^2}\delta w\right\}dydxdt.$$

4 Variatsion potensial energiyani aniqlash

Yuqoridagi Gamilton-Ostrogradskiy variatsion tamoyiliga asosan, ikkinchi bosqichda variatsion potensial energiyaning umumiy ko‘rinishini keltirib chiqaramiz. Buning uchun quyidagi ketma-ketlikdagi amallarni bajaramiz.

Variatsiya ostidagi potensial energiyani aniqlashning umumiy ko‘rinishi.

$$\delta\Pi = \int_V \left[\sigma_{11}\delta\varepsilon_{11} + \sigma_{22}\delta\varepsilon_{22} + \sigma_{12}\delta\varepsilon_{12}\right] dV, \tag{8}$$

bu yerda: $\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{22}$ – deformatsiyalar, $\sigma_{11}, \sigma_{12}, \sigma_{22}$ – kuchlanishlar.

Koshi munosabatlaridan nochiziqli deformatsiyalanish quyidagi formula bilan aniqlanadi.

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right), (i = 1,2;\ j = 1,2; k = 1,2,3.). \tag{9}$$

Koshi munosabatiga ko‘ra nochiziqli deformatsiya elementlari tenzor ko‘rinishidan va (2) Kirxgof-Lyav gipotezasidan foydalanib quyidagilarga ega bo‘lamiz:

$$\begin{cases} \varepsilon_{11} = \dfrac{\partial u}{\partial x} - z\dfrac{\partial^2 w}{\partial x^2} + \dfrac{1}{2}\dfrac{\partial w}{\partial x}\cdot\dfrac{\partial w}{\partial x}; \\ \varepsilon_{22} = \dfrac{\partial v}{\partial y} - z\dfrac{\partial^2 w}{\partial y^2} + \dfrac{1}{2}\dfrac{\partial w}{\partial y}\cdot\dfrac{\partial w}{\partial y}; \\ \varepsilon_{12} = \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} - 2z\dfrac{\partial^2 w}{\partial x\partial y} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}\right). \end{cases} \tag{10}$$

Plastina qalinligi bo‘yicha birlashtiriladi ya’ni $z \in \left[-h/2; h/2\right]$ bo‘yicha integrallanadi.

$$\begin{aligned} \delta\Pi = \int_y\int_x \Bigg[& \int_z \sigma_{11}dz\delta\frac{\partial u}{\partial x} - \int_z z\sigma_{11}dz\delta\frac{\partial^2 w}{\partial x^2} + \int_z \sigma_{11}dz\frac{\partial w}{\partial x}\delta\frac{\partial w}{\partial x} + \int_z \sigma_{22}dz\delta\frac{\partial v}{\partial y} - \\ & -\int_z z\sigma_{22}dz\delta\frac{\partial^2 w}{\partial y^2} + \int_z \sigma_{22}dz\frac{\partial w}{\partial y}\delta\frac{\partial w}{\partial y} + \frac{1}{2}\int_z \sigma_{12}dz\delta\frac{\partial u}{\partial y} + \frac{1}{2}\int_z \sigma_{12}dz\delta\frac{\partial v}{\partial x} - \\ & -\int_z z\sigma_{12}dz\delta\frac{\partial^2 w}{\partial x\partial y} + \frac{1}{2}\int_z \sigma_{12}dz\frac{\partial w}{\partial x}\delta\frac{\partial w}{\partial y} + \frac{1}{2}\int_z \sigma_{12}dz\frac{\partial w}{\partial y}\delta\frac{\partial w}{\partial x}\Bigg]dxdy. \end{aligned} \tag{11}$$

Bu yerda quyidagicha belgilashlar kiritiladi:

$$\begin{aligned} & N_{11} = \int_{-h/2}^{h/2}\sigma_{11}dz,\ N_{22} = \int_{-h/2}^{h/2}\sigma_{22}dz,\ N_{12} = \int_{-h/2}^{h/2}\sigma_{12}dz; \\ & M_{11} = \int_{-h/2}^{h/2} z\sigma_{11}dz,\ M_{22} = \int_{-h/2}^{h/2} z\sigma_{22}dz,\ \ M_{12} = \int_{-h/2}^{h/2} z\sigma_{12}dz. \end{aligned} \tag{12}$$

