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ABSTRACTS

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Linear homogeneous inequalities and routes of trajectories of Lotka–Volterra operators

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The discrete version of the Lotka-Volterra operator on the simplex S^{m-1} is determined by specifying a real skew-symmetric matrix $A = (a_{ki})$ with the condition $|a_{ki}| \leq 1$ and acts according to the following law

$x'_k = x_k(1 + \sum_{i=1}^{m} a_{ki}x_i), k = \overline{1, m}$, where, $x' = (x'_1, ..., x'_m) = Vx$ and $V : S^{m-1} \to S^{m-1}$. It is known [1-2] that $V : S^{m-1} \to S^{m-1}$ for any $|a_{ki}| \leq 1$ is a homeomorphism, therefore, along with a positive trajectory $\{x^n\}, n \in \mathbb{N}$ where $x^{n+1} = Vx^n, x^0 \in S^{m-1,}$ can be consider also negative trajectories defined by the mapping $V^{-1} : S^{m-1} \to S^{m-1}$.

In this part of the work we will consider one of the most interesting representatives of the Lotka–Volterra map, preserving the four-dimensional simplex $V : S^4 \to S^4$, in case $\delta_1, \delta_2, \delta_3 > 0$. Let the skew-symmetric matrix have the form

$$A = \begin{pmatrix} 0 & 0 & a & -b & c \\ 0 & 0 & -d & e & -f \\ -a & d & 0 & 0 & 0 \\ b & -e & 0 & 0 & 0 \\ -c & f & 0 & 0 & 0 \end{pmatrix}, where 0 < a, b, c, d, e, f \leq 1. \quad (1)$$

Next, we introduce the following notation: vertices of the simplex S^4 by $e_1, ..., e_5$, $\Gamma_{12} = co\{e_1, e_2\}$ – edge, $\Gamma_{345}\{e_3, e_4, e_5\}$ – face of the simplex, as well as $\delta_1 = ce - bf$, $\delta_2 = cd - af$, $\delta_3 = bd - ae$, $\Delta = \delta_1 + \delta_2 + \delta_3$. Note that $Fix(V) = \Gamma_{12} \cup \Gamma_{345}$. Taking into account the positivity of the expressions $\delta_1 = ce - bf, \delta_2 = cd - af, \delta_3 = bd - ae$, we mark the points $M_1 = \left(\frac{d}{a+d}, \frac{a}{a+d}, 0, 0, 0\right), M_2 = \left(\frac{e}{b+e}, \frac{b}{b+e}, 0, 0, 0\right), M_3 = \left(\frac{f}{c+f}, \frac{c}{c+f}, 0, 0, 0), M_4 = (0, 0, \frac{b}{a+b}, \frac{a}{a+b}, 0\right)$,
$M_5 = (0, 0, \frac{e}{d+e}, \frac{d}{d+e}, 0), M_6 = (0, 0, 0, \frac{c}{b+c}, \frac{b}{b+c}), M_7 = (0, 0, 0, \frac{f}{e+f}, \frac{e}{e+f}), M_8 = \left(0, 0, \frac{\delta_1}{\Delta}, \frac{\delta_2}{\Delta}, \frac{\delta_3}{\Delta}\right)$, with the help of which we determine the increase or decrease of this coordinate under the action of the operator V. Consider $\delta_1, \delta_2, \delta_3 > 0$ we obtain the following picture of the partition of the faces Γ_{12} and Γ_{345}. In respect that $a\delta_1 - b\delta_2 + c\delta_3 = 0$ and $-d\delta_1 + e\delta_2 - f\delta_3 = 0$ we obtain solutions to the inequalities $P = \{x \in S^4 : Ax \geq 0\} = co\{M_6, M_7, M_8\}, Q = \{x \in S^4 : Ax \leq 0\} = co\{M_4, M_5, M_8\}$

Theorem. Under the condition $\delta_1, \delta_2, \delta_3 > 0$, any trajectory converges, and if $x^{(0)} \in riS^4$, then $\alpha(x^{(0)}) \in P, \omega(x^{(0)}) \in Q$.

Lemma. The intersection of any two polytopes is either empty or a common face.

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Nevanlinna-Ostrovsky class for $A(z)$–analytic functions

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Let $A(z)$ be an antianalytic function, i.e. $\frac{\partial A}{\partial z} = 0$ in the convex domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq c < 1$ for all $z \in D$, where $c =$const. The function $f(z)$ is said to be $A(z)$–analytic in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z} \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)$–analytic functions defined in the domain D.

According to, the function

$$\psi\left(a,z\right) = z - a + \overline{\int\limits_{\gamma(a,z)} \overline{A(\tau)}d\tau}$$

is an $A(z)$–analytic function.

The following set is an open subset of arbitrary convex domain D :

$$L\left(a,r\right) = \left\{ \left|\psi\left(a,z\right)\right| = \left| z - a + \overline{\int\limits_{\gamma(a,z)} \overline{A(\tau)}d\tau} \right| < r \right\}.$$

For sufficiently small $r > 0$, this set compactly lies in D (we denote this fact by $L(a,r) \subset\subset D$) and contains the point a. This set $L(a,r)$ is called the $A(z)$–lemniscate centered at the point a. The lemniscate $L(a,r)$ is a simply - connected set (see [2]).

