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Using the formula $f(h,\theta) = \tanh(\theta \tanh h) = \frac{1}{2}\ln\frac{(1+\theta)e^{2h}+(1-\theta)}{(1-\theta)e^{2h}+(1+\theta)}$, and introducing the notations $\alpha = \frac{1-\theta}{1+\theta}$, $z_i = e^{2h_i}$, $i = 1,2$, we have $\alpha \in (0,\,1) \cup (1,\,+\infty)$.It is clear that $h_1 = h_2 = 0$ is a solution of (7). Denote $h_0 := 0$. By the last notations, (7) yields the following system of equations:

$$\begin{cases} z_1 = \dfrac{z_2+\alpha}{\alpha z_2+1}, \\ z_2 = \left(\dfrac{z_1+\alpha}{\alpha z_1+1}\right)^3. \end{cases} \tag{8}$$

Theorem. For the Gibbs measures for the Ising model corresponding to $h = \{h_x, x \in V\}$ which defined by rule **(A)** on the Cayley tree of order three the following statements hold:

1. if $\alpha \in (0; 2-\sqrt{3}) \cup (2+\sqrt{3};\infty)$, then there are three Gibbs measures, moreover, two of them are non-translation-invariant Gibbs measures;

2. if $\alpha \in (2-\sqrt{3};1) \cup (1;2+\sqrt{3})$, then there exists unique translation-invariant Gibbs measure.

Remark. If $\alpha \in (2-\sqrt{3};1) \cup (1;2+\sqrt{3})$, the translation-invariant measure corresponding to $(z_1, z_2) = (1,1)$ is studied in [1,3,4].

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Analytic continuation of Cauchy-type integrals for $A(z)$-analytic functions

Zhabborov N.M.[1], Husenov B.E.[2]

[1]Tashkent State University of Economics, Tashkent, Uzbekistan;
[2]Bukhara State University, Bukhara, Uzbekistan;
e-mail1: jabborov61@mail.ru, e-mail2: b.e.husenov@buxdu.uz

Let $A(z)$ be an antianalytic function, i.e. $\frac{\partial A}{\partial z} = 0$ in the convex domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq c < 1$ for all $z \in D$, where c =const. The function $f(z)$ is said to be $A(z)$–*analytic* in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z}. \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)$–analytic functions defined in the domain D.

According to, the function $\psi\left(a,z\right)=z-a+\overline{\int\limits_{\gamma(a,z)}\overline{A(\tau)}d\tau}$ is an $A(z)$–analytic function. The following set is an open subset of arbitrary convex domain D:

$$L\left(a,r\right)=\left\{\left|\psi\left(a,z\right)\right|<r\right\}\cdot$$

For sufficiently small $r>0$, this set compactly lies in D (we denote this fact by $L(a,r)\subset\subset D$) and contains the point a. This set $L(a,r)$ is called the $A(z)$–lemniscate centered at the point a. The lemniscate $L(a,r)$ is a simply - connected set (see [1]).

Now we assume that the domain $D\subset\mathbb{C}$ is convex, and $\xi\in D$ is a fixed point in it. Consider the function

$$K(z,\xi)=\frac{1}{2\pi i}\frac{1}{z-\xi+\overline{\int\limits_{\gamma(\xi,z)}\overline{A(\tau)}d\tau}},\tag{2}$$

where $\gamma(\xi,z)$ is a smooth curve which points of $\xi,z\in D$. Since the domain is simply connected and the function $\overline{A}(z)$ is holomorphic, the integral

$$I(z)=\overline{\int\limits_{\gamma(\xi,z)}\overline{A(\tau)}d\tau}$$

does not depend on a path of integration; it coincides with a primitive, i. e. $I'(z)=\overline{A}(z)$. (see [2]).

Now, we define the concepts of angular and radial limits of $A(z)$–analytic functions in lemniscate $L(a,r)\subset\subset D$. The lemniscate $L(a,r)$ is a simply connected domain with a real analytic boundary, then according to Riemann's theorem there exists a conformal map $\chi(z):U\to L(a,r)$, which is also conformal in some neighborhood of closure $\bar{U}$. Let χ maps the boundary point $\lambda\in\partial U$ to the boundary point $\zeta\in\partial L(a,r)$. Then the curve γ_ζ has the property that it connects points a,ζ and is perpendicular to all lines of level $\partial L(a,\rho)=\{|\psi(a,z)|=\rho\},0<\rho\leq r$. In the theory of $A(z)$–analytic functions, the curve $\gamma_\zeta=\chi(\tau_\lambda)$ plays the role of the radial direction, and the image of the angle $\chi(<)$ plays the role of the angular set at the point $\zeta\in\partial L(a,r)$. We will denote this angle by $<=<_\zeta$. The limit $f^*(\zeta)=\lim\limits_{z\to\zeta,z\in\gamma_\zeta}f(z)$ is called *the radial limit*, and $f^*_<(\zeta)=\lim\limits_{z\to\zeta,z\in<_\zeta}f(z)$ is *the angular limit* of the function $f(z)$ at the point $\zeta\in\partial L(a,r)$ (see [3]).

