O‘ZBEKISTON RESPUBLIKASI OLIY TA’LIM, FAN VA INNOVATSIYALAR VAZIRLIGI

TERMIZ DAVLAT UNIVERSITETI

O‘ZBEKISTON RESPUBLIKASI FANLAR AKADEMIYASI

V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI

ALGEBRA VA ANALIZNING DOLZARB MASALALARI

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МИНИСТЕРСТВО ВЫСШЕГО ОБРАЗОВАНИЯ, НАУКИ И ИННОВАЦИЙ РЕСПУБЛИКИ УЗБЕКИСТАН

ТЕРМЕЗСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ИНСТИТУТ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

АКАДЕМИИ НАУК РЕСПУБЛИКИ УЗБЕКИСТАН

СБОРНИК МАТЕРИАЛОВ

Международной научно-практической конференция по

АКТУАЛЬНЫМ ПРОБЛЕМАМ АЛГЕБРЫ И АНАЛИЗА

Термез, 20–21 октября 2025 года

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MINISTRY OF HIGHER EDUCATION, SCIENCE AND INNOVATIONS OF THE REPUBLIC OF UZBEKISTAN

TERMEZ STATE UNIVERSITY

INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

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International Scientific-Practical Conference on

CURRENT PROBLEMS OF ALGEBRA AND ANALYSIS

Termez, October 20–21, 2025

2 – SHO‘BA. MATEMATIK TAHLIL MASALALARI VA ULARNING TADBIQLARI

СЕКЦИЯ 2. ЗАДАЧИ МАТЕМАТИЧЕСКОГО АНАЛИЗА И ИХ ПРИМЕНЕНИЯ

1.**A. Tassaddiq, W. Ahmed, S. Zaman, A. Raza, U. Islam and K. Nantomah** *A Modified Iterative Approach for Fixed Point Problem in Hadamard Spaces.* Journal of Function Spaces, 2024(2024) 10
2.**B. Martinet.** *Régularisation d'inéuations variationnelles par approximations successives.* Rev. Fr. Inform. Rech. Oper. 4 (1970)154-158,

UDC 519.853.3

Modified proximal point algorithm for dealing with fixed point problems and convex minimization problems in CAT(0) Space

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Lemma. Assume that a complete CAT(0) space $\mathcal{Z}$ has a nonempty closed and convex subset $\mathcal{D}$. The single-valued nonexpansive mappings are denoted by $\mathfrak{T}_i : \mathcal{D} \to \mathcal{D}$; the multi-valued nonexpansive mappings are denoted by $\mathcal{S}_i : \mathcal{D} \to CB(\mathcal{D})$ for $i = 1:3$ and $\mathfrak{G}, h : \mathcal{D} \to (-\infty, \infty]$ are two proper convex and lower semi-continuous functions. Suppose that

$$\Omega = \mathfrak{U}(\mathfrak{T}_1) \cap \mathfrak{U}(\mathfrak{T}_2) \cap \mathfrak{U}(\mathfrak{T}_3) \cap \mathfrak{U}(\mathcal{S}_1) \cap \mathfrak{U}(\mathcal{S}_2) \cap \mathfrak{U}(\mathcal{S}_3) \cap \arg\min_{y\in\mathcal{D}} \mathfrak{G}(y) \cap \arg\min_{\zeta\in\mathcal{D}} h(\zeta) \neq \emptyset$$

and $\mathcal{S}_i q = \{q\}$, $i = 1:3$ for $q \in \Omega$. For $u_1 \in \mathcal{D}$, let the sequence $\{u_k\}$ is generated in the following manner:

$$\begin{cases} s_k = \arg\min_{y\in\mathcal{D}}[\mathfrak{G}(y) + \frac{1}{2\lambda_k} d^2(y, u_k)], \\ v_k = \arg\min_{\zeta\in\mathcal{D}}[h(\zeta) + \frac{1}{2\sigma_k} d^2(\zeta, s_k)], \\ \varphi_k = \varrho_k u_k \oplus \varsigma_k v_k' \oplus \gamma_k v_k'', \\ \Psi_k = \psi_k u_k \oplus \kappa_k v_k''' \oplus \phi_k \mathfrak{T}_1 u_k, \\ u_{k+1} = \delta_k u_k \oplus \eta_k \mathfrak{T}_2 u_k \oplus \xi_k \mathfrak{T}_3 \Psi_k, \quad for \quad all \quad k \in \mathbb{N}. \end{cases} \tag{1}$$

where $\{\varrho_k\}$, $\{\varsigma_k\}$, $\{\gamma_k\}$, $\{\psi_k\}$, $\{\kappa_k\}$, $\{\phi_k\}$, $\{\delta_k\}$, $\{\eta_k\}$ and $\{\xi_k\}$ are sequences in $(0, 1)$ such that

$$0 < a \leq \{\varrho_k\}, \{\varsigma_k\}, \{\gamma_k\}, \{\psi_k\}, \{\kappa_k\}, \{\phi_k\}, \{\delta_k\}, \{\eta_k\}, \{\xi_k\} \leq b < 1,$$
$$\varrho_k + \varsigma_k + \gamma_k = 1, \psi_k + \kappa_k + \phi_k = 1, \delta_k + \eta_k + \xi_k = 1, \tag{2}$$

for all $k \in \mathbb{N}$ and $\{\lambda_k\}$ is a sequence such that $\lambda_k \geq \lambda > 0$ for all $k \in \mathbb{N}$ and some λ. Then, the subsequent claims are true:

