ABSTRACTS

of the international conference

MATHEMATICAL ANALYSIS AND ITS APPLICATIONS IN MODERN MATHEMATICAL PHYSICS

PART I

Samarkand
September 23-24, 2022

holds for all $n \in \mathbb{N}$ *and* $x \in \mathfrak{X}$.

Proposition 3. Let $\Phi : t \mapsto \Phi(t)$ be a function from $\mathbb{R}_0^+$ to contractions on $\mathfrak{X}$ such that $\Phi(0) = I$. Let $\{U_C(t)\}_{t\geq 0}$ be a contraction C_0-semigroup, and let domain $D \subset \mathrm{dom}(C)$ be a core of related generator C.

If the function $\Phi(t)$ has a strong right-derivative $\Phi'(+0)$ at $t = 0$ *(*that is, $\Phi'(+0)x$ exists for any $x \in \mathrm{dom}(\Phi'(+0))$*)* and if

$$\Phi'(+0)\, x := \lim_{t\to+0} \frac{1}{t}(\Phi(t) - I)\, x = -\, C\, x\,,$$

for all $x \in D$, then

$$\lim_{n\to\infty} [\Phi(t/n)]^n\, x = U_C(t)\, x\ , \tag{1}$$

for all $t \in \mathbb{R}_0^+$ and $x \in \mathfrak{X}$, where $U_C(t) = e^{-tC}$ in (1).

Proposition 4. Let A, B and C be generators of contraction C_0-semigroups on $\mathfrak{X}$. Suppose that algebraic sum

$$Cx = Ax + Bx\ , \tag{2}$$

is valid for all $x \in D$, where domain $D = \mathrm{core}\,(C)$. Then the semigroup $\{U_C(t)\}_{t\geq 0}$ can be approximated on $\mathfrak{X}$ in the operator-norm topology by the Trotter-Kato product formula*:*

$$\lim_{n\to\infty} \|(\Phi(t/n))^n - e^{-tC}\| = 0\ , \tag{3}$$

for all $t \in \mathbb{R}_0^+$, where $C := \overline{(A + B)}$ is closure of the operator sum in (2) and $\Phi(t) := e^{-tA}e^{-tB}$ in (3).

References

1. *Zagrebnov V.A.* Notes on the Chernoff estimate, ArXiv:2205.04794v1 [math.FA], 10 May 2022.
2. *Zagrebnov V.A.* Operator-norm Trotter product formula on Banach spaces, ArXiv:2205.04807v1 [math.FA], 10 May 2022.

The Poisson representation for the class of H_A^1 functions

[1]**Zhabborov N.**, [2]**Husenov B.**

[1] *Joint Belarusian-Uzbek Intersectoral Institute of Applied Technical Qualifications in Tashkent,*
e-mail: jabborov61@mail.ru

[2] *Bukhara State University, Bukhara, Uzbekistan,*
e-mail: husenovbehzod@mail.ru

Let $A(z)$ be an antianalytic function, i. e. $\frac{\partial A}{\partial z} = 0$ in the domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq C < 1$ for all $z \in D$. The function $f(z)$ is said to be $A(z)-$analytic in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z} \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)-$analytic functions defined in the domain D. According to, the function

$$\psi\,(z;a) = z - a + \overline{\int_{\gamma(a;z)} \overline{A(\tau)}d\tau}$$

is an $A(z)-$analytic function.

The following set is an open subset of arbitrary convex domain D :

$$L(a;r)=\left\{|\psi(z;a)|=\left|z-a+\overline{\int\limits_{\gamma(a;z)}\overline{A(\tau)}d\tau}\right|<r\right\}.$$

For sufficiently small $r>0$, this set compactly lies in D (we denote this fact by $L(a;r)\subset\subset D$) and contains the point a. This set $L(a;r)$ is called the $A(z)-$lemniscate centered at the point a. The lemniscate $L(a;r)$ is a simply - connected set (see [2]).

Hardy classes H^p were introduced by F. Riesz's. The Hardy class $H_A^p, p>0$ for $A(z)$-analytic functions is given in [4]. Before we will introduce this class for $A(z)$-analytic functions in the case $p=1$.

Definition 1. *$f(z)\in O_A(L(a;r))$ is said to be in H_A^1, if*

$$\frac{1}{2\pi\rho}\int\limits_{|\psi(z;a)|=\rho}|f(z)||dz+A(z)d\bar{z}| \tag{2}$$

is bounded in lemniscate $L(a;r)$, where $\rho<r, z\in L(a;r)$.

Let $f=u+iv$.

Theorem 1. (see [3]). *The real part of the $A(z)-$analytic functions of $f(z)\in O_A(D)$ satisfies equation*

$$\Delta_A u=\frac{\partial}{\partial z}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial\bar{z}}-2A\frac{\partial u}{\partial z}\right)\right)+\frac{\partial}{\partial\bar{z}}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial z}-2\bar{A}\frac{\partial u}{\partial\bar{z}}\right)\right)=0 \tag{3}$$

in the domain of D.

In connection with Theorem 1, it is natural to define the $A(z)-$harmonic function as follows.

Definition 2 (see [3]). *A double differentiable function $u\in C^2(D),\quad u:D\to R^1$ is called $A(z)-$harmonic in the D domain if the D domain if it satisfies the differential equation (3).*

The class of $A(z)-$harmonic functions in the domain of D is denoted as $h_A(D)$. Thus, the real part and hence the imaginary part, of the $A(z)-$harmonic function in the domain of D. The inverse theorem is also true for simply connected domains.

