Министерство высшего
Образования, науки и инноваций

Национальный университет
Узбекистана имени Мирзо Улугбека

Международная
научная конференция

# АКТУАЛЬНЫЕ ПРОБЛЕМЫ СОВРЕМЕННОЙ ГЕОМЕТРИИ И ТОПОЛОГИИ

## Материалы конференции

27 - 29 октября 2024 года
г. Ташкент, Узбекистан

cpmgt2024.nuu.uz

germs of plane curves and the corresponding links. There exists no conceptual proof of this relation. All proofs consist of independent computation of the both sides and comparison of the results.

One can consider the complex plane as the complexification of the real one. This way one can consider algebraic links in the three-sphere $S^3_\varepsilon$ with the additional structure: the complex conjugation. In this setting, there are at least two versions of the Poincaré series. There is a problem to define an analogue (or analogues?) of the Alexander polynomial in this setting and to compare it with the Poincaré series.

## APPLICATION TO $A(z)$–ANALYTIC FUNCTIONS CLASS $N_A$

**Husenov B. E.**[1], **Hikmatova M. M.**[2]
Bukhara State University, Bukhara, Uzbekistan;
b.e.husenov@buxdu.uz
Bukhara State University, Bukhara, Uzbekistan;
mokhinisohikmatova@mail.ru

Let $A(z)$ be an antianalytic function, i.e. $\frac{\partial A}{\partial z} = 0$ in the convex domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq c < 1$ for all $z \in D$, where $c$ =const. The function $f(z)$ is said to be $A(z)$–*analytic* in the domain $D$ if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z} \tag{17}$$

We denote by $O_A(D)$ the class of all $A(z)$–analytic functions defined in the domain $D$.

According to, the function

$$\psi\left(a, z\right) = z - a + \overline{\int\limits_{\gamma(a,z)} \overline{A(\tau)} d\tau}$$

is an $A(z)$–analytic function.

The following set is an open subset of arbitrary convex domain $D$ :

$$L\left(a, r\right) = \left\{\left|\psi\left(a, z\right)\right| < r\right\}.$$

For sufficiently small $r > 0$, this set compactly lies in $D$ (we denote this fact by $L(a, r) \subset\subset D$) and contains the point $a$. This set $L(a, r)$ is called the $A(z)$–lemniscate centered at the point $a$. The lemniscate $L(a, r)$ is a simply - connected set (see [2]).

Initially, we introduce the Nevanlinna class for $A(z)$–analytic functions.

Let $L(a, r) \subset\subset D$ and $f(z) \in O_A(L(a, r))$. This function belongs to the class Nevanlinna $N_A$ if its mean

$$\frac{1}{2\pi\rho} \int\limits_{|\psi(a,z)|=\rho} \ln^+ |f(z)||dz + A(z)d\bar{z}| \tag{18}$$

is uniformly bounded, $\sup\limits_{\rho<r}\left\{\frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho}\ln^{+}|f(z)||dz+A(z)d\bar{z}|\right\}<+\infty.$

Now, we define the concepts of angular and radial limits of $A(z)$–analytic functions in lemniscate $L(a,r)\subset\subset D$. The radial limits of the function $f(z)$ at some point $\zeta\in\partial L(a,r)$ is denoted as $f^{*}(\zeta)$, and the angular limits is denoted as $f^{*}_{<}(\zeta)$.

In the classical case of the disk $U=\{|w|<1\}\subset\mathbb{C}_w$, the limit by the radius $\tau_\zeta=\{w=t\zeta\}, 0\le t\le 1, \zeta\in\partial U$ of the function $\varphi(w)$,

$$\varphi^{*}(\zeta)=\lim_{w\to\zeta, w\in\tau_\zeta}\varphi(w)$$

is called the *radial limit*, and the limit by the angle $<\subset U$, leaving the point $\zeta\in<$, is called the *angular limit*,

$$\varphi^{*}_{<}(\zeta)=\lim_{w\to\zeta, w\in<_\zeta}\varphi(w)\cdot$$

Since lemniscate $L(a,r)$ is a simply connected domain with a real analytic boundary, then according to Riemann's theorem there exists a conformal map $\chi(z):U\to L(a,r)$, which is also conformal in some neighborhood of closure $\bar{U}$. Let $\chi$ maps the boundary point $\lambda\in\partial U$ to the boundary point $\zeta\in\partial L(a,r)$. Then the curve $\gamma_\zeta$ has the property that it connects points $a,\zeta$ and is perpendicular to all lines of level $\partial L(a,\rho)=\{|\psi(a,z)|=\rho\}, 0<\rho\le r$. In the theory of $A(z)$–analytic functions, the curve $\gamma_\zeta=\chi(\tau_\lambda)$ plays the role of the radial direction, and the image of the angle $\chi(<)$ plays the role of the angular set at the point $\zeta\in\partial L(a,r)$. We will denote this angle by $<=<_\zeta$. The limit $f^{*}(\zeta)=\lim\limits_{z\to\zeta, z\in\gamma_\zeta}f(z)$ is called the radial limit, and $f^{*}_{<}(\zeta)=\lim\limits_{z\to\zeta, z\in<_\zeta}f(z)$ is the angular limit of the function $f(z)$ at the point $\zeta\in\partial L(a,r)$ (see [3]).

**Theorem 1.** *Any function $f(z)\in N_A(L(a,r))$ is represented as a ratio of two bounded functions,*

$$f(z)=\frac{f_1(z)}{f_2(z)},\quad f_1(z),f_2(z)\in O_A(L(a,r)),\quad |f_1(z)|,|f_2(z)|\le C<\infty,\quad \forall z\in L(a,r)\cdot$$

As is known, the bounded $A(z)$–analytic function in lemniscate $L(a,r)$ has radial (angular) limits $f^{*}(\zeta)$ almost everywhere on the boundary $\partial L(a,r)$ (see [3]). From Theorem 1 we have a stronger statement:

**Corollary 1.** *A function of class $f(z)\in N_A(L(a,r))$ has radial(angular) limits $f^{*}_{<}(\zeta)$ almost everywhere on the boundary $\partial L(a,r)$.*

*REFERENCES*

1. Gavrilov V.I., Subbotin A.V., Efimov D.A., Boundary Properties of Analytic Functions (Further Contribution). Moscow.: Moscow University Press, 2012.

2. Sadullaev A., Zhabborov N.M., On a class of $A$–analytic functions. J. Siberian Fed. Univ. volume 9, issue 3, p. 374-383, 2016.

3. Zhabborov N.M., Khursanov Sh.Y., Husenov B.E. Existence of boundary values of Hardy class functions $H^1_A$. Bulleten of the National University of Uzbekistan, volume 5, issue 2, p. 79-90, 2022.