(12) formuladagi $N_{11}, N_{22}, N_{12}, M_{11}, M_{22}, M_{12}$ larni formulaga keltirib qo‘yamiz.

$$\begin{aligned} \delta\Pi = \int_y\int_x \Bigg[& N_{11}\delta\frac{\partial u}{\partial x} - M_{11}\delta\frac{\partial^2 w}{\partial x^2} + N_{11}\frac{\partial w}{\partial x}\delta\frac{\partial w}{\partial x} + N_{22}\delta\frac{\partial v}{\partial y} - M_{22}\delta\frac{\partial^2 w}{\partial y^2} + \\ & + N_{22}\frac{\partial w}{\partial y}\delta\frac{\partial w}{\partial y} + \frac{1}{2}N_{12}\delta\frac{\partial u}{\partial y} + \frac{1}{2}N_{12}\delta\frac{\partial v}{\partial x} - M_{12}\delta\frac{\partial^2 w}{\partial x\partial y} + \frac{1}{2}N_{12}\frac{\partial w}{\partial x}\delta\frac{\partial w}{\partial y} + \\ & + \frac{1}{2}N_{12}\frac{\partial w}{\partial y}\delta\frac{\partial w}{\partial x}\Bigg]\Bigg\}dxdy. \end{aligned} \tag{13}$$

Variatsiya potensial energiyaning har bir hadini alohida bo‘laklab integrallaymiz. Keyin, hosil bo‘lgan variatsion potensial energiyaning hadlarini ixchamlaymiz.

$$\begin{aligned} \delta\Pi = \int_y \Bigg\{ & N_{11}\delta u - M_{11}\delta\frac{\partial w}{\partial x} + \frac{\partial M_{11}}{\partial x}\delta w + N_{11}\frac{\partial w}{\partial x}\delta w + \frac{1}{2}N_{12}\delta v - \\ & -\frac{1}{2}M_{12}\delta\frac{\partial w}{\partial y} + \frac{1}{2}\frac{\partial M_{12}}{\partial y}\delta w + \frac{1}{2}N_{12}\frac{\partial w}{\partial y}\delta w\Bigg\}dy\Bigg|_x + \int_x \big\{N_{22}\delta v - \\ & -M_{22}\delta\frac{\partial w}{\partial y} + \frac{\partial M_{22}}{\partial y}\delta w + N_{22}\frac{\partial w}{\partial y}\delta w + \frac{1}{2}N_{12}\delta u + \frac{1}{2}\frac{\partial M_{12}}{\partial x}\delta w - \end{aligned}$$

$$-\frac{1}{2}M_{12}\delta\frac{\partial w}{\partial x}+\frac{1}{2}N_{12}\frac{\partial w}{\partial x}\delta w\Big\}dx\Big|_y-\int_y\int_x\Big\{\frac{\partial N_{11}}{\partial x}\delta u+\frac{1}{2}\frac{\partial N_{12}}{\partial y}\delta u+$$
$$+\frac{\partial N_{22}}{\partial y}\delta v+\frac{1}{2}\frac{\partial N_{12}}{\partial x}\delta v+\frac{\partial}{\partial x}\left(N_{11}\frac{\partial w}{\partial x}+\frac{1}{2}N_{12}\frac{\partial w}{\partial y}\right)\delta w+$$
$$+\frac{\partial}{\partial y}\left(N_{22}\frac{\partial w}{\partial y}+\frac{1}{2}N_{12}\frac{\partial w}{\partial x}\right)\delta w+\frac{\partial^2 M_{11}}{\partial x^2}\delta w+\frac{\partial^2 M_{12}}{\partial x\partial y}\delta w+\frac{\partial^2 M_{22}}{\partial y^2}\delta w\Big\}dxdy.$$

Bunda: M_{11}, M_{22}, M_{12}- egilish va burilish momentlari; N_{11}, N_{22}, N_{12}- normal va urunma kuchlari.

