ISSN 1066-369X, Russian Mathematics, 2023, Vol. 67, No. 8, pp. 27–35. © Allerton Press, Inc., 2023.
Russian Text © The Author(s), 2023, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2023, No. 8, pp. 45–55.

Study of Oscillatory Flows of a Viscoelastic Fluid in a Flat Channel Based on the Generalized Maxwell Model

K. Navruzov[a,*], A. Sh. Begjanov[a,], Sh. B. Sharipova[a,***], and J. Jumayev[b,****]**

[a] *Kuralbay Navruzov Urganch State University, Urganch, 220100 Republic of Uzbekistan*

[b] *Jura Jumayev Bukhara State University, Bukhara, 200018 Republic of Uzbekistan*

**e-mail: qurol_46@mail.ru*

***e-mail: Amirbek_beg@mail.ru*

****e-mail: shohistasharipova0@gmail.com*

*****e-mail: j.jumaev@buxdu.uz*

Received March 29, 2023; revised March 29, 2023; accepted May 29, 2023

Abstract—The problems of the oscillatory flow of a viscoelastic fluid in a flat channel for a given harmonic oscillation of the fluid flow rate are solved on the basis of the generalized Maxwell model. The transfer function of the amplitude-phase frequency characteristics is determined. These functions make it possible to evaluate the hydraulic resistance under a given law, the change in the longitudinal velocity averaged over the channel section, as well as during the flow of a viscoelastic fluid in a nonstationary flow, and allow determining the dissipation of mechanical energy in a nonstationary flow of the medium, which are important in the regulation of hydraulic and pneumatic systems. Its real part allows determining the active hydraulic resistance, and the imaginary part is reactive or inductance of the oscillatory flow.

Keywords: viscoelastic fluid, unsteady flow, transfer function, oscillatory flow, amplitude, phase