(i) $\lim_{k\to\infty} d(u_k, q)$ exists for all $q \in \Omega$;
(ii) $\lim_{k\to\infty} d(u_k, s_k) = 0$; $\lim_{k\to\infty} d(s_k, v_k) = 0$;
(iii) $\lim_{k\to\infty} dist(u_k, \mathcal{S}_i u_k) = 0, i = 1, 2, 3$;
(iv) $\lim_{k\to\infty} d(u_k, \mathfrak{T}_i u_k) = 0, i = 1, 2, 3$;
(v) $\lim_{k\to\infty} d(u_k, J_\lambda u_k) = 0$, $\lim_{k\to\infty} d(u_k, J_\sigma u_k) = 0$.

Theorem. *assume that a complete CAT(0) space $\mathcal{Z}$ has a nonempty closed and convex subset $\mathcal{D}$. The single-valued nonexpansive mappings are denoted by $\mathfrak{T}_i : \mathcal{D} \to \mathcal{D}$; the multi-valued nonexpansive mappings are denoted by $\mathcal{S}_i : \mathcal{D} \to CB(\mathcal{D})$ for $i = 1:3$ and $\mathfrak{G}, h : \mathcal{D} \to (-\infty, \infty]$ are two proper convex and lower semi-continuous functions. Consider that*

$$\Omega = \mathfrak{U}(\mathfrak{T}_1) \cap \mathfrak{U}(\mathfrak{T}_2) \cap \mathfrak{U}(\mathfrak{T}_3) \cap \mathfrak{U}(\mathcal{S}_1) \cap \mathfrak{U}(\mathcal{S}_2) \cap \mathfrak{U}(\mathcal{S}_3) \cap \arg\min_{y\in\mathcal{D}} \mathfrak{G}(y) \cap \arg\min_{\zeta\in\mathcal{D}} h(\zeta) \neq \emptyset$$

and $\mathcal{S}_i q = \{q\}$, $i = 1:3$ for $q \in \Omega$. For $u_1 \in \mathcal{D}$, let the sequence $\{u_k\}$ is generated by (1), where $\{\varrho_k\}$, $\{\varsigma_k\}$, $\{\gamma_k\}$, $\{\psi_k\}$, $\{\kappa_k\}$, $\{\phi_k\}$, $\{\delta_k\}$, $\{\eta_k\}$ and $\{\xi_k\}$ are sequences in $(0,1)$ such that it satisfies (2) and $\{\lambda_k\}$ is a sequence such that $\lambda_k \geq \lambda > 0$ for all $n \in \mathbb{N}$ and some λ. In turn, the sequence $\{u_k\}$ Δ-converges to a point in Ω.

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Smooth boundary points of the matrix ball

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This work presents a proposition that plays an important role in proving the boundary Schwarz lemma for the matrix ball and characterizes properties related to smooth boundary points.

Let $\mathbb{C}^n[m \times m]$ be the set of all $Z = (Z_1, \dots Z_n)$ vectors, consist of $Z_q (q = 1, 2, ..., n) \in \mathbb{C}[m \times m]$ matrices of orders m. For any $Z, W \in \mathbb{C}^n[m \times m]$, the inner product defined as

$$\langle Z, W \rangle = Z_1 W_1^* + ... + Z_n W_n^*$$

where W^* is transpose complex conjugate of W and for any $\mathrm{A}, \mathrm{B} \in \mathbb{C}[m, m]$, the inner product is given by

$$(\mathrm{A}, \mathrm{B}) = \sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{m}} a_{ij} \bar{b}_{ij},$$

where $\mathrm{A} = (a_{ij})_{m \times m}$ and $\mathrm{B} = (b_{ij})_{m \times m}$. The matrix ball of type $\mathcal{I}$, denoted by $B_{m,n}$, is defined as

$$B_{m,n} = \{Z \in \mathbb{C}^n[m \times m] : I - \langle Z, Z \rangle > 0\} \tag{1}$$

where I is the unit square matrix of order m, and the inequality sign means that the left-hand side is positive definite[1].

Now, let us introduce the following notation

$$L_Z = \sum_{q=1}^{n} Z_q Z_q^*, \quad Z = (Z_1, \dots, Z_n) \in \mathbb{C}^n[m \times m]$$

It is easy to verify that L_Z is self-adjoint, and using this, we can rewrite (**??**) as follows

$$B_{m,n} = \{Z \in \mathbb{C}^n[m \times m] : I - L_Z > 0\}.$$