5 Natijalar

Magnitelastik anizotrop yupqa plastinalarning harakat tenglamalari

Gamilton-Ostrogradskiy variatsion tamoyiliga asosan aniqlanilgan, kinetik va potensial energiyaning o‘zgarishi va elektromagnit maydon kuchlarni hisobga olgan holda tashqi kuchlar bajargan ishning variatsiyalarini keltirib o‘rniga qo‘yamiz. Natijada magnitelastik yupqa plastinalar uchun quyidagi boshlang‘ich va chegara shartlarga ega bo‘lgan harakat tenglamalari tizimi hosil bo‘ladi.

$$\begin{cases}-\rho h\frac{\partial^2 u}{\partial t^2}+\frac{\partial N_{11}}{\partial x}+\frac{1}{2}\frac{\partial N_{12}}{\partial y}+N_x+R_x+q_x+T_{zx}=0,\\ -\rho h\frac{\partial^2 v}{\partial t^2}+\frac{1}{2}\frac{\partial N_{12}}{\partial x}+\frac{\partial N_{22}}{\partial y}+N_y+R_y+q_y+T_{zy}=0,\\ -\rho h\frac{\partial^2 w}{\partial t^2}+\frac{\partial}{\partial x}\left(N_{11}\frac{\partial w}{\partial x}+\frac{1}{2}N_{12}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{1}{2}N_{12}\frac{\partial w}{\partial x}+N_{22}\frac{\partial w}{\partial y}\right)+\\ +\frac{\partial^2 M_{11}}{\partial x^2}+\frac{\partial^2 M_{12}}{\partial x\partial y}+\frac{\partial^2 M_{22}}{\partial y^2}+N_z+R_z+q_z+T_{zz}=0.\end{cases}\qquad(15)$$

Boshlag‘ich shart:

$$\rho h\frac{\partial u}{\partial t}\delta u\Big|_t=0,\ \rho h\frac{\partial v}{\partial t}\delta v\Big|_t=0,\ \rho h\frac{\partial w}{\partial t}\delta w\Big|_t=0,$$
$$\rho\frac{h^3}{12}\frac{\partial^2 w}{\partial t\partial x}\delta w\Big|_x\Big|_t=0,\ \rho\frac{h^3}{12}\frac{\partial^2 w}{\partial t\partial y}\delta w\Big|_y\Big|_t=0;\qquad(16)$$

Chegaraviy shart:

$$N_{11}\delta u\big|_x=0,\ \frac{1}{2}N_{12}\delta v\Big|_x=0,\ -M_{11}\delta\frac{\partial w}{\partial x}\Big|_x=0,\ -\frac{1}{2}M_{12}\delta\frac{\partial w}{\partial y}\Big|_x=0,$$
$$\left[N_{11}\frac{\partial w}{\partial x}+\frac{1}{2}N_{12}\frac{\partial w}{\partial y}-\frac{\partial M_{11}}{\partial x}-\frac{1}{2}\frac{\partial M_{12}}{\partial y}\right]\delta w\Big|_x=0,$$
$$\left[\left(N_{Px}+N_{Txx}\right)\delta u+\left(N_{Py}+N_{Txy}\right)\delta v+\left(N_{Pz}+N_{Txz}\right)\delta w\right]\Big|_x=0,\qquad(17)$$
$$N_{22}\delta v\big|_y=0,\ \frac{1}{2}N_{12}\delta u\big|_y=0,\ -M_{22}\delta\frac{\partial w}{\partial y}\Big|_y=0,\ -\frac{1}{2}M_{12}\delta\frac{\partial w}{\partial x}\Big|_y=0,$$
$$\left[N_{22}\frac{\partial w}{\partial y}+\frac{1}{2}N_{12}\frac{\partial w}{\partial x}-\frac{\partial M_{22}}{\partial y}-\frac{1}{2}\frac{\partial M_{12}}{\partial x}\right]\delta w\Big|_y=0,$$
$$\left[\left(N_{Fx}+N_{Tyx}\right)\delta u+\left(N_{Fy}+N_{Tyy}\right)\delta v+\left(N_{Fz}+N_{Tyz}\right)\delta w\right]\Big|_y=0.$$