DOI: 10.3103/S1066369X23080066

INTRODUCTION

Of practical interest is the study of pulsating viscoelastic flows in a flat channel and a cylindrical pipe under the influence of harmonic oscillations of the pressure gradient or when harmonic oscillations of fluid flow are superimposed on the flow. In [1], the motion of a viscoelastic fluid along a long pipe under the action of an oscillatory pressure gradient was studied. The distinctive features of this motion are shown in comparison with the corresponding motion of a Newtonian fluid. The inertia-free oscillatory flow of a viscoelastic fluid in a round infinite pipe under the influence of an oscillatory pressure gradient was studied in [2], which showed that in an oscillating flow the longitudinal velocity profiles are symmetrical and there is a significant phase shift between the pressure gradient and velocity. In pulsating flows, there was in fact no phase shift, and the axial velocity varied asymmetrically relative to its average over the period of oscillation. Laminar oscillatory flows of Maxwell and Oldroyd-B viscoelastic fluids were studied in [3], which demonstrates many interesting features that are absent in the flows of Newtonian fluids. The results of study [3] show that in the inertialess regime, when Re 1, the properties of the flow depend on three characteristic lengths: the wavelength λ_0; the decay length of viscoelastic shear waves $x_0 = \left(\frac{2\nu}{\omega_0}\right)^{1/2}$, where ν is the kinematic viscosity and ω_0 is the oscillation frequency; and the characteristic transverse dimension of the system a. In this regard, the lengths are accordingly divided into three scales and three independent dimensionless groups: $\frac{t_\vartheta}{\lambda}$ (viscosity before relaxation time), De (relaxation time before the period of oscillation), and X (viscosity coefficient). At the same time, the oscillatory regions of the flow are divided into two systems corresponding to “wide” $\left(\frac{a}{x_0} > 1\right)$ and “narrow” $\left(\frac{a}{x_0} < 1\right)$ systems. In wide systems, oscillations are limited to near-wall flows, and in the central core, to inviscid flows. In narrow systems, shear waves cross the entire system and also its center, ultimately leading to constructive resonances that lead to a sharp increase in the amplitude of the velocity profile. In [4], unsteady flows of a viscoelastic fluid were analyzed using the Oldroyd-B model in a round infinite cylindrical pipe under the influence of a time-dependent pressure gradient in the following cases: (a) the pressure gradient changes with time in accordance with the exponential law; (b) the pressure gradient changes according to harmonic laws; and (c) the pressure gradient is constant. In all cases, the formulas were obtained for the distribution of velocity, fluid flow rate, and other hydrodynamic quantities in a pulsating flow. Based on the Maxwell model, the problem of nonstationary oscillatory flow of a viscoelastic fluid in a round cylindrical pipe was considered in [5]. The formulas for determining the dynamic and frequency characteristics were obtained. With the help of numerical experiments, the influence of the oscillation frequency and the relaxation properties of the liquid on the tangential shear stress on the wall was studied. It was shown that the viscoelastic properties of the fluid, as well as its acceleration, are the limiting factors for the use of the quasi-stationary approach. In recent decades, electrokinetic phenomena, including electroosmosis, flow potential, electrophoresis, and sedimentation potential, have attracted much attention and have many applications in micro- and nanochannels. In this regard, in [6], the electrokinetic flow of viscoelastic fluids in a flat channel under the influence of an oscillatory pressure gradient was studied. It is assumed that the motion of the liquid occurs laminarly and unidirectionally; in this regard, the motion of the liquid is in a linear mode. Surface potentials are considered small, so the Poisson—Boltzmann equation is linearized. Resonant behavior appears in the flow when the elastic properties of the Maxwellian fluid predominate. The resonance phenomenon enhances electrokinetic effects, and at the same time, the efficiency of electrokinetic energy conversion increases. In the works listed above, the field of fluid velocities was mainly investigated under various modes of change in the pressure gradient. Changes in the tangential and normal stresses that arise during motion have been studied relatively little. In most cases, in hydrodynamic models of nonstationary flows, fluids were replaced by a sequence of flows with a quasi-stationary distribution of hydrodynamic quantities. However, the structures of nonstationary flows differ from the structure of stationary flows, and in such cases such a replacement must be justified in each specific case. At present, the question of the legitimacy of studying the quasi-stationary characteristics for determining the field of shear stresses in nonstationary flows of viscous and viscoelastic fluids is far from being resolved. Naturally, under such conditions there is a need to use hydrodynamic models of nonstationary processes that take into account time-dependent changes in the hydrodynamic characteristics of the flow. It should be noted that in the general case, hydrodynamic characteristics in pipeline transport cannot be determined from characteristics that correspond to stationary flow conditions. In this paper, we study the oscillatory flow of a viscoelastic fluid using the generalized Maxwell model in a flat channel when harmonic oscillations of fluid flow are superimposed on the flow. The transfer function of the amplitude-phase frequency characteristics is determined. Using this function, the change in hydrodynamic resistance during an oscillatory flow of an elastic-viscous fluid is analyzed depending on the dimensionless oscillation frequency.

1. PROBLEM SETTING AND SOLUTION METHOD

Let us consider the problem of slow oscillatory flow of a viscoelastic incompressible fluid between two stationary parallel planes extending in both directions to infinity. Let us denote the distance between the walls by $2h$. The Ox axis runs horizontally in the middle of the channel along the flow. The Oy axis is directed perpendicular to the Ox axis. The flow of viscoelastic fluid occurs symmetrically along the axis of the channel. The differential equation of motion of a viscoelastic incompressible fluid in terms of stresses has the following form [7—10]:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\partial \tau}{\partial y}, \tag{1}$$

where u is the longitudinal velocity, p is the pressure, ρ is the density, τ is the tangential stress, and t is the time.

The rheological equations of state of a liquid are taken in form of the generalized Maxwell equation [3, 6]:

$$\tau = \tau_s + \tau_p, \quad \tau_s = -\eta_s \frac{\partial u}{\partial y}, \quad \lambda \frac{\partial \tau_p}{\partial y} + \tau_p = -\eta_p \frac{\partial u}{\partial y}. \tag{2}$$

Here, λ is the relaxation time, τ_s is the tangential stress of the Newtonian fluid, τ_p is the tangential stress of the Maxwellian fluid, τ is the tangential stress of the solution, η_s is the dynamic viscosity of a Newtonian fluid, and η_p is the dynamic viscosity of the Maxwellian fluid. The equality between the dynamic viscosities is satisfied [3, 6]:

$$\eta_0 = \eta_s + \eta_p,$$

where η_0 is the dynamic viscosity of the solution. Substituting (2) into the equation of motion (1) for the fluid velocity, we obtain

$$\rho\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} + \eta_s\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \eta_p\frac{\partial^2 u}{\partial y^2}. \tag{3}$$

