



# **ABSTRACTS**

**of the international conference**

**MATHEMATICAL ANALYSIS AND ITS  
APPLICATIONS IN MODERN  
MATHEMATICAL PHYSICS**

## **PART I**

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<b>Khalkhuzhaev A.M., Usmonov L.S.</b> On the number of the eigenvalues of the two-particle Schrödinger operator on a lattice .....	69
<b>Khalxujayev A.M., Khayitova K.G.</b> Analytic description of the essential spectrum of A operator matrix in fermionic fock space .....	71
<b>Ko'chimov A., Kilichev N.</b> Ikki o'lchamli Fridrixs modelidagi operatorlar uchun manfiy xos qiymatning mavjudligi .....	72
<b>Kuliev K., Kuchiboyeva D., Ismoilov M.</b> Diskret Hardi tipidagi tengsizliklar .....	73
<b>Kuliev K.D., Kulieva G., Eshimova M.K.</b> Reverse discrete Hardy type inequalities with variable limits of summation .....	75
<b>Lakaev S.N., Abdukhakimov S.Kh., Azizova M.A.</b> On the number and location of eigenvalues of the two particle Schrodinger operator on a lattice .....	77
<b>Latipov H.M.</b> Gershgorin's bounds for a 4x4 operator matrix in cut Fock space .....	78
<b>Mamatov T., Rashidov A.</b> Mixed fractional differential operators in Holder spaces .....	81
<b>Masharipov S., Eshniyazov A.</b> Invariant of nonlinear operators and their interpretation for quadratic stochastic operators .....	83
<b>Muminov M., Shadiev U.</b> On existence eigenvalues of the generalized Friedrichs model .....	84
<b>Muminov M.I., Jurakulova F.M.</b> Description of the essential spectrum of operator matrix in bosonic Fock space. One dimensional case .....	85
<b>Muminov Z., Ismoilov G.</b> Asymptotics of the eigenvalue of a non-local discrete Schrodinger operator on two-dimensional lattice .....	87
<b>Muminov Z., Kulzhanov U., Ismoilov G.</b> Three Dimensional One-Particle Shrödinger Operator with Point Interaction .....	88
<b>Mustafoyeva Z., Yarashova O'.</b> Ground states for p-SOS model on the Cayley tree .....	90
<b>Nodirov Sh., Raximov F.</b> On the number of fixed points of a fourth degree operator .....	92
<b>Qushaqov H., Yusupov I., Muhammadjonov A.</b> About one monotonic function related matrix .....	93
<b>Rahmatullaev M., Askarov J.</b> Periodic Ground States for the one modified SOS model .....	95
<b>Rahmatullaev M., Pulatov B.</b> On $p$ -adic quasi Gibbs measure for the Potts model on a Cayley tree of order two .....	97
<b>Rahmatullaev M., Tukhtabaev A., Mamadjonov R.</b> On $p$ -adic generalized Gibbs measure for the Ising model with external field on a Cayley tree .....	100
<b>Rahmatullaev M.M., Karshiboev O.Sh.</b> Description of the translation-invariant splitting Gibbs measures for the three-state SOS model on the binary tree .....	102
<b>Rasulov T., Sharipova M.</b> Usual, quadratic and cubic numerical ranges corresponding to a $3 \times 3$ operator matrices .....	104
<b>Rasulov T., Umirkulova G.</b> Analysis of the essential spectrum of a Hamiltonian related to a system of three particles on a1D lattice .....	107
<b>Rasulov T.H.</b> Dominance order of the diagonally dominant $n \times n$ operator matrices .....	109
<b>Ruzhansky M., Safarov A.R., Khasanov G.A.</b> Uniform estimates for oscillatory integrals with homogeneous polynomial phases of degree 4 .....	111
<b>Sadullayev A.</b> On Weierstrass preparation theorem .....	113
<b>Satliqov G'R.</b> Separat garmonik funksiyalar uchun o'rta qiymat xossalari .....	114
<b>Sattorov E.N., Rustamov S., Boboxonova G.</b> On the continuation of solution of the generalized Cauchy-riemann system with quaternion parameter .....	115
<b>Sayliyeva G.R.</b> Essential spectrum of a $3 \times 3$ operator matrix with non compact perturbation .....	116
<b>Shokhrukh Kh. Yu.</b> Bound states of Schrödinger-type operators on one and two dimensional lattices .....	119
<b>Shoyimardonov S.K.</b> Occurrence of the Neimark-Sacker bifurcation in the phytoplankton-zooplankton system .....	119
<b>Tosheva N.A.</b> Threshold analysis for the family of generalized Friedrichs models .....	121
<b>Xudayarov S.S.</b> On invariant sets of a quadratic non-stochastic operator .....	123
<b>Zagrebnov V. A.</b> Comments on Chernoff and Trotter-Kato product formulae .....	124
<b>Zhabborov N., Husenov B.</b> The Poisson representation for the Hardy class of functions .....	125