bu yerda $u,v,w-$ plastinaning egilishlari; $h-$ plastinaning qalinligi; $M_{11}, M_{22}, M_{12}-$ egilish va burilish momentlari; $N_{11}, N_{22}, N_{12}-$ normal va urinma kuchlari, $R_x, R_y, R_z, N_x, N_y, N_z-$

elektromagnit maydon va hajmiy kuchlari; $T_{zx}, T_{zy}, T_{zz}, q_x, q_y, q_z -$ sirt kuchlarini tashkil etuvchilari; $N_{Px}, N_{Py}, N_{Pz}, N_{Fx}, N_{Fy}, N_{Fz}, N_{Txx}, N_{Txy}, N_{Txz}, N_{Tyx}, N_{Tyy}, N_{Tyz} -$ kontur kuchlarini tashkil etuvchilari.

Shunday qilib, Kirxgof-Lyav gipotezasi, Koshi munosabatlari, Guk qonuni hamda Maksvell elektromagnit tenzor ko‘rinishlaridan foydalanib, Gamilton-Ostrogradskiy variatsion tamoyili asosida magnitelastik yupqa plastinaning harakat tenglamasi, boshlang‘ich va chegaraviy shartlari keltirib chiqarildi.

Anizotrop yupqa elektr o‘tkazuvchan jismlarga ta’sir etuvchi elektromagnit maydon kuchlari

Elastik o‘tkazuvchi muhitning statsionar elektromagnit maydon bilan o‘zaro ta’siri shu muhitda hajmiy Lorens kuchlarining paydo bo‘lishidan yuzaga keladi va uning taqsimot zichligi quyidagi formula bilan aniqlanadi.

$$f = j \times B. \tag{18}$$

Dekart kordinatalar tizimda quyidagicha ifodalanadi:

$$f_i = \frac{\partial T_{ij}}{\partial x_i}, \tag{19}$$

bunda T_{ij} – Maksvell magnit tenzor kuchlanishi.

Erkin elektr zaryadlari yo‘qligini va siljish tokini hisobga olmagan holda faraz qilib, Maksvell tenglamalarining quyidagi shakliga ega bo‘lamiz.

$$\begin{aligned} &rot\, H = j, \quad rot\, E = -\frac{\partial B}{\partial t}, \\ &div\, H = 0, \quad div\, E = 0, \\ &j = \sigma\left(E + \frac{\partial u}{\partial t} \times B\right). \end{aligned} \tag{20}$$

Ideal o‘tkazuvchi muhit ($\sigma \to \infty$, o‘tkazuvchanlik) deb hisoblasak, tenglamalar tizimining yechimi juda soddalashadi.

$$j = \frac{c}{4\pi} rot\, h, \quad h = rot\left(u \times H\right). \tag{21}$$

Bunday holatda umumiy hajmiy kuchlarga qo‘shilgan elektromagnit maydon ta’siri natijasida kelib chiqadigan hajmiy kuchlar quyidagicha ifodalanadi.

$$R = \rho \mathrm{K} = \frac{1}{4\pi}\ (\mathrm{rot}(\ \mathrm{rot}\ (\mathrm{U} \times \mathrm{H}))) \times \mathrm{H}, \tag{22}$$

Bunda: ***U****(u1,u2,u3)* – ko‘chish vektori; ***H****(Hx,Hy,Hz)* – magnit maydon kuchlanish vektori.