Theorem 2. (see [3]). *If the function is $u(z)\in h_A(D)$, where D is a simply connected domain, then $f\in O_A(D):u=Re\,f$.*

For $A(z)-$analytic and $A(z)-$harmonic functions, the following Dirichlet problem is naturally considered:

Dirichlet problem. *A bounded domain of $G\subset D$ is given and a continuous function of $\omega(\zeta)$ is set at the boundary of ∂G. It is required to find $A(z)-$harmonic in the domain of G, continuous on the closure of $\bar{G}$ the function of $u(z)\in h_A(G)\cap C(\bar{G}):u|_{\partial G}=\omega$.*

Theorem 3. (see [3]) (an analogue of the Poisson formula for $A(z)-$harmonic functions). *If the $\omega(\zeta)$ function is continuous on the boundary of the lemniscate of $L(a;r)\subset D$, then the function*

$$u(z)=\frac{1}{2\pi r}\int\limits_{|\psi(\zeta;a)|=r}\omega(\zeta)\frac{r^2-|\psi(z;a)|^2}{|\psi(\zeta;z)|^2}|d\zeta+A(\zeta)d\bar{\zeta}| \tag{4}$$

is the solution of the Dirichlet problem in $L(a;r)$.

The $f(\zeta;z)=\frac{\psi(a;\zeta)+\psi(a;z)}{\psi(z;\zeta)}$ function is an $A(z)-$analytic function for $z\in L(a;r)$, where $\zeta\in\partial L(a;r)$. Then

$$P(\zeta;z)=\frac{1}{2\pi r}(f(\zeta;z)+\bar{f}(z;\zeta))=\frac{1}{2\pi r}\left(\frac{\psi(a;\zeta)+\psi(a;z)}{\psi(a;\zeta)-\psi(a;z)}+\frac{\bar{\psi}(a;\zeta)+\bar{\psi}(a;z)}{\bar{\psi}(a;\zeta)-\bar{\psi}(a;z)}\right)=$$

$$= \frac{1}{2\pi r}\left(\frac{|\psi(a;\zeta)|^2 - |\psi(a;z)|^2}{|\psi(z;\zeta)|^2}\right) = \frac{1}{2\pi r}\left(\frac{r^2 - |\psi(a;z)|^2}{|\psi(z;\zeta)|^2}\right).$$

Formula (4) is called an analogue of the Poisson formula for $A(z)-$harmonic functions.
Initially we will introduce Hardy class for also $A(z)$-harmonic functions in the case $p = 1$.

Statement 1. *$u(z) \in h_A(L(a;r))$ is said to be in H_A^1, if the average integral*

$$\frac{1}{2\pi\rho}\int\limits_{|\psi(z;a)|=\rho} |u(z)||dz + A(z)d\bar{z}| \tag{5}$$

is bounded in lemniscate $L(a;r)$.

Now we give the Poisson representation for the class of functions H_A^1.

Theorem 4. *Let $u(z) \in H_A^1(L(a;R))$. If $u(z)$ is a $A(z)-$harmonic function in the lemniscate $L(a;r)$, then*

$$u(z) = \int\limits_{|\psi(\zeta;a)|=r} P(\zeta;z)u(\zeta)(d\zeta + A(\zeta)d\bar{\zeta}), \tag{6}$$

where $z \in L(a;r)$.

References

1. *P.Koosis.* Introduction to H^p spaces, United Kingdom. Cambridge University Press, 1998.
2. *A.Sadullayev, N.M.Zhabborov.* On a class of A-analytic functions, J. Siberian Fed. Univ., no. 3(9), 2016, 374-383 p.
3. *N.M.Zhabborov, T.U.Otaboyev and Sh.Ya.Khursanov.* Schwarz inequality and Schwarz formula for A-analytic functions, J. Modern math. Fundamental directions., no. 4(64), 2018, 637-649 p. (Russian)
4. *B.E.Husenov.* Generalization of the Hardy class for $A(z)-$analytic functions, J. Scientific Reports of Bukhara State Univ., no. 4(86), 2021, 29-46 p.

$\overrightarrow{\alpha}$ - сепаратно субгармонические функции

[1]Абдикадиров С.М.

[1] *Каракалпакский государственный университет имени Бердаха, Узбекистан. Институт математики имени В.И.Романовского Академии наук Республики Узбекистан,*
e-mail: abdikadirov1983@inbox.ru

Пусть $\alpha-$произвольная замкнутая, строго положительная дифференциальная форма бистепени $(n-1,\, n-1)$ в области $D \subset \mathbb{C}^n$:

$$\alpha = \left(\frac{i}{2}\right)^{n-1}\sum_{j,k=1}^{n}\alpha_{jk}(z)dz\,[j]\wedge d\bar{z}\,[k],\;\; \alpha_{jk}(z) \in C^1\,(D)\,,\;\; d\alpha = 0,$$

здесь $dz\,[j] = dz_1 \wedge ... \wedge dz_{j-1} \wedge dz_{j+1} \wedge ... \wedge dz_n$, $d\bar{z}\,[k] = d\bar{z}_1 \wedge ... \wedge d\bar{z}_{k-1} \wedge d\bar{z}_{k+1} \wedge ... \wedge d\bar{z}_n$.

Строго положительность α означает, что для любой компактной области $\Omega \Subset D$ существует число $\varepsilon > 0$ такое что дифференциальная форма $\alpha - \varepsilon\beta^{n-1} \geq 0$, где $\beta = dd^c|z|^2-$ форма объема в пространстве $\mathbb{C}^n$.

Определение 1 (см. [1]). *Функция $u\,(z) \in L^1_{loc}\,(D)$ называется α-субгармонической в области $D \subset \mathbb{C}^n$, если*
1) она полунепрерывна сверху в D, т. е. $u(z^0) \geq \overline{\lim\limits_{z\to z^0}}\, u(z)$, $\forall z^0 \in D$;