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Using the formula $f(h, \theta) = \tanh(\theta \tanh h) = \frac{1}{2} \ln \frac{(1+\theta)e^{2h} + (1-\theta)}{(1-\theta)e^{2h} + (1+\theta)}$, and introducing the notations $\alpha = \frac{1-\theta}{1+\theta}$, $z_i = e^{2h_i}$, $i = 1, 2$, we have $\alpha \in (0, 1) \cup (1, +\infty)$. It is clear that $h_1 = h_2 = 0$ is a solution of (7). Denote $h_0 := 0$. By the last notations, (7) yields the following system of equations:

$$\begin{cases} z_1 = \frac{z_2 + \alpha}{\alpha z_2 + 1}, \\ z_2 = \left(\frac{z_1 + \alpha}{\alpha z_1 + 1} \right)^3. \end{cases} \quad (8)$$

Theorem. For the Gibbs measures for the Ising model corresponding to $h = \{h_x, x \in V\}$ which defined by rule (A) on the Cayley tree of order three the following statements hold:

1. if $\alpha \in (0; 2 - \sqrt{3}) \cup (2 + \sqrt{3}; \infty)$, then there are three Gibbs measures, moreover, two of them are non-translation-invariant Gibbs measures;
2. if $\alpha \in (2 - \sqrt{3}; 1) \cup (1; 2 + \sqrt{3})$, then there exists unique translation-invariant Gibbs measure.

Remark. If $\alpha \in (2 - \sqrt{3}; 1) \cup (1; 2 + \sqrt{3})$, the translation-invariant measure corresponding to $(z_1, z_2) = (1, 1)$ is studied in [1,3,4].

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ANALYTIC CONTINUATION OF CAUCHY-TYPE INTEGRALS FOR $A(z)$ -ANALYTIC FUNCTIONS

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Let $A(z)$ be an antianalytic function, i.e. $\frac{\partial A}{\partial z} = 0$ in the convex domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq c < 1$ for all $z \in D$, where $c = \text{const}$. The function $f(z)$ is said to be $A(z)$ -analytic in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z) \frac{\partial f}{\partial z}. \quad (1)$$

We denote by $O_A(D)$ the class of all $A(z)$ -analytic functions defined in the domain D .

According to, the function $\psi(a, z) = z - a + \overline{\int_{\gamma(a, z)} A(\tau) d\tau}$ is an $A(z)$ -analytic function.

The following set is an open subset of arbitrary convex domain D :

$$L(a, r) = \{|\psi(a, z)| < r\}.$$

For sufficiently small $r > 0$, this set compactly lies in D (we denote this fact by $L(a, r) \subset\subset D$) and contains the point a . This set $L(a, r)$ is called the $A(z)$ -lemniscate centered at the point a . The lemniscate $L(a, r)$ is a simply - connected set (see [1]).

Now we assume that the domain $D \subset \mathbb{C}$ is convex, and $\xi \in D$ is a fixed point in it. Consider the function

$$K(z, \xi) = \frac{1}{2\pi i} \frac{1}{z - \xi + \overline{\int_{\gamma(\xi, z)} A(\tau) d\tau}}, \quad (2)$$

where $\gamma(\xi, z)$ is a smooth curve which points of $\xi, z \in D$. Since the domain is simply connected and the function $\overline{A(z)}$ is holomorphic, the integral

$$I(z) = \overline{\int_{\gamma(\xi, z)} A(\tau) d\tau}$$

does not depend on a path of integration; it coincides with a primitive, i. e. $I'(z) = \overline{A(z)}$. (see [2]).

Now, we define the concepts of angular and radial limits of $A(z)$ -analytic functions in lemniscate $L(a, r) \subset\subset D$. The lemniscate $L(a, r)$ is a simply connected domain with a real analytic boundary, then according to Riemann's theorem there exists a conformal map $\chi(z) : U \rightarrow L(a, r)$, which is also conformal in some neighborhood of closure $\bar{U}$. Let χ maps the boundary point $\lambda \in \partial U$ to the boundary point $\zeta \in \partial L(a, r)$. Then the curve γ_ζ has the property that it connects points a, ζ and is perpendicular to all lines of level $\partial L(a, \rho) = \{|\psi(a, z)| = \rho\}, 0 < \rho \leq r$. In the theory of $A(z)$ -analytic functions, the curve $\gamma_\zeta = \chi(\tau_\lambda)$ plays the role of the radial direction, and the image of the angle $\chi(<)$ plays the role of the angular set at the point $\zeta \in \partial L(a, r)$. We will denote this angle by $< = <_\zeta$. The limit $f^*(\zeta) = \lim_{z \rightarrow \zeta, z \in \gamma_\zeta} f(z)$ is called *the radial limit*, and $f^*_{<}(\zeta) = \lim_{z \rightarrow \zeta, z \in <_\zeta} f(z)$ is *the angular limit* of the function $f(z)$ at the point $\zeta \in \partial L(a, r)$ (see [3]).