$$\begin{aligned} \mathrm{R_x} &= \int_z \rho \mathrm{K_x dz} = \frac{\mathrm{h}}{4\pi}\Bigg[(H_y^2 + H_z^2)\frac{\partial^2 u}{\partial x^2} + H_y^2 \frac{\partial^2 u}{\partial y^2} - H_x H_y \frac{\partial^2 v}{\partial x^2} + \\ &+ H_z^2 \frac{\partial^2 v}{\partial x \partial y} - H_x H_y \frac{\partial^2 v}{\partial y^2} - H_x H_z \frac{\partial^2 w}{\partial x^2} - H_y H_z \frac{\partial^2 w}{\partial x \partial y} + H_x H_z \frac{\partial^2 w}{\partial y^2}\Bigg], \\ \mathrm{R_y} &= \int_z \rho \mathrm{K_y dz} = \frac{\mathrm{h}}{4\pi}\Bigg[-H_x H_y \frac{\partial^2 u}{\partial x^2} + H_x^2 \frac{\partial^2 u}{\partial x \partial y} - H_x H_y \frac{\partial^2 u}{\partial y^2} + \\ &+ H_x^2 \frac{\partial^2 v}{\partial x^2} + (H_x^2 + H_z^2)\frac{\partial^2 v}{\partial y^2} + H_y H_z \frac{\partial^2 w}{\partial x^2} - 2H_x H_z \frac{\partial^2 w}{\partial x \partial y} - H_y H_z \frac{\partial^2 w}{\partial y^2}\Bigg], \\ \mathrm{R_z} &= \int_z \rho \mathrm{K_z dz} = \frac{\mathrm{h}}{4\pi}\Bigg[-H_x H_z \frac{\partial^2 u}{\partial x^2} - H_y H_z \frac{\partial^2 u}{\partial x \partial y} - H_x H_z \frac{\partial^2 v}{\partial x \partial y} - \\ &- H_y H_z \frac{\partial^2 v}{\partial y^2} + (H_y^2 - H_x^2)\frac{\partial^2 w}{\partial x^2} + 4H_x H_y \frac{\partial^2 w}{\partial x \partial y} - (H_y^2 - H_x^2)\frac{\partial^2 w}{\partial y^2}\Bigg]. \end{aligned} \tag{23}$$

Bu yerda $h -$ plastinaning qalinligi.

Maksvellning elektrodinamik kuchlanish tenzorlari umumiy sirt va kontur (chegara) kuchlariga qo‘shiladi.

$$\begin{cases} T_{ik} = \dfrac{1}{4\pi}\left[H_i h_k + h_i H_k\right] - \dfrac{б_{ik}}{4\pi}\vec{h}\vec{H}, \\ T_{ik}^e = \dfrac{1}{4\mu\pi}\left[H_i^e h_k^e + h_i^e H_k^e\right] - \dfrac{б_{ik}}{4\pi}\vec{h}^e\vec{H}^e. \end{cases} \quad (24)$$

Bunda

$$б_{ik} = \begin{cases} 0, i \neq k, \\ 1, i = k. \end{cases}$$

Sirt va kontur (chegaraviy) kuchlarni mos ravishda qo‘yidagicha ifodalanadi:

Plastina yuzasida:

$$\begin{aligned} T_{zx} &= T_{zx}{}^{+} + T_{zx}{}^{-} = T_{31}{}^{+} + T_{31}{}^{e+} + T_{31}{}^{-} + T_{31}{}^{e-}, \\ T_{zy} &= T_{zy}{}^{+} + T_{zy}{}^{-} = T_{32}{}^{+} + T_{32}{}^{e+} + T_{32}{}^{-} + T_{32}{}^{e-}, \\ T_{zz} &= T_{zz}{}^{+} + T_{zz}{}^{-} = T_{33}{}^{+} + T_{33}{}^{e+} + T_{33}{}^{-} + T_{33}{}^{e-}. \end{aligned} \quad (25)$$