holds for all  $n \in \mathbb{N}$  and  $x \in \mathfrak{X}$ .

**Proposition 3.** Let  $\Phi : t \mapsto \Phi(t)$  be a function from  $\mathbb{R}_0^+$  to contractions on  $\mathfrak{X}$  such that  $\Phi(0) = I$ . Let  $\{U_C(t)\}_{t \geq 0}$  be a contraction  $C_0$ -semigroup, and let domain  $D \subset \text{dom}(C)$  be a core of related generator  $C$ .

If the function  $\Phi(t)$  has a strong right-derivative  $\Phi'(+0)$  at  $t = 0$  (that is,  $\Phi'(+0)x$  exists for any  $x \in \text{dom}(\Phi'(+0))$ ) and if

$$\Phi'(+0)x := \lim_{t \rightarrow +0} \frac{1}{t}(\Phi(t) - I)x = -Cx,$$

for all  $x \in D$ , then

$$\lim_{n \rightarrow \infty} [\Phi(t/n)]^n x = U_C(t)x, \tag{1}$$

for all  $t \in \mathbb{R}_0^+$  and  $x \in \mathfrak{X}$ , where  $U_C(t) = e^{-tC}$  in (1).

**Proposition 4.** Let  $A, B$  and  $C$  be generators of contraction  $C_0$ -semigroups on  $\mathfrak{X}$ . Suppose that algebraic sum

$$Cx = Ax + Bx, \tag{2}$$

is valid for all  $x \in D$ , where domain  $D = \text{core}(C)$ . Then the semigroup  $\{U_C(t)\}_{t \geq 0}$  can be approximated on  $\mathfrak{X}$  in the operator-norm topology by the Trotter-Kato product formula:

$$\lim_{n \rightarrow \infty} \|(\Phi(t/n))^n - e^{-tC}\| = 0, \tag{3}$$

for all  $t \in \mathbb{R}_0^+$ , where  $C := \overline{(A + B)}$  is closure of the operator sum in (2) and  $\Phi(t) := e^{-tA}e^{-tB}$  in (3).

References

1. Zagrebnov V.A. Notes on the Chernoff estimate, ArXiv:2205.04794v1 [math.FA], 10 May 2022.
2. Zagrebnov V.A. Operator-norm Trotter product formula on Banach spaces, ArXiv:2205.04807v1 [math.FA], 10 May 2022.

The Poisson representation for the class of  $H_A^1$  functions

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Let  $A(z)$  be an antianalytic function, i. e.  $\frac{\partial A}{\partial z} = 0$  in the domain  $D \subset \mathbb{C}$ ; moreover, let  $|A(z)| \leq C < 1$  for all  $z \in D$ . The function  $f(z)$  is said to be  $A(z)$ -analytic in the domain  $D$  if for any  $z \in D$ , the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z) \frac{\partial f}{\partial z} \tag{1}$$

We denote by  $O_A(D)$  the class of all  $A(z)$ -analytic functions defined in the domain  $D$ . According to, the function

$$\psi(z; a) = z - a + \int_{\gamma(a; z)} \overline{A(\tau)} d\tau$$

is an  $A(z)$ -analytic function.