We consider that the oscillatory flow of a viscoelastic fluid occurs due to a given harmonic oscillation of the fluid flow rate or the longitudinal velocity averaged over the channel cross-section

$$Q = a_Q\cos\omega t = \operatorname{Re} a_Q e^{i\omega t}, \quad \langle u\rangle = a_u\cos\omega t = \operatorname{Re} a_u e^{i\omega t},$$

where a_Q and a_u are, respectively, the amplitude of the fluid flow rate and the amplitude of the longitudinal velocity averaged over the channel cross-section. In this case, the flow occurs symmetrically along the channel axis, and the no-slip condition is satisfied on the channel wall, i.e., the longitudinal velocity on the channel wall is zero. Then the boundary conditions reads

$$u = 0 \text{ at } y = h, \quad \frac{\partial u}{\partial y} = 0 \text{ at } y = 0. \tag{4}$$

Due to the linearity of Eq. (3), the longitudinal velocity, pressure, and shear stress on the wall can be written as follows:

$$u(y,t) = \operatorname{Re} u_1(y)e^{i\omega t}, \quad p(x,t) = \operatorname{Re} p_1(x)e^{i\omega t}, \quad \tau(t) = \operatorname{Re}\tau_1 e^{i\omega t}. \tag{5}$$

Substituting (5) into Eq. (3), we obtain

$$\frac{\partial^2 u_1(y)}{\partial y^2} - \frac{\rho i\omega}{\eta_0}\left(X + \frac{Z}{1+i\omega\lambda}\right)^{-1} = \frac{1}{\eta_0}\left(\left(X+\frac{Z}{1+i\omega\lambda}\right)^{-1}\right)\frac{\partial p_1(x)}{\partial x}. \tag{6}$$

Here,

$$X = \frac{\eta_s}{\eta_0}, \quad Z = \frac{\eta_p}{\eta_0}, \quad X + Z = \frac{\eta_s}{\eta_0} + \frac{\eta_p}{\eta_0} = 1.$$

The fundamental solutions to Eq. (6) without the right-hand side are the functions

$$\cos\left(\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)y\right), \quad \sin\left(\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)y\right).$$

Here,

$$\eta(i\omega) = \left(\frac{1+i\omega\lambda}{1+i\omega\lambda X}\right)^{1/2}, \quad \alpha_0 = \sqrt{\frac{\rho\omega}{\eta_0}}h,$$

and the solution to the inhomogeneous part has the form

$$\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right).$$

Thus, the general solution to Eq. (6) reads

$$u_1(y) = C_1\cos\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right) + C_2\sin\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right) + \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right). \tag{7}$$

To determine constant coefficients, we use the boundary conditions (4):

$$\frac{\partial u_1(y)}{\partial y} = C_1\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)\sin\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right) + C_2\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)\cos\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right). \tag{8}$$

At $y = 0$ relation (8) has the form

$$0 = C_2 \frac{i^{3/2}\alpha_0}{h}\eta(i\omega).$$

It is easy to find $C_2 = 0$ from here. Provided that $u_1 = 0$ at $y = h$, from (7) we determine

$$C_1 = -\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\frac{1}{\cos\left(i^{3/2}\alpha_0\eta(i\omega)\right)}.$$

As a result, to determine the speed, we have

$$u_1(y) = \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{\cos\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right)}{\cos\left(i^{3/2}\alpha_0\eta(i\omega)\right)}\right), \quad (9)$$

where $\alpha_0\sqrt{\frac{\omega}{\nu_0}}h$ is the Womersley vibration number (dimensionless vibration frequency) and ν_0 is the kinematic viscosity of the solution.

Using the equation

$$\tau_1(i\omega) = -\frac{\eta_0}{\eta^2(i\omega)}\frac{\partial u_1(y)}{\partial y}\bigg|_{y=h}$$

we find the shear stress on the wall

$$\tau_1(i\omega) = -h\left(-\frac{\partial P}{\partial x}\right)\frac{1}{i\alpha_0^2}\left(\frac{i^{\frac{3}{2}}\alpha_0\eta(i\omega)\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}{\cos\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}\right). \quad (10)$$

Having integrated both sides of formula (9) over the variable y ranging from $-h$ to h, we find the formulas for the fluid flow rate:

$$Q_1 = 2h\left[\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}{\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)\cos\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}\right)\right]. \quad (11)$$