Now we assume that the domain $D \subset \mathbb{C}$ is convex, and $\xi \in D$ is a fixed point in it. Consider the function

$$K(z,\xi) = \frac{1}{2\pi i}\frac{1}{z - \xi + \overline{\int\limits_{\gamma(\xi,z)} \overline{A(\tau)}d\tau}}, \tag{2}$$

where $\gamma(\xi,z)$ is a smooth curve which points of $\xi, z \in D$. Since the domain is simply connected and the function $\overline{A}(z)$ is holomorphic, the integral

$$I(z) = \overline{\int\limits_{\gamma(\xi,z)} \overline{A(\tau)}d\tau}$$

does not depend on a path of integration; it coincides with a primitive, i. e. $I'(z) = \overline{A}(z)$. (see [2]).

Initially, we introduce the Hardy class for $A(z)$–analytic functions. Let $L(a,r) \subset\subset D$ and $f(z) \in O_A(L(a,r))$.

Definition 1 (see [3]). *The Hardy class $H^p, p > 0$ for $A(z)$–analytic functions is the set of all functions $f(z) \in O_A(L(a,r))$ such that its averages*

$$\frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho} |f(z)|^p\left|dz + A(z)d\bar{z}\right|$$

are uniformly bounded for $\rho < r$, $\sup\limits_{\rho<r}\left\{ \frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho} |f(z)|^p|dz + A(z)d\bar{z}| \right\} < \infty.$

Now we introduce a class of functions close to the Hardy space:

Definition 2. *Let $f(z) \in O_A(L(a,r))$. This function belongs to the class Nevanlinna N_A if its mean*

$$\frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho} \ln^+|f(z)||dz + A(z)d\bar{z}| \tag{3}$$

is uniformly bounded, $\sup\limits_{\rho<r}\left\{ \frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho} \ln^+|f(z)||dz + A(z)d\bar{z}| \right\} < +\infty.$

Considering that the geometric mean $\frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho} \ln^+|f(z)||dz + A(z)d\bar{z}|$ is no greater than the arithmetic mean $\left(\frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho} \ln^+|f(z)||dz + A(z)d\bar{z}|\right)^{\frac{1}{p}}$ for any $p > 0$, we conclude that class H^p_A is contained in class N_A, $H^p_A \subset N_A$.

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On the continuation of the Hartogs series with harmonic coefficients

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The paper considers the question of continuation of sums of Hartogs series, allowing harmonic continuation along a fixed direction, assuming only the harmonicity of the coefficients. Many works of famous mathematicians on the theory of functions of a complex variable are devoted to the study of this issue.

Let us consider the following Hartogs formal series:

$$u\left(x,w\right)=\sum_{k=-\infty}^{+\infty}c_k(x)\rho^{|k|}e^{ik\varphi},w=\rho e^{ik\varphi},x\in D, \tag{1}$$

where D is a domain from $\mathbb{R}^n$. The main result is the following theorem.

Theorem. *Let series* (1) *satisfy the following conditions*

1. *all coefficients* $c_k(x)\in h(D)$ *are harmonic functions,*
2. *for each fixed* $x\in D$ *the inequality*

$$\overline{\lim_{k\to\infty}}\ \sqrt[|k|]{|c_k(x)|}\leq\frac{1}{R}\ \ ,\ R>0.$$

Then there exists a nowhere dense closed set $S\subset D$ *such that series* (1) *locally uniformly converges in the domain* $(D\backslash S)\times\{w:\ |w|<R\}$ *and the sum* of the series u(x,w) belongs to the class h((D\S)×{w: |w|<R}) harmonic functions.

The proof of this theorem essentially uses the following lemma.

Lemma [1]. *Consider the space* $\mathrm{R}^n(x)$ *embedded in* $\mathrm{C}^n(z)=\mathrm{R}^n(x)+i\cdot\mathrm{R}^n(y)$, *where* $z=(z_1,...,z_n)$, $z_j=x_j+i\cdot y_j, j=1,...,n$, *and let* D *be some bounded domain of* $\mathrm{R}^n(x)$. *Then there exists a domain* $\hat{D}\subset\mathrm{C}^n(z)$ *such that* $D\subset\hat{D}$ *and for any function* $u(x)\in h(D)$ *there exists a function* $\hat{u}(z)$ *holomorphic in the domain* $\hat{D}$ *such that* $\hat{u}|_D=u$. *In addition, for any number* $M>1$ *there is a subdomain* $\hat{D}_M\subset\hat{D}$, $D\subset\hat{D}_M$, *such that* $\|\hat{u}\|_{\hat{D}_M}\leq M\|u\|_D,\quad\forall u\in h(D)\bigcap L_\infty(D)$.

Comment. A similar problem for Hartogs series with holomorphic coefficients was considered in [2].

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