Plastina konturlarida:

a) x o‘qi uchun normal hisoblanadi.

$$\begin{aligned} T_{xx} &= T_{11}{}^{+} + T^{e}{}_{11}, \\ T_{xy} &= T_{12}{}^{+} + T^{e}{}_{12}, \\ T_{xz} &= T_{13}{}^{+} + T^{e}{}_{13} \end{aligned} \quad (26)$$

b) y o‘qi uchun normal hisoblanadi.

$$\begin{aligned} T_{yx} &= T_{21}{}^{+} + T^{e}{}_{21}, \\ T_{yy} &= T_{22}{}^{+} + T^{e}{}_{22}, \\ T_{yz} &= T_{23}{}^{+} + T^{e}{}_{23} \end{aligned} \quad (27)$$

Kuchlanish tenzori komponentlarini to‘liq ko‘rinishdagi ifodalarga almashtirishdan, elektromagnit sirt va kontur kuchlari uchun quyidagilarga ega bo‘lamiz:

$$\begin{aligned} T_{zx} &= \frac{1}{4\pi}\left[H_1 h_3 + h_1 H_3\right]^{+} + \frac{1}{4\pi}\left[H_1 h_3 + h_1 H_3\right]^{-} + \frac{1}{4\mu\pi}\left[H_1^e h_3^e + h_1^e H_3^e\right]^{+} + \\ &+ \frac{1}{4\mu\pi}\left[H_1^e h_3^e + h_1^e H_3^e\right]^{-}, \\ T_{zy} &= \frac{1}{4\pi}\left[H_2 h_3 + h_2 H_3\right]^{+} + \frac{1}{4\pi}\left[H_2 h_3 + h_2 H_3\right]^{-} + \frac{1}{4\mu\pi}\left[H_2^e h_3^e + h_2^e H_3^e\right]^{+} + \\ &+ \frac{1}{4\mu\pi}\left[H_2^e h_3^e + h_2^e H_3^e\right]^{-}, \\ T_{zz} &= \frac{1}{4\pi}\left[h_3 H_3 - (H_1 h_1 + h_2 H_2)\right]^{+} + \frac{1}{4\pi}\left[h_3 H_3 - (H_1 h_1 + h_2 H_2)\right]^{-} + \\ &+ \frac{1}{4\mu\pi}\left[h_3^e H_3^e - H_1^e h_1^e - H_2^e h_2^e\right]^{+} + \frac{1}{4\mu\pi}\left[h_3^e H_3^e - H_1^e h_1^e - H_2^e h_2^e\right]^{-}. \end{aligned} \quad (28)$$

$$\begin{aligned} T_{xx} &= \frac{1}{4\pi}\left[h_1 H_1 - (H_2 h_2 + H_3 h_3)\right] + \frac{1}{4\mu\pi}\left[H_1^e h_1^e - H_2^e h_2^e - H_3^e h_3^e\right], \\ T_{xy} &= \frac{1}{4\pi}\left[H_1 h_2 + h_1 H_2\right] + \frac{1}{4\mu\pi}\left[H_1^e h_2^e + H_1^e h_2^e\right], \\ T_{xz} &= \frac{1}{4\pi}\left[H_1 h_3 + h_1 H_3\right] + \frac{1}{4\mu\pi}\left[H_1^e h_3^e + H_1^e h_3^e\right]. \end{aligned} \quad (29)$$

$$T_{yx} = \frac{1}{4\pi}\left[H_1 h_2 + h_1 H_2\right] + \frac{1}{4\mu\pi}\left[H_1^e h_2^e + H_1^e h_2^e\right],$$
$$T_{yy} = \frac{1}{4\pi}\left[h_2 H_2 - (H_1 h_1 + H_3 h_3)\right] + \frac{1}{4\mu\pi}\left[H_2^e h_2^e - H_1^e h_1^e - H_3^e h_3^e\right], \quad (30)$$
$$T_{yz} = \frac{1}{4\pi}\left[H_2 h_3 + h_2 H_3\right] + \frac{1}{4\mu\pi}\left[H_2^e h_3^e + H_3^e h_2^e\right].$$

Egilish va burilish momentlari hamda normal va urinma kuchlarini aniqlash

Endi yupqa anizotrop plastinalarning magnitelastiklik masalasini geometrik nochiziqli matematik modelidagi M_{11}, M_{22}, M_{12} - egilish va burilish momentlarini va N_{11}, N_{22}, N_{12} - normal va urinma kuchlarini aniqlaymiz.