Let $L(a, r) \subset\subset D$. The following theorem relates to the question of the analytic continuation of Cauchy-type integrals for $A(z)$ -analytic functions

$$f(z) = \int_{\partial L(a, r)} f^*(\zeta) K(\zeta, z) (d\zeta + A(\zeta) d\bar{\zeta}). \quad (3)$$

First, we show the $A(z)$ -analyticity of the function represented by this integral:

Lemma 1. *If the following integral is bounded:*

$$\int_{\partial L(a, r)} \frac{|f^*(\zeta)|}{1 + |\psi(a, \zeta)|} |d\zeta + A(\zeta) d\bar{\zeta}| < \infty,$$

then integral (3) uniformly converges along z on any closed set that does not contain boundary points $\partial L(a, r)$.

Hence, by virtue of the lemma condition, we obtain using the Weierstrass sign the uniform convergence of the integral we are interested in.

Corollary 1. *The integral (3) represents an $A(z)$ -analytic function in any compact lemniscate $L(a, r)$ that does not contain boundary points $\partial L(a, r)$.*

The following generalized theorem solves the problem of the analytical continuation of the Cauchy-type integral for $A(z)$ -analytic functions through the points of the integration boundary.

Theorem 1. *Let $f(z) \in O_A(L(a, r))$, contain a piece M boundary $\partial L(a, r)$ (containing no other points M). Also, M divides lemniscate $L(a, r)$ into two parts G_1 and G_2 , lying, respectively, to the left of M and to the right of M , and $f_1(z)$ and $f_2(z)$ are functions represented by the integral (3) in G_1 and G_2 . Then the function $f_1(z) + f(z)$ gives an analytical continuation of the function $f_1(z)$ in the lemniscate $L(a, r)$ through the arc M .*

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CONSISTENCY THEOREM FOR A MODEL ON A CAYLEY TREE

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In this talk, we consider a model which was introduced in [1] on the Cayley tree of order k where the spin takes values in the set $\Phi = \{-1, +1\}$.

The Cayley tree Γ^k of order $k \geq 1$ infinite tree, i.e., a graph without cycles, such that exactly $k+1$ edges originate from each vertex. Let $\Gamma^k = (V, L)$ where V is the set of vertices and L is the set of edges. Let i be the incidence function setting its endpoints $x, y \in V$ into correspondence with each edge $l \in L$. Two vertices x and y are called *nearest-neighbors* and we denote $l(i) = \langle x, y \rangle$. A collection of nearest neighbor pairs $\langle x, x_1 \rangle, \langle x_1, x_2 \rangle, \dots, \langle x_{d-1}, y \rangle$ is called *a path* from x to y . The distance $d(x, y)$ on the Cayley tree is the number of edges of the shortest path from x and y .

Let Φ be a finite set. A configuration σ on V is defined as a function $x \in V \rightarrow \sigma(x) \in \Phi$. The set of all configurations coincides with $\Omega = \Phi^V$. Let $A \subset V$. We then let Ω_A denote the space of configurations defined on A . We let $|A|$ denote the cardinality of the set A . We consider the generalized Kronecker symbol to be the function

$$U(\sigma_A) : \Omega_A \rightarrow \{|A| - 1, |A| - 2, \dots, |A| - \min\{|A|, |\Phi|\}\}$$

defined as $U(\sigma_A) = |A| - |\sigma_A \cap \Phi|$, where $A \subset V$ and $\sigma_A \cap \Phi$ is the number of different values of $\sigma_A(x)$, $x \in A$. Note that if $|A| = 2$, say, $A = \{x, y\}$, then $U_0(\sigma(x), \sigma(y)) = \delta_{\sigma(x), \sigma(y)}$ [1-5].

Here, we consider the case where $\Phi = \{-1, +1\}$ and $|A| = k + 2$. We let M denote the set of all balls $b(x) = \{y \in V : d(x, y) \leq 1\}$ of unit radius. We define the Hamiltonian as

$$H(\sigma) = -J \sum_{b \in M} U(\sigma_b), \quad (1)$$