Let $L(a,r)\subset\subset D$. The following theorem relates to the question of the analytic continuation of Cauchy-type integrals for $A(z)$–analytic functions

$$f(z)=\int\limits_{\partial L(a,r)}f^*(\zeta)K(\zeta,z)(d\zeta+A(\zeta)d\bar{\zeta})\cdot\tag{3}$$

First, we show the $A(z)$–analyticity of the function represented by this integral:

Lemma 1. *If the following integral is bounded:*

$$\int\limits_{\partial L(a,r)}\frac{|f^*(\zeta)|}{1+|\psi(a,\zeta)|}|d\zeta+A(\zeta)d\bar{\zeta}|<\infty,$$

then integral (3) uniformly converges along z on any closed set that does not contain boundary points $\partial L(a,r)$.

Hence, by virtue of the lemma condition, we obtain using the Weierstrass sign the uniform convergence of the integral we are interested in.

Corollary 1. *The integral (3) represents an $A(z)$–analytic function in any compact lemniscate $L(a,r)$ that does not contain boundary points $\partial L(a,r)$.*

The following generalized theorem solves the problem of the analytical continuation of the Cauchy-type integral for $A(z)$–analytic functions through the points of the integration boundary.

Theorem 1. *Let $f(z) \in O_A(L(a,r))$, contain a piece M boundary $\partial L(a,r)$ (containing no other points M). Also, M divides lemniscate $L(a,r)$ into two parts G_1 and G_2, lying, respectively, to the left of M and to the right of M, and $f_1(z)$ and $f_2(z)$ are functions represented by the integral (3) in G_1 and G_2. Then the function $f_1(z)+f(z)$ gives an analytical continuation of the function $f_1(z)$ in the lemniscate $L(a,r)$ through the arc M.*

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CONSISTENCY THEOREM FOR A MODEL ON A CAYLEY TREE

Mustafoyeva Z.
V.I.Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences, Tashkent, Uzbekistan,
mustafoyeva53@gmail.com

In this talk, we consider a model which was introduced in [1] on the Cayley tree of order $\mathbf{k}$ where the spin takes values in the set $\mathbf{\Phi} = \{\mathbf{-1},\ \mathbf{+1}\}$.

The Cayley tree $\mathbf{\Gamma^k}$ of order $\mathbf{k} \geq \mathbf{1}$ infinite tree, i.e., a graph without cycles, such that exactly $\mathbf{k+1}$ edges originate from each vertex. Let $\mathbf{\Gamma^k}=(\mathbf{V},\mathbf{L})$ where $\mathbf{V}$ is the set of vertices and $\mathbf{L}$ is the set of edges. Let $\mathbf{i}$ be the incidence function setting its endpoints $\mathbf{x},\mathbf{y} \in \mathbf{V}$ into correspondence with each edge $\mathbf{l} \in \mathbf{L}$. Two vertices $\mathbf{x}$ and $\mathbf{y}$ are called *nearest-neighbors* and we denote $\mathbf{l}(\mathbf{i}) = <\mathbf{x},\mathbf{y}>$. A collection of nearest neighbor pairs $<\mathbf{x},\mathbf{x_1}>$, $<\mathbf{x_1},\mathbf{x_2}>$,...,$<\mathbf{x_{d-1}},\mathbf{y}>$ is called *a path* from $\mathbf{x}$ to $\mathbf{y}$. The distance $\mathbf{d}(\mathbf{x},\mathbf{y})$ on the Cayley tree is the number of edges of the shortest path from $\mathbf{x}$ and $\mathbf{y}$.

Let Φ be a finite set. A configuration σ on V is defined as a function $x \in V \ \to \sigma(x) \in \Phi$. The set of all configurations coincides with $\Omega = \Phi^V$. Let $A \subset V$. We then let Ω_A denote the space of configurations defined on A. We let $|A|$ denote the cardinality of the set A.
We consider the generalized Kronecker symbol to be the function

$$U(\sigma_A) : \Omega_A \ \to \{|\mathrm{A}| - 1,\ |\mathrm{A}| - 2, ...,\ |\mathrm{A}| - \min\{|\mathrm{A}|,\ |\Phi|\}\ \}$$

defined as $U(\sigma_A) = |A| - |\sigma_A \cap|$, where $A \subset V$ and $\sigma_A \cap \Phi$ is the number of different values of $\sigma_A(x)$, $x \in A$. Note that if $|A| = 2$, say, $A = \{x, y\}$, then $U_0(\sigma(x), \sigma(y)) = \delta_{\sigma(x),\ \sigma(y)}$ [1-5].
Here, we consider the case where $\Phi = \{-1,\ +1\}$ and $|A| = k+2$. We let M denote the set of all balls $b(x) = \{y \in V :\ d(x,y) \leq 1\}$ of unit radius. We define the Hamiltonian as

$$H(\sigma) = -J \sum_{b \in M} U(\sigma_b), \tag{1}$$