Biz tadqiq qilayotgan plastina anizotrop materialligini hisobga olgan holda, Guk qonuni quyidagicha ifodalanadi.

$$\sigma_{11} = B_{11}\varepsilon_{11} + B_{12}\varepsilon_{22} + B_{16}\varepsilon_{12},$$
$$\sigma_{22} = B_{12}\varepsilon_{11} + B_{22}\varepsilon_{22} + B_{26}\varepsilon_{12}, \quad (31)$$
$$\sigma_{12} = B_{16}\varepsilon_{11} + B_{26}\varepsilon_{22} + B_{66}\varepsilon_{12}.$$

Bunda $\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{22} -$ deformatsiya tenzori komponentalari; $\sigma_{11}, \sigma_{12}, \sigma_{22} -$ kuchlanish tenzori komponentalari; $B_{ij}\left(i, j = 1,2,6\right) -$ doimiylar.

Bu yerda $B_{ij}\left(i, j = 1,2,6\right) -$ doimiylari $a_{ij}\left(i, j = 1,2,6\right)$ bilan quyidagicha ifodalanadi.

$$B_{11} = \frac{a_{22}a_{66} - a_{26}^2}{\Delta}, \quad B_{12} = \frac{a_{16}a_{26} - a_{12}a_{66}}{\Delta}, \quad B_{22} = \frac{a_{11}a_{66} - a_{16}^2}{\Delta},$$
$$B_{16} = \frac{a_{12}a_{26} - a_{22}a_{16}}{\Delta}, \quad B_{26} = \frac{a_{12}a_{16} - a_{11}a_{26}}{\Delta}, \quad B_{66} = \frac{a_{11}a_{22} - a_{12}^2}{\Delta},$$
$$\Delta = \left(a_{11}a_{22} - a_{12}^2\right)a_{66} + 2a_{12}a_{16}a_{26} - a_{11}a_{26}^2 - a_{22}a_{16}^2.$$

Bunda tadqiq qilayotgan plastina anizotrop material bo‘lib, bu o‘z navbatida ortotrop yoki transversal-izotrop material bo‘lishi mumkin.

Agar *ortotrop* material bo‘lsa, unda $a_{ij}\left(i, j = 1,2,6\right) -$ plastina materialining elastik koeffitsiyentlari quyidagicha bo‘ladi.

$$a_{11} = \frac{1}{E_1}, \quad a_{22} = \frac{1}{E_2}, \quad a_{12} = -\frac{\nu_{12}}{E_2} = -\frac{\nu_{21}}{E_1} = -\frac{\nu_2}{E_2} = -\frac{\nu_1}{E_1},$$
$$a_{66} = \frac{1}{G_{12}}, \quad a_{16} = 0, \quad a_{26} = 0; \quad \nu_1 = \nu_{21}, \quad \nu_2 = \nu_{12}. \quad (32)$$

Demak, (32) dan kelib chiqqan holda, (38) formulalar bo‘yicha $B_{ij}\left(i, j = 1,2,6\right)$larni topsak:

$$B_{11} = \frac{E_1}{1 - \nu_1\nu_2}, \quad B_{22} = \frac{E_2}{1 - \nu_1\nu_2}, \quad B_{12} = \frac{\nu_2 E_1}{1 - \nu_1\nu_2} = \frac{\nu_1 E_2}{1 - \nu_1\nu_2}, \quad (33)$$
$$B_{66} = G_{12}, B_{16} = B_{26} = 0;$$

