O‘ZBEKISTON RESPUBLIKASI OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

TERMIZ DAVLAT UNIVERSITETI

O‘ZBEKISTON RESPUBLIKASI FANLAR AKADEMIYASI

V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI

ALGEBRA VA ANALIZNING DOLZARB MASALALARI

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Termiz, 2025-yil, 20-21- oktabr

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МИНИСТЕРСТВО ВЫСШЕГО ОБРАЗОВАНИЯ, НАУКИ И ИННОВАЦИЙ РЕСПУБЛИКИ УЗБЕКИСТАН

ТЕРМЕЗСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ИНСТИТУТ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

АКАДЕМИИ НАУК РЕСПУБЛИКИ УЗБЕКИСТАН

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MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATIONS OF THE REPUBLIC OF UZBEKISTAN

TERMEZ STATE UNIVERSITY

INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

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2 – SHO‘BA. MATEMATIK TAHLIL MASALALARI VA ULARNING TADBIQLARI

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Convergence analysis of a modified proximal point algorithm for a nearly asymptotically quasi-nonexpansive mapping

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Let $\mathcal{C}$ be a nonempty closed convex subset of complete CAT(0) space and $\mathcal{T} : \mathcal{C} \to \mathcal{C}$ be a mapping. Let $x_1 \in \mathcal{T}$ be arbitrary and the sequence generated iteratively by

$$\begin{aligned} c_n &= \mathcal{T}^n((1-\rho_n)x_n \oplus \rho_n \mathcal{T}^n x_n), \\ b_n &= \mathcal{T}^n(\mathcal{T}^n c_n), \\ a_n &= \mathcal{T}^n(\mathcal{T}^n b_n), \\ x_{n+1} &= \mathcal{T}^n a_n, \quad n \geq 1. \end{aligned} \tag{1}$$

Let $(\mathcal{X}, d)$ be a metric space and $f : \mathcal{X} \to (\infty, \infty]$ be a proper and convex function. One of the major problems in optimization is to find $x \in \mathcal{X}$ such that

$$f(x) = \min_{y \in \mathcal{X}} f(y). \tag{2}$$

The set of minimizers of f is denoted by $\arg\min_{y \in \mathcal{X}} f(y)$.
We provide the following modified proximal point approach for nearly asympotically quasi-nonexpansive mappings in CAT(0) spaces utilising the iteration process defined by Asifa et al. :

$$\begin{aligned} &x_1 \in \mathcal{C} \\ v_n &= \arg\min_{c \in \mathcal{C}} \big(\mathcal{L}(c) + \frac{1}{2\lambda_n} d^2(c, x_n)\big), \\ u_n &= \arg\min_{b \in \mathcal{C}} \big(\mathcal{G}(b) + \frac{1}{2\varphi_n} d^2(b, v_n)\big), \\ c_n &= ((1-\rho_n)x_n \oplus \rho_n \mathcal{T}^n u_n), \\ b_n &= \mathcal{T}^n(\mathcal{T}^n c_n), \\ a_n &= \mathcal{T}^n(\mathcal{T}^n b_n), \\ x_{n+1} &= \mathcal{T}^n a_n, \quad n \geq 1. \end{aligned} \tag{3}$$

The proposed problem is illustrate convergence outcomes of the proposed process under several moderate conditions based on previous work.

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Modified proximal point algorithm for dealing with fixed point problems and convex minimization problems in CAT(0) Space

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Lemma. Assume that a complete CAT(0) space $\mathcal{Z}$ has a nonempty closed and convex subset $\mathcal{D}$. The single-valued nonexpansive mappings are denoted by $\mathfrak{T}_i : \mathcal{D} \to \mathcal{D}$; the multi-valued nonexpansive mappings are denoted by $\mathcal{S}_i : \mathcal{D} \to CB(\mathcal{D})$ for $i = 1:3$ and $\mathfrak{G}, h : \mathcal{D} \to (-\infty, \infty]$ are two proper convex and lower semi-continuous functions. Suppose that

$$\Omega = \mathfrak{U}(\mathfrak{T}_1) \cap \mathfrak{U}(\mathfrak{T}_2) \cap \mathfrak{U}(\mathfrak{T}_3) \cap \mathfrak{U}(\mathcal{S}_1) \cap \mathfrak{U}(\mathcal{S}_2) \cap \mathfrak{U}(\mathcal{S}_3) \cap \underset{y\in\mathcal{D}}{\arg\min}\, \mathfrak{G}(y) \cap \underset{\zeta\in\mathcal{D}}{\arg\min}\, h(\zeta) \neq \emptyset$$