Taking into account $Q_1 = 2h\langle u_1\rangle$ in formula (11), we find the longitudinal velocity averaged over the channel cross-section

$$\langle U_1(i\omega)\rangle = \frac{1}{pi\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}{\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)\cos\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}\right), \quad (12)$$

$$\rho i\omega = i\frac{\omega}{v}h^2\frac{\eta_0}{h^2} = i\alpha_0^2\frac{\eta_0}{h^2}.$$

Then formula (12), taking into account (10), takes the form

$$\langle U_1(i\omega)\rangle = -\frac{3}{3\eta_0}\tau_1\frac{3\alpha_0\eta(i\omega)\cos(i^{\frac{3}{2}}\alpha_0\eta(i\omega) - \sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}{\left(i^{\frac{3}{2}}\alpha_0\right)^2\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}. \quad (13)$$

Using (13), for the shear stress on the wall we determine the transfer function

$$W_{\eta_1,u_1}(i\omega) = \frac{h}{3\eta_0}\frac{\tau_1(i\omega)}{u_1(i\omega)}. \tag{14}$$

Taking into account (14), from Eq. (13) we obtain

$$W_{\tau_1,u_1}(i\omega) = \frac{h}{3\eta_0}\frac{\tau_1(i\omega)}{\langle u_1(i\omega)\rangle} = -\frac{\left(i^{\frac{3}{2}}\right)^2 \sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}{3\left(\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)\cos\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega) - \sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)\right)\right)} = \chi + \beta i. \tag{15}$$

The transfer function (15) is sometimes called the amplitude-phase frequency response. These functions make it possible to estimate the hydraulic resistance for a given law of change in the longitudinal velocity averaged over the channel cross-section. Its real part allows determining the active hydraulic resistance, and its imaginary part allows determining the reactivity or inductance of the oscillatory flow.

2. CALCULATION RESULTS AND ANALYSIS

The hydrodynamic resistance during oscillatory flow in Newtonian and also viscoelastic flows is determined by the ratio of the pressure gradient to the average velocity; sometimes, this ratio is called the "impedance" of the flow. The ratio of pressure gradient to average speed is found from formula (12):

$$Z = \frac{\left(-\frac{\partial p}{\partial x}\right)}{R_0\langle u_1(i\omega)\rangle} = \frac{1}{i\alpha_0^2}\left(1 - \frac{\sin\left(M_1 - i\overline{M}_1\right)}{\left(M_1 - i\overline{M}_1\right)\cos\left(M_1 - i\overline{M}_1\right)}\right)^{-1} = R_n^0 + iL_0. \tag{16}$$

Here, $R_0 = \frac{\eta}{h^2}$ is the hydrodynamic resistance of a Newtonian fluid at a stationary flow. Separating the real and imaginary parts of formula (16), we determine the total hydrodynamic resistance $\overline{R}$ and inductance $\overline{L}$:

$$R = \frac{\alpha_0^2\left(A_1^2 + B_1^2\right)}{\left(A_2^2 + B_2^2\right)}B_2, \quad \frac{\overline{L}}{\alpha_0^2} = \frac{\left(A_1^2 + B_1^2\right)}{A_2^2 + B_2^2}A_2,$$

where

$$A_1 = \overline{A}\overline{M}_1 + \overline{B}M_1, \quad B_1 = \overline{A}M_1 - \overline{B}\overline{M}_1,$$

$$A_2 = \left(A_1^2 + B_1^2\right) - A_1C - B_1D, \quad B_2 = \left(B_1C - A_1D\right),$$

$$C = \sin M_1 \cosh \overline{M}_1, \quad D = -\cos M_1 \sinh \overline{M}_1;$$

$$\overline{A} = \sin M_1 \sinh \overline{M}_1, \quad \overline{B} = \cos M_1 \cosh \overline{M}_1, \quad M_1 = \frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_1, \quad \overline{M}_1 = \frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_2,$$

$$\overline{\overline{G}}_1 = \overline{\overline{G}}_1 + \overline{\overline{G}}_2, \quad \overline{\overline{G}}_2 = \overline{G}_1 - \overline{G}_2, \quad \sqrt{\frac{1}{\eta^*(i\omega)}} = \sqrt{G_1 + G_2 i} = \overline{G}_1 + \overline{G}_2 i,$$

$$\overline{G}_1 = \sqrt{\sqrt{G_1^2 + G_2^2}}\cos\frac{\varphi + 2n\pi}{2}, \quad \overline{G}_2 = \sqrt{\sqrt{G_1^2 + G_2^2}}\sin\frac{\varphi + 2n\pi}{2}, \quad n = 0,1;$$

