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ABSTRACTS

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is a max-min linear functional on  $C_b(X)$ . Conversely, for any max-min linear functional  $\tilde{\nu}: C_b(X) \rightarrow \mathbb{R}$  there exists a unique  $\tau$ -smooth idempotent probability measure  $\tilde{\mu}(X) \neq 0$  on  $\mathfrak{B}(X)$  such that

$$\tilde{\nu}(\tilde{\varphi}) = \ln \left( \frac{1}{\tilde{\mu}(X)} \int_X^{\oplus} e^{\tilde{\varphi}(x)} d\tilde{\mu} \right), \tilde{\varphi} \in C_b(X),$$

where  $C_b(X)$  denote the set of real-valued bounded continuous functions on  $X$ .

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## CONTINUATION OF $A(z)$ -ANALYTIC FUNCTIONS ACCORDING TO PENVEL

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We investigate this thesis of the continuation of the  $A(z)$ -analytic function according to Penvel. Here, we will use the principle of continuity.

Let  $A(z)$  be an antianalytic function in the domain  $D \subset \mathbb{C}$ ; moreover, let  $|A(z)| \leq c$  for all  $z \in D$ , where  $c < 1$ . The function  $f(z)$  is said to be  $A(z)$ -analytic in the domain  $D$  if for any  $z \in D$ , the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z) \frac{\partial f}{\partial z} \quad (1)$$

We denote by  $O_A(D)$  the class of all  $A(z)$ -analytic functions defined in the domain  $D$  (see [1]).

According to, the function  $\psi(a, z) = z - a + \int_{\gamma(a, z)}^{\overline{A(\tau)}} d\tau$  is an  $A(z)$ -analytic function.

The following set  $L(a, r) = \{|\psi(a, z)| < r\}$  is an open subset of arbitrary convex domain  $D$ . For sufficiently small  $r > 0$ , this set compactly lies in  $D$  and contains the point  $a$ . This set  $L(a, r)$  is called the  $A(z)$ -lemniscate centered at the point  $a$ . The lemniscate  $L(a, r)$  is a simply - connected set (see [1]).

If  $f(z) \in O_A(L(a, r)) \cap C(\bar{L}(a, r))$ , where  $L(a, r) \subset D$  is a fixed  $A(z)$ -lemniscate, then in  $L(a, r)$  the function  $f(z)$  is expanded in a Taylor series:

$$f(z) = \sum_{k=0}^{\infty} c_k \psi^k(a, z), \quad (2)$$

where  $c_k = \frac{1}{2\pi i} \int_{|\psi(a, \xi)|=\rho} \frac{f(\xi)}{(\psi(a, \xi))^{k+1}} (d\xi + A(\xi)d\bar{\xi}), 0 < \rho < r, k = 0, 1, 2, \dots$  (see [1]).

Suppose that two lemniscates  $L(a, r_1)$  and  $L(b, r_2)$ , whose intersection is the empty set,  $L(a, r_1) \cap L(b, r_2) = \emptyset$ , have a common boundary section consisting of a smooth curve  $l$ , and consider the domain  $D = L(a, r_1) \cup L(b, r_2) \cup l$ .

**The principle of continuity for  $A(z)$ -analytic functions.** *If functions  $f_1(\zeta)$  and  $f_2(\zeta)$  are  $A(z)$ -analytic in lemniscates  $L(a, r_1)$  and  $L(b, r_2)$  respectively, continuous up to  $l$  and, in addition,*

$$f_1(\zeta) = f_2(\zeta), \zeta \in l,$$

*then function  $f_2(\zeta)$  is an  $A(z)$ -analytic continuation of function  $f_1(\zeta)$  from lemniscate  $L(a, r_1)$  to lemniscate  $L(b, r_2)$  through curve  $l$ .*

One important consequence follows from the principle of continuity. Let  $l$ , the boundary of the lemniscate  $L(c, r_3)$ , contain a smooth curve  $l_0$ , and let the function  $f(\zeta)$  be  $A(z)$ -analytic in  $L(c, r_3)$  and continuous up to  $l_0$ . Let's also assume that  $f(\zeta) = 0, \zeta \in l_0$ , then  $f(\zeta) \equiv 0$  everywhere up to  $L(c, r_3)$  (see [2]).

In fact, let us attach to lemniscate  $L(c, r_3)$  along subset  $l_0$  of its border  $l$  lemniscate  $L(d, r_4)$ ,  $L(c, r_3) \cap L(d, r_4)$ , and consider function

$$F(\zeta) = \begin{cases} f(\zeta), & \zeta \in L(c, r_3), \\ 0, & \zeta \in l_0, \\ 0, & \zeta \in L(d, r_4). \end{cases}$$

By virtue of the principle of continuity we conclude that function  $F(\zeta)$   $A(z)$ -analytic in the domain of  $D = L(c, r_3) \cup L(d, r_4) \cup l_0$ . On the other hand, since  $F(\zeta) = 0$  is in the lemniscate  $L(d, r_4)$ , which lies inside  $D$ , then by the property of uniqueness of  $A(z)$ -analytic functions  $F(\zeta) = 0$  is everywhere in  $D$ , and therefore also in  $L(c, r_3)$ , and consequently  $f(\zeta) \equiv 0$  is in  $L(c, r_3)$  (see [2]).

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## THE EXISTENCE OF EIGENVALUES OF A SECOND-ORDER OPERATOR MATRIX

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By  $\mathbb{T} := (-\pi; \pi]$  we denote an one dimensional torus. Let  $\mathcal{H}_0 := \mathbb{C}$  be the one-dimensional complex space,  $\mathcal{H}_1 := L_2(\mathbb{T})$  be the Hilbert space of square-integrable (complex-valued) functions defined on  $\mathbb{T}$ .

We introduce the Hilbert space

$$\mathcal{F}_{\text{as}}^{(1)}(L_2(\mathbb{T})) := \mathcal{H}_0 \oplus \mathcal{H}_1.$$

An element of the space  $\mathcal{F}_{\text{as}}^{(1)}(L_2(\mathbb{T}))$  is a vector function of the form  $f = \{f_0, f_1\}$ , where  $f_\alpha \in \mathcal{H}_\alpha, \alpha = 0, 1$ .