and $\mathcal{S}_i q = \{q\}$, $i = 1:3$ for $q \in \Omega$. For $u_1 \in \mathcal{D}$, let the sequence $\{u_k\}$ is generated in the following manner:

$$\begin{cases} s_k = \underset{y\in\mathcal{D}}{\arg\min}[\mathfrak{G}(y) + \frac{1}{2\lambda_k} d^2(y, u_k)], \\ v_k = \underset{\zeta\in\mathcal{D}}{\arg\min}[h(\zeta) + \frac{1}{2\sigma_k} d^2(\zeta, s_k)], \\ \varphi_k = \varrho_k u_k \oplus \varsigma_k v_k' \oplus \gamma_k v_k'', \\ \Psi_k = \psi_k u_k \oplus \kappa_k v_k''' \oplus \phi_k \mathfrak{T}_1 u_k, \\ u_{k+1} = \delta_k u_k \oplus \eta_k \mathfrak{T}_2 u_k \oplus \xi_k \mathfrak{T}_3 \Psi_k, \quad for \quad all \quad k \in \mathbb{N}. \end{cases} \tag{1}$$

where $\{\varrho_k\}$, $\{\varsigma_k\}$, $\{\gamma_k\}$, $\{\psi_k\}$, $\{\kappa_k\}$, $\{\phi_k\}$, $\{\delta_k\}$, $\{\eta_k\}$ and $\{\xi_k\}$ are sequences in $(0,1)$ such that

$$0 < a \le \{\varrho_k\}, \{\varsigma_k\}, \{\gamma_k\}, \{\psi_k\}, \{\kappa_k\}, \{\phi_k\}, \{\delta_k\}, \{\eta_k\}, \{\xi_k\} \le b < 1,$$

$$\varrho_k + \varsigma_k + \gamma_k = 1, \psi_k + \kappa_k + \phi_k = 1, \delta_k + \eta_k + \xi_k = 1, \tag{2}$$

for all $k \in \mathbb{N}$ and $\{\lambda_k\}$ is a sequence such that $\lambda_k \ge \lambda > 0$ for all $k \in \mathbb{N}$ and some λ. Then, the subsequent claims are true:

(i) $\lim\limits_{k\to\infty} d(u_k, q)$ exists for all $q \in \Omega$;
(ii) $\lim\limits_{k\to\infty} d(u_k, s_k) = 0$; $\lim\limits_{k\to\infty} d(s_k, v_k) = 0$;
(iii) $\lim\limits_{k\to\infty} dist(u_k, \mathcal{S}_i u_k) = 0, i = 1, 2, 3$;
(iv) $\lim\limits_{k\to\infty} d(u_k, \mathfrak{T}_i u_k) = 0, i = 1, 2, 3$;
(v) $\lim\limits_{k\to\infty} d(u_k, J_\lambda u_k) = 0$, $\lim\limits_{k\to\infty} d(u_k, J_\sigma u_k) = 0$.

Theorem. *assume that a complete CAT(0) space $\mathcal{Z}$ has a nonempty closed and convex subset $\mathcal{D}$. The single-valued nonexpansive mappings are denoted by $\mathfrak{T}_i : \mathcal{D} \to \mathcal{D}$; the multi-valued nonexpansive mappings are denoted by $\mathcal{S}_i : \mathcal{D} \to CB(\mathcal{D})$ for $i = 1:3$ and $\mathfrak{G}, h : \mathcal{D} \to (-\infty, \infty]$ are two proper convex and lower semi-continuous functions. Consider that*

$$\Omega = \mathfrak{U}(\mathfrak{T}_1) \cap \mathfrak{U}(\mathfrak{T}_2) \cap \mathfrak{U}(\mathfrak{T}_3) \cap \mathfrak{U}(\mathcal{S}_1) \cap \mathfrak{U}(\mathcal{S}_2) \cap \mathfrak{U}(\mathcal{S}_3) \cap \underset{y\in\mathcal{D}}{\arg\min}\, \mathfrak{G}(y) \cap \underset{\zeta\in\mathcal{D}}{\arg\min}\, h(\zeta) \neq \emptyset$$