$$\phi = \arctan\frac{G_2}{G_1}, \quad \frac{1}{\eta^*(i\omega)} = \frac{1 + \mathrm{De}^2 X_1\alpha_0^4 + i\mathrm{De}\alpha_0^2(1 - X_1)}{1 + \mathrm{De}^2 X_1^2\alpha_0^4} = G_1 + G_2 i,$$

$$G_1 = \frac{1 + \mathrm{De}^2 X_1\alpha_0^4}{1 + \mathrm{De}^2 X_1^2\alpha_0^4}, \quad G_2 = \frac{\mathrm{De}\,\alpha_0^2(1 - X_1)}{1 + \mathrm{De}^2 X_1^2\alpha_0^4},$$

$$\eta^*(i\omega) = \left(\frac{\eta_s}{\eta} + \frac{\eta_p}{\eta}\frac{1}{1+i\mathrm{De}\alpha_0^2}\right) = \left(X_1 + Z_1\frac{1}{1+i\mathrm{De}\alpha_0^2}\right) = \frac{1+i\mathrm{De}X_1\alpha_0^2}{1+i\mathrm{De}\alpha_0^2}, \quad \frac{\eta_s}{\eta} + \frac{\eta_p}{\eta} = 1,$$

$$X_1 = \frac{\eta_s}{\eta}, \quad Z_1 = \frac{\eta_p}{\eta}, \quad X_1 + Z_1 = 1,$$

$$\mathrm{De} = \frac{\lambda\eta}{\rho h^2}, \quad \alpha_0^2 = \frac{\omega}{\nu}h^2.$$

The results of studying Eq. (16) for a Newtonian fluid are presented in many works [7, 9, 10]. Figure 1 shows the dependence of hydrodynamic resistance on the dimensionless oscillation frequency α_0, when the elastic number is De = 0.05 and at different concentrations of Newtonian fluid in the solution.

In the graph of Fig. 1 at $X = 1$ changes in the total hydrodynamic resistance of a Newtonian fluid in an oscillatory flow are presented, coinciding with the results of other researchers [7, 9]. From this graph it can be seen that, with an increase in the dimensionless oscillation frequency α_0, the total hydrodynamic resistance of a Newtonian fluid increases monotonically. Curves *2–5* shown in Fig. 1 characterize the change in hydrodynamic resistance in the oscillatory flow of an viscoelastic fluid with a low elastic Deborah number, with the addition of its Newtonian fluid. Indeed, curves *2–5* differ little from curve *1*. This means that in this case, instead of the hydrodynamic resistance of an elastic-viscous fluid, the hydrodynamic resistance of a Newtonian fluid can be taken. However, with an increase in the elastic Deborah number, a significant difference is observed between the hydrodynamic resistance of an viscoelastic fluid and a Newtonian fluid. This difference is shown in Figs. 2–4 with increasing values of the elastic Deborah number. Starting from the elastic Deborah number De = 0.5, a decrease in the hydrodynamic resistance is observed depending on the concentration of the Newtonian fluid (curves *3–5* in Fig. 3). When there is no Newtonian fluid in a solution, then the solution has viscoelastic properties. In such cases, in an oscillatory flow, the hydrodynamic resistance of a viscoelastic fluid changes in an oscillatory manner depending on the dimensionless oscillation frequency α_0, and it increases with increasing elastic Deborah number (curves *5* in Figs. 2–4). The content of Newtonian fluid in the solution smoothes out the oscillatory mode of change in hydrodynamic resistance (curves *3* and *4* in Figs. 2–4). In the general case, during oscillatory flow of a viscoelastic fluid, the hydrodynamic resistance decreases as much as possible in the