The following set is an open subset of arbitrary convex domain  $D$  :

$$L(a; r) = \left\{ |\psi(z; a)| = \left| z - a + \int_{\gamma(a; z)} \overline{A(\tau)} d\tau \right| < r \right\}.$$

For sufficiently small  $r > 0$ , this set compactly lies in  $D$  (we denote this fact by  $L(a; r) \subset\subset D$ ) and contains the point  $a$ . This set  $L(a; r)$  is called the  $A(z)$ -lemniscate centered at the point  $a$ . The lemniscate  $L(a; r)$  is a simply - connected set (see [2]).

Hardy classes  $H^p$  were introduced by F. Riesz's. The Hardy class  $H^p_A, p > 0$  for  $A(z)$ -analytic functions is given in [4]. Before we will introduce this class for  $A(z)$ -analytic functions in the case  $p = 1$ .

**Definition 1.**  $f(z) \in O_A(L(a; r))$  is said to be in  $H^1_A$ , if

$$\frac{1}{2\pi\rho} \int_{|\psi(z; a)|=\rho} |f(z)| |dz + A(z)d\bar{z}| \quad (2)$$

is bounded in lemniscate  $L(a; r)$ , where  $\rho < r, z \in L(a; r)$ .

Let  $f = u + iv$ .

**Theorem 1.** (see [3]). *The real part of the  $A(z)$ -analytic functions of  $f(z) \in O_A(D)$  satisfies equation*

$$\Delta_A u = \frac{\partial}{\partial z} \left( \frac{1}{1-|A|^2} \left( (1+|A|^2) \frac{\partial u}{\partial \bar{z}} - 2A \frac{\partial u}{\partial z} \right) \right) + \frac{\partial}{\partial \bar{z}} \left( \frac{1}{1-|A|^2} \left( (1+|A|^2) \frac{\partial u}{\partial z} - 2\bar{A} \frac{\partial u}{\partial \bar{z}} \right) \right) = 0 \quad (3)$$

in the domain of  $D$ .

In connection with Theorem 1, it is natural to define the  $A(z)$ -harmonic function as follows.

**Definition 2** (see [3]). *A double differentiable function  $u \in C^2(D)$ ,  $u : D \rightarrow R^1$  is called  $A(z)$ -harmonic in the  $D$  domain if the  $D$  domain if it satisfies the differential equation (3).*

The class of  $A(z)$ -harmonic functions in the domain of  $D$  is denoted as  $h_A(D)$ . Thus, the real part and hence the imaginary part, of the  $A(z)$ -harmonic function in the domain of  $D$ . The inverse theorem is also true for simply connected domains.

**Theorem 2.** (see [3]). *If the function is  $u(z) \in h_A(D)$ , where  $D$  is a simply connected domain, then  $f \in O_A(D) : u = \operatorname{Re} f$ .*

For  $A(z)$ -analytic and  $A(z)$ -harmonic functions, the following Dirichlet problem is naturally considered:

**Dirichlet problem.** *A bounded domain of  $G \subset D$  is given and a continuous function of  $\omega(\zeta)$  is set at the boundary of  $\partial G$ . It is required to find  $A(z)$ -harmonic in the domain of  $G$ , continuous on the closure of  $\bar{G}$  the function of  $u(z) \in h_A(G) \cap C(\bar{G}) : u|_{\partial G} = \omega$ .*

**Theorem 3.** (see [3]) (an analogue of the Poisson formula for  $A(z)$ -harmonic functions). *If the  $\omega(\zeta)$  function is continuous on the boundary of the lemniscate of  $L(a; r) \subset D$ , then the function*

$$u(z) = \frac{1}{2\pi r} \int_{|\psi(\zeta; a)|=r} \omega(\zeta) \frac{r^2 - |\psi(z; a)|^2}{|\psi(\zeta; z)|^2} |d\zeta + A(\zeta)d\bar{\zeta}| \quad (4)$$

is the solution of the Dirichlet problem in  $L(a; r)$ .