Agar material *transversal-izotrop* bo‘lsa, (32) dagi parametrlar uchun $E = E_1 = E_2$, $G = G_{12}$, $\nu = \nu_1 = \nu_{21} = \nu_2 = \nu_{12}$ tengliklar o‘rinli bo‘ladi va (31) formulalar bo‘yicha $B_{ij}\left(i, j = 1,2,6\right)$ lar quyidagicha ifodalanadi.

$$B_{11} = B_{22} = \frac{E}{1-\nu^2}, \quad B_{12} = \frac{\nu E}{1-\nu^2}, \quad B_{66} = G = \frac{E}{2(1-\nu)}, \quad B_{16} = B_{26} = 0; \qquad (34)$$

Bunda $E, E_1, E_2 -$ yung moduli; $\nu, \nu_1, \nu_2, \nu_{12}, \nu_{21} -$Puasson koeffisiyenti; $G, G_{12} -$ siljish moduli.

Kirxgof-Lyav gipotezasini hisobga olgan holda, quyidagicha aniqlanadi.

$$\begin{aligned}
\sigma_{11} &= B_{11}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + B_{12}\left(\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + B_{16}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right), \\
\sigma_{22} &= B_{12}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + B_{22}\left(\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + B_{26}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right), \qquad (35) \\
\sigma_{12} &= B_{16}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + B_{26}\left(\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + B_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right).
\end{aligned}$$

Yuqoridagilarga asoslanib, momentlar va kuchlar quyidagicha mos ravishda (34) va (35) munosabatlari bilan aniqlanadi.

$$\begin{aligned}
N_{11} &= h\left[B_{11}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + B_{12}\left(\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + B_{16}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right], \\
N_{22} &= h\left[B_{12}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + B_{22}\left(\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + B_{26}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right], \qquad (36) \\
N_{12} &= h\left[B_{16}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + B_{26}\left(\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + B_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right].
\end{aligned}$$

$$\begin{aligned}
M_{11} &= -\frac{h^3}{12}\left(B_{11}\frac{\partial^2 w}{\partial x^2} + 2B_{16}\frac{\partial^2 w}{\partial x \partial y} + B_{12}\frac{\partial^2 w}{\partial y^2}\right), \\
M_{22} &= -\frac{h^3}{12}\left(B_{12}\frac{\partial^2 w}{\partial x^2} + 2B_{26}\frac{\partial^2 w}{\partial x \partial y} + B_{22}\frac{\partial^2 w}{\partial y^2}\right), \qquad (37) \\
M_{12} &= -\frac{h^3}{12}\left(B_{16}\frac{\partial^2 w}{\partial x^2} + 2B_{66}\frac{\partial^2 w}{\partial x \partial y} + B_{26}\frac{\partial^2 w}{\partial y^2}\right).
\end{aligned}$$

6 Xulosa

Yuqorida elektromagnit maydonda joylashgan yupqa anizotrop plastinalar manitelastiklik masalasining matematik modeli ishlab chiqildi. Gamilton-Ostrogradskiy variatsion tamoyili asosida Kirxgof-Lyav gipotezasi, Koshi munosabatlari, Guk qonuni hamda Maksvell elektromagnit tenzor ko‘rinishlaridan foydalanib, potensial energiya, kinetik energiya va tashqi kuchlar bajargan ishning variatsion ko‘rinishlari aniqlanildi. Natijada ko‘chishga nibatan boshlang‘ich va chegaraviy shartlarga ega, xususiy hosilali nochiziqli differensial tenglamalar tizimi ko‘rinishidagi matematik modeli olindi.

Ishlab chiqilgan matematik modeldan elektromagnit maydonda joylashgan yupqa murakkab shaklli anizotrop plastinalarning magnitelastiklik masalalarini tadqiq qilishda foydalanish mumkin.

Adabiyotlar

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