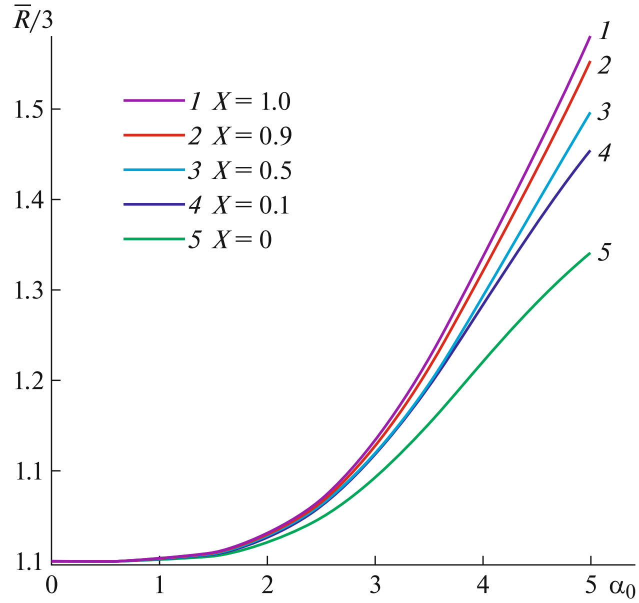

Fig. 1. Hydrodynamic resistance over dimensionless oscillation frequency α_0 at different concentrations of Newtonian fluid for De = 0.05.

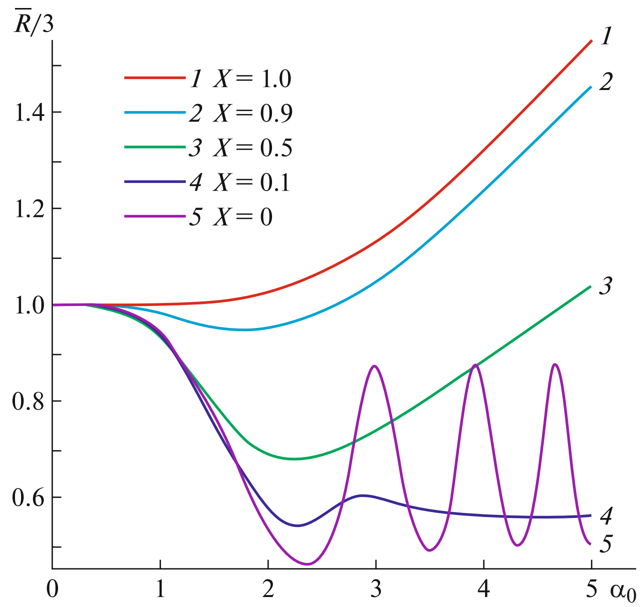

Fig. 2. Hydrodynamic resistance over dimensionless oscillation frequency α_0 at different concentrations of Newtonian fluid for De = 0.5.

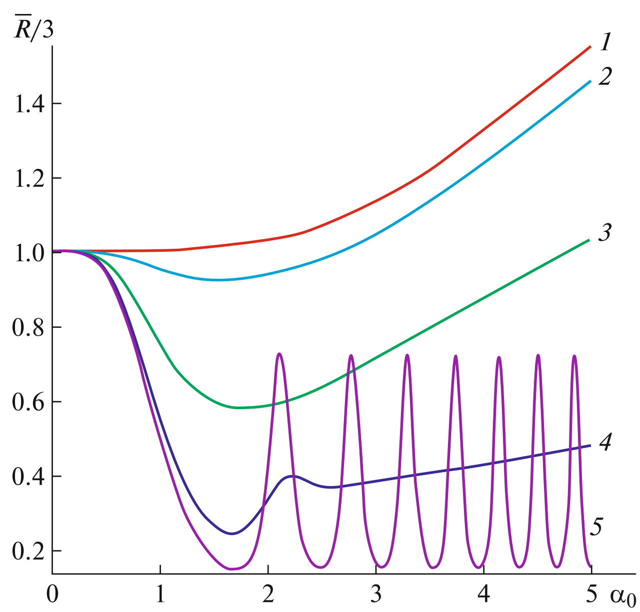

Fig. 3. Hydrodynamic resistance over dimensionless oscillation frequency α_0 at different concentrations of Newtonian fluid for De = 1.

range of values $1 < \alpha_0 < 3$ of the dimensionless vibration frequency and then increases as this frequency increases. The resulting effect makes it possible to estimate the hydrodynamic resistance for a given law of change in the longitudinal velocity averaged over the channel cross-section, as well as for a viscoelastic fluid in a nonstationary flow. Thus, it is possible to determine the dissipation of the mechanical energy of the medium, which is important in regulating hydraulic and pneumatic systems.