The  $f(\zeta; z) = \frac{\psi(a; \zeta) + \psi(a; z)}{\psi(z; \zeta)}$  function is an  $A(z)$ -analytic function for  $z \in L(a; r)$ , where  $\zeta \in \partial L(a; r)$ . Then

$$P(\zeta; z) = \frac{1}{2\pi r} (f(\zeta; z) + \bar{f}(z; \zeta)) = \frac{1}{2\pi r} \left( \frac{\psi(a; \zeta) + \psi(a; z)}{\psi(z; \zeta)} + \frac{\bar{\psi}(a; \zeta) + \bar{\psi}(a; z)}{\bar{\psi}(a; \zeta) - \bar{\psi}(a; z)} \right) =$$

$$= \frac{1}{2\pi r} \left( \frac{|\psi(a; \zeta)|^2 - |\psi(a; z)|^2}{|\psi(z; \zeta)|^2} \right) = \frac{1}{2\pi r} \left( \frac{r^2 - |\psi(a; z)|^2}{|\psi(z; \zeta)|^2} \right).$$

Formula (4) is called an analogue of the Poisson formula for  $A(z)$ -harmonic functions. Initially we will introduce Hardy class for also  $A(z)$ -harmonic functions in the case  $p = 1$ .

**Statement 1.**  $u(z) \in h_A(L(a; r))$  is said to be in  $H_A^1$ , if the average integral

$$\frac{1}{2\pi\rho} \int_{|\psi(z;a)|=\rho} |u(z)||dz + A(z)d\bar{z}| \tag{5}$$

is bounded in lemniscate  $L(a; r)$ .

Now we give the Poisson representation for the class of functions  $H_A^1$ .

**Theorem 4.** Let  $u(z) \in H_A^1(L(a; R))$ . If  $u(z)$  is a  $A(z)$ -harmonic function in the lemniscate  $L(a; r)$ , then

$$u(z) = \int_{|\psi(\zeta;a)=r} P(\zeta; z)u(\zeta)(d\zeta + A(\zeta)d\bar{\zeta}), \tag{6}$$

where  $z \in L(a; r)$ .

References

1. *P.Koosis*. Introduction to  $H^p$  spaces, United Kingdom. Cambridge University Press, 1998.
2. *A.Sadullayev, N.M.Zhabborov*. On a class of A-analytic functions, J. Siberian Fed. Univ., no. 3(9), 2016, 374-383 p.
3. *N.M.Zhabborov, T.U.Otaboyev and Sh.Ya.Khursanov*. Schwarz inequality and Schwarz formula for A-analytic functions, J. Modern math. Fundamental directions., no. 4(64), 2018, 637-649 p. (Russian)
4. *B.E.Husenov*. Generalization of the Hardy class for  $A(z)$ -analytic functions, J. Scientific Reports of Bukhara State Univ., no. 4(86), 2021, 29-46 p.

$\vec{\alpha}$  - сепаратно субгармонические функции  
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Пусть  $\alpha$ -произвольная замкнутая, строго положительная дифференциальная форма бистепени  $(n - 1, n - 1)$  в области  $D \subset \mathbb{C}^n$ :

$$\alpha = \left(\frac{i}{2}\right)^{n-1} \sum_{j,k=1}^n \alpha_{jk}(z) dz [j] \wedge d\bar{z} [k], \alpha_{jk}(z) \in C^1(D), d\alpha = 0,$$

здесь  $dz [j] = dz_1 \wedge \dots \wedge dz_{j-1} \wedge dz_{j+1} \wedge \dots \wedge dz_n$ ,  $d\bar{z} [k] = d\bar{z}_1 \wedge \dots \wedge d\bar{z}_{k-1} \wedge d\bar{z}_{k+1} \wedge \dots \wedge d\bar{z}_n$ .

Строго положительность  $\alpha$  означает, что для любой компактной области  $\Omega \Subset D$  существует число  $\varepsilon > 0$  такое что дифференциальная форма  $\alpha - \varepsilon\beta^{n-1} \geq 0$ , где  $\beta = dd^c|z|^2$  - форма объема в пространстве  $\mathbb{C}^n$ .

**Определение 1 (см. [1]).** Функция  $u(z) \in L_{loc}^1(D)$  называется  $\alpha$ -субгармонической в области  $D \subset \mathbb{C}^n$ , если

1) она полунепрерывна сверху в  $D$ , т. е.  $u(z^0) \geq \overline{\lim}_{z \rightarrow z^0} u(z), \forall z^0 \in D;$