V.I.Romanovskiy Institute of Mathematics

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ABSTRACTS OF THE CONFERENCE

OPERATOR ALGEBRAS, NON-ASSOCIATIVE STRUCTURES AND RELATED PROBLEMS

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$$v^{(j)}(x,\,0)=v_0^{(j)}(x),\;\; x\in\overline{b_j^-},\, j=1,2,...,n, \tag{5}$$

$$y^{(j)}(x,\,0)=y_0^{(j)}(x),\;\; x\in\overline{b_j^0},\, j=1,2,...,(n-1) \tag{6}$$

boundary conditions

$$u^{(1)}(0,t)=g_0^{(n)}(t),\;\; v^{(1)}(0,t)=d_0^{(n)}(t),\, t\geq 0, \tag{7}$$

$$u^{(n)}(L,t)=h_0^{(n)}(t),\;\; v^{(n)}(L,t)=r_0^{(n)}(t),\, t\geq 0 \tag{8}$$

on external bonds, respectively.
At the vertex point the solution satisfies the following gluing (Kirchhoff) conditions [6]:

$$u^{(j-1)}\left(L,t\right)+u^{(j)}\left(0,t\right)+y^{(j-1)}\left(0,t\right), u_x^{(j-1)}\left(L,t\right)=u_x^{(j)}\left(0,t\right)=y_x^{(j-1)}\left(0,t\right), \tag{9}$$

$$v^{(j-1)}\left(L,t\right)+v^{(j)}\left(0,t\right)+y^{(j-1)}\left(L,t\right), v_x^{(j-1)}\left(L,t\right)=v_x^{(j)}\left(0,t\right)=y_x^{(j-1)}\left(L,t\right), j=\overline{2,n}.$$

The last conditions usually called continuity and flux conservation (Kirchhoff) conditions on branching point of the graphs. We suppose, that initial data are smooth enough functions and they satisfies the conditions (7)–(9).

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RADIAL LIMITS FOR $A(z)$-ANALYTIC FUNCTIONS

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Let $A(z)$ be an antianalytic function, i. e. $\frac{\partial A}{\partial z}=0$ in the domain $D\subset\mathbb{C}$; moreover, let $|A(z)|\leq C<1$ for all $z\in D$. The function $f(z)$ is said to be $A(z)-$analytic in the domain D if for any $z\in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}}=A(z)\frac{\partial f}{\partial z} \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)-$analytic functions defined in the domain D.

According to, the function

$$\psi\left(z;a\right)=z-a+\overline{\int\limits_{\gamma(a;z)}\overline{A(\tau)}d\tau}$$

is an $A(z)-$analytic function.

The following set is an open subset of arbitrary convex domain D :

$$L\left(a;r\right)=\left\{\left|\psi\left(z;a\right)\right|=\left|z-a+\overline{\int\limits_{\gamma(a;z)}\overline{A(\tau)}d\tau}\right|<r\right\}.$$

For sufficiently small $r>0$, this set compactly lies in D (we denote this fact by $L(a;r)\subset\subset D$) and contains the point a. This set $L(a;r)$ is called the $A(z)-$lemniscate centered at the point a. The lemniscate $L(a;r)$ is a simply - connected set (see [2]).

Hardy classes H^p were introduced by F. Riesz's. The Hardy class $H^p_A, p>0$ for $A(z)$-analytic functions is given in [4]. Before we will introduce this class for $A(z)$-analytic functions in the case $p=1$.

Definition 1. *$f(z)\in O_A(L(a;r))$ is said to be in H^1_A, if*

$$\frac{1}{2\pi\rho}\int\limits_{|\psi(z;a)|=\rho}|f(z)||dz+A(z)d\bar{z}| \qquad (2)$$

is bounded in lemniscate $L(a;r)$, where $\rho<r, z\in L(a;r)$.

Let $f=u+iv$.

Theorem 1. (see [3]). *The real part of the $A(z)-$analytic functions of $f(z)\in O_A(D)$ satisfies equation*

$$\Delta_A u=\frac{\partial}{\partial z}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial\bar{z}}-2A\frac{\partial u}{\partial z}\right)\right)+\frac{\partial}{\partial\bar{z}}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial z}-2\bar{A}\frac{\partial u}{\partial\bar{z}}\right)\right)= \qquad (3)$$

in the domain of D.

In connection with Theorem 1, it is natural to define the $A(z)-$harmonic function as follows.

Definition 2. (see [3]). *A double differentiable function $u\in C^2(D),\quad u:D\to R^1$ is called $A(z)-$harmonic in the D domain if the D domain if it satisfies the differential equation (3).*

The class of $A(z)-$harmonic functions in the domain of D is denoted as $h_A(D)$. Thus, the real part and hence the imaginary part, of the $A(z)-$harmonic function in the domain of D. The inverse theorem is also true for simply connected domains.

Now we give the radial limits for $A(z)$-analytic functions. The classical theorem was first proved P. Fatou in 1906. This suggestion about radial limits for $A(z)$-analytic functions consists in the following statement:

Statement 1. *Let $A(z)$-analytical function of $f(z)$ in lemniscate $L(a;r)$. If the lemniscate $L(a;r)$ is bounded in this ($\exists E > 0, |f(z)| < E < \infty$, for $\forall z \in L(a;r)$), then the radial limits of the $f(\zeta) = \lim_{z\to\zeta} f(z)$ exist for all points of the ζ at $|\psi(\zeta;a)| = r$, except possibly for a set of Lebesgue measure zero.*

Corollary 1. *The function $u(z)$ is $A(z)$-harmonic and bounded in $L(a;r)$, then the radial limits $u(\zeta) = \lim_{z\to\zeta} u(z)$ exist for all points $\zeta \in L(a;r)$, except possibly for a set of Lebesgue measure zero.*

For $u(z)$ is the real part of a function $f(z)$ which is $A(z)-$harmonic in $|\psi(z;a)| < r$. For a finite real number $\exists B > 0, u(z) < B$ for all $z \in L(a;r)$ the function $(f(z) - B)^{-1}$ is $A(z)-$analytic and bounded and therefore has radial limits at almost all points $\zeta \in \partial L(a;r)$. The same is therefore true of $f(z)$ and of $u(z) = Ref(z)$.

We also give this corollary for the Hardy class H_A^1 :

Corollary 2. *If $f(z) \in H_A^1(L(a;r))$, then the radial limit of $f(\zeta) = \lim_{z\to\zeta} f(z)$ exists and is finite for almost all $\zeta \in \partial L(a;r)$.*

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ESTIMATES FOR GENERALIZATION OSCILLATORY INTEGRALS WITH POLYNOMIAL PHASE

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Many problems of harmonic analysis, analytic number theory and mathematical physics, including estimates for oscillatory (trigonometric) integrals, a common problem integration of rational polynomials instead of integrating functions. In particular, we may encounter such problems in papers [1]-[6]. This work is analogous [4] and application for oscillatory integrals with Mittag-Leffler functions.

The function $E_\alpha(z)$ is named after the great Swedish mathematican Gösta Magnus Mittag-Leffler (1846-1927) who defined it by a power series

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \quad \alpha \in \mathbb{C}, Re(\alpha) > 0, \tag{1}$$

and studied its properties in 1902-1905 in five subsequent notes [7] in connection with his summation method for divergent series.