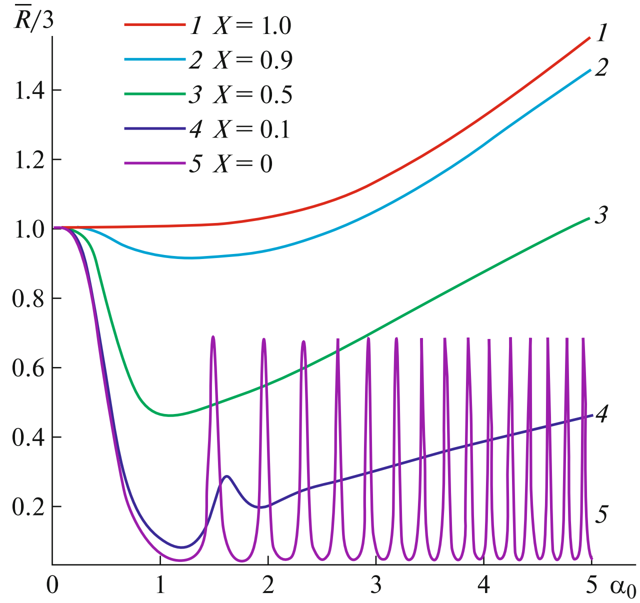

Fig. 4. Hydrodynamic resistance over dimensionless oscillation frequency α_0 at different concentrations of the Newtonian fluid for De = 2.

CONCLUSIONS

The problems of oscillatory flow of a viscoelastic fluid in a flat channel for a given harmonic oscillation of fluid flow are solved based on the generalized Maxwell model. The transfer function of the amplitude-phase frequency response is determined. Using this function, the dependence of the hydrodynamic resistance on the dimensionless oscillation frequency was studied for various values of the elastic Deborah number and the concentration of the Newtonian fluid. It is shown that in the oscillatory flow of a viscoelastic fluid, the hydrodynamic resistance decreases depending on the Deborah number. As this number increases, the decrease becomes even more pronounced. This effect makes it possible to estimate the hydrodynamic resistance for a given law of change in the longitudinal velocity averaged over the channel cross-section, as well as for the flow of a viscoelastic fluid in a nonstationary flow, which allows determining the dissipation of the mechanical energy of the medium, which is important in regulating hydraulic and pneumatic systems.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

REFERENCES

1. J. R. Jones and T. S. Walters, "Flow of elastico-viscous liquids in channels under the influence of a periodic pressure gradient, part 1," Rheol. Acta **6**, 240–245 (1967). https://doi.org/10.1007/bf01976440
2. E. M. Khabakhpasheva, V. I. Popov, A. N. Kekalov, and E. S. Mikhailova, "Pulsating flow of viscoelastic fluids in tubes," J. Non-Newtonian Fluid Mech. **33**, 289–304 (1989). https://doi.org/10.1016/0377-0257(89)80003-5
3. L. Casanellas and J. Ortín, "Laminar oscillatory flow of Maxwell and Oldroyd-B fluids: Theoretical analysis," J. Non-Newtonian Fluid Mech. **166**, 1315–1326 (2011). https://doi.org/10.1016/j.jnnfm.2011.08.010
4. A. Abu-El Hassan and E. M. El-Maghawry, "Unsteady axial viscoelastic pipe flows of an Oldroyd B fluid," in *Rheology-New Concepts, Applications and Methods* (IntechOpen, 2013). https://doi.org/10.5772/53638

5. Zh. A. Akilov, M. S. Dzhabbarov, and B. Kh. Khuzhayorov, "Tangential shear stress under the periodic flow of a viscoelastic fluid in a cylindrical tube," Fluid Dyn. **56**, 189–199 (2021). https://doi.org/10.1134/S0015462821020014
6. Z. Ding and Yo. Jian, "Electrokinetic oscillatory flow and energy conversion of viscoelastic fluids in microchannels: A linear analysis," J. Fluid Mech. **919**, 1–31 (2021). https://doi.org/10.1017/jfm.2021.380
7. D. N. Popov, *Nonstationary Hydromechanical Processes* (Mashinostroenie, Moscow, 1982).
8. G. Astarita and G. Marrucci, *Principles of Non-Newtonian Fluid Mechanics* (McGraw-Hill, London, 1974).
9. L. G. Loitsyanskii, *Mechanics of Liquids and Gases* (Drofa, Moscow, 2003; Pergamon, Oxford, 2014).
10. V. I. Kolesnichenko and A. N. Sharifulin, *Introduction to the Mechanics of Incompressible Liquid* (Izd-vo Perm. Nats. Issled. Politekh. Univ., Perm, 2019).