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MATEMATIK FIZIKA VA ZAMONAVIY ANALIZNING TURDOSH MASALALARI

professor D.K. Durdiyev tavalludining 60 yilligiga bag‘ishlangan xalqaro ilmiy anjumani

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Bukhara State University

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MATHEMATICAL PHYSICS AND RELATED PROBLEMS OF MODERN ANALYSIS

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dedicated to the 60th anniversary of the birth of Professor D.K. Durdiev
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ABSTRACTS

PART I

is a max-min linear functional on $C_b(X)$. Conversely, for any max-min linear functional $\tilde{\nu}\colon C_b(X) \to \mathbb{R}$ there exists a unique τ-smooth idempotent probability measure $\tilde{\mu}(X) \neq 0$ on $\mathfrak{B}(X)$ such that

$$\tilde{\nu}(\tilde{\varphi}) = \ln \left(\frac{1}{\tilde{\mu}(X)} \int\limits_X^{\oplus} e^{\tilde{\varphi}(x)} d\tilde{\mu} \right), \tilde{\varphi} \in C_b(X),$$

where $C_b(X)$ denote the set of real-valued bounded continuous functions on X.

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CONTINUATION OF $A(z)$–ANALYTIC FUNCTIONS ACCORDING TO PENVEL

Husenov B. E.
Bukhara State University, Bukhara, Uzbekistan;
b.e.husenov@buxdu.uz

We investigate this thesis of the continuation of the $A(z)$–analytic function according to Penvel. Here, we will use the principle of continuity.

Let $A(z)$ be an antianalytic function in the domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq c$ for all $z \in D$, where $c < 1$. The function $f(z)$ is said to be $A(z)$*–analytic* in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z) \frac{\partial f}{\partial z} \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)$–analytic functions defined in the domain D (see [1]).

According to, the function $\psi(a,z) = z - a + \overline{\int\limits_{\gamma(a,z)} \overline{A(\tau)} d\tau}$ is an $A(z)$–analytic function. The following set $L(a,r) = \{|\psi(a,z)| < r\}$ is an open subset of arbitrary convex domain D. For sufficiently small $r > 0$, this set compactly lies in D and contains the point a. This set $L(a,r)$ is called the $A(z)$*–lemniscate* centered at the point a. The lemniscate $L(a,r)$ is a simply - connected set (see [1]).

If $f(z) \in O_A(L(a,r)) \bigcap C\left(\bar{L}(a,r)\right)$, where $L(a,r) \subset D$ is a fixed $A(z)$-lemniscate, then in $L(a,r)$ the function $f(z)$ is expanded in a Taylor series:

$$f(z) = \sum_{k=0}^{\infty} c_k \psi^k(a,z), \tag{2}$$

where $c_k = \frac{1}{2\pi i} \int\limits_{|\psi(a,\xi)|=\rho} \frac{f(\xi)}{(\psi(a,\xi))^{k+1}} \left(d\xi + A(\xi) d\bar{\xi}\right), 0 < \rho < r, k = 0, 1, 2, \ldots$ (see [1]).

Suppose that two lemniscates $L(a, r_1)$ and $L(b, r_2)$, whose intersection is the empty set, $L(a, r_1) \bigcap L(b, r_2) = \varnothing$, have a common boundary section consisting of a smooth curve l, and consider the domain $D = L(a, r_1) \bigcup L(b, r_2) \bigcup l$.

The principle of continuity for $A(z)$–analytic functions. *If functions $f_1(\zeta)$ and $f_2(\zeta)$ are $A(z)$–analytic in lemniscates $L(a, r_1)$ and $L(b, r_2)$ respectively, continuous up to l and, in addition,*

$$f_1(\zeta) = f_2(\zeta), \zeta \in l,$$

then function $f_2(\zeta)$ is an $A(z)$–analytic continuation of function $f_1(\zeta)$ from lemniscate $L(a, r_1)$ to lemniscate $L(b, r_2)$ through curve l.

One important consequence follows from the principle of continuity. Let l, the boundary of the lemniscate $L(c, r_3)$, contain a smooth curve l_0, and let the function $f(\zeta)$ be $A(z)$–analytic in $L(c, r_3)$ and continuous up to l_0. Let's also assume that $f(\zeta) = 0, \zeta \in l_0$, then $f(\zeta) \equiv 0$ everywhere up to $L(c, r_3)$ (see [2]).

In fact, let us attach to lemniscate $L(c, r_3)$ along subset l_0 of its border l lemniscate $L(d, r_4), L(c, r_3) \bigcap L(d, r_4)$, and consider function

$$F(\zeta) = \begin{cases} f(\zeta), & \zeta \in L(c, r_3), \\ 0, & \zeta \in l_0, \\ 0, & \zeta \in L(d, r_4). \end{cases}$$

By virtue of the principle of continuity we conclude that function $F(\zeta)$ $A(z)$–analytic in the domain of $D = L(c, r_3) \bigcup L(d, r_4) \bigcup l_0$. On the other hand, since $F(\zeta) = 0$ is in the lemniscate $L(d, r_4)$, which lies inside D, then by the property of uniqueness of $A(z)$–analytic functions $F(\zeta) = 0$ is everywhere in D, and therefore also in $L(c, r_3)$, and consequently $f(\zeta) \equiv 0$ is in $L(c, r_3)$ (see [2]).

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THE EXISTENCE OF EIGENVALUES OF A SECOND-ORDER OPERATOR MATRIX

Ismoilova D. E

Bukhara State University, Bukhara, Uzbekistan;
dilierkinovna9@gmail.com

By $\mathbb{T} := (-\pi; \pi]$ we denote an one dimensional torus. Let $\mathcal{H}_0 := \mathbb{C}$ be the one-dimensional complex space, $\mathcal{H}_1 := L_2(\mathbb{T})$ be the Hilbert space of square-integrable (complex-valued) functions defined on $\mathbb{T}$.

We introduce the Hilbert space

$$\mathcal{F}_{\text{as}}^{(1)} (L_2(\mathbb{T})) := \mathcal{H}_0 \oplus \mathcal{H}_1.$$

An element of the space $\mathcal{F}_{\text{as}}^{(1)} (L_2(\mathbb{T}))$ is a vector function of the form $f = \{f_0, f_1\}$, where $f_\alpha \in \mathcal{H}_\alpha, \alpha = 0, 1$.