



# ABSTRACTS

of the international conference

**MATHEMATICAL ANALYSIS AND ITS  
APPLICATIONS IN MODERN  
MATHEMATICAL PHYSICS**

## PART I

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**Non-tangential boundary values for  $A(z)$ -analytic functions  
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Let  $A(z)$  be an antianalytic function, i. e.  $\frac{\partial A}{\partial z} = 0$  in the domain  $D \subset \mathbb{C}$ ; moreover, let  $|A(z)| \leq C < 1$  for all  $z \in D$ . The function  $f(z)$  is said to be  $A(z)$ -analytic in the domain  $D$  if for any  $z \in D$ , the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z) \frac{\partial f}{\partial z} \tag{1}$$

We denote by  $O_A(D)$  the class of all  $A(z)$ -analytic functions defined in the domain  $D$ . According to, the function

$$\psi(z; a) = z - a + \int_{\gamma(a; z)} \overline{A(\tau)} d\tau$$

is an  $A(z)$ -analytic function.

The following set is an open subset of arbitrary convex domain  $D$  :

$$L(a; r) = \left\{ |\psi(z; a)| = \left| z - a + \int_{\gamma(a; z)} \overline{A(\tau)} d\tau \right| < r \right\}.$$

For sufficiently small  $r > 0$ , this set compactly lies in  $D$  (we denote this fact by  $L(a; r) \subset\subset D$ ) and contains the point  $a$ . This set  $L(a; r)$  is called the  $A(z)$ -lemniscate centered at the point  $a$ . The lemniscate  $L(a; r)$  is a simply - connected set (see [2]).

Hardy classes  $H^p$  were introduced by F. Riesz's. The Hardy class  $H^p_A, p > 0$  for  $A(z)$ -analytic functions is given in [4]. Before we will introduce this class for  $A(z)$ -analytic functions in the case  $p = 1$ .

**Definition 1.**  $f(z) \in O_A(L(a; r))$  is said to be in  $H^1_A$ , if

$$\frac{1}{2\pi\rho} \int_{|\psi(z; a)|=\rho} |f(z)||dz + A(z)d\bar{z}| \tag{2}$$

is bounded in lemniscate  $L(a; r)$ , where  $\rho < r, z \in L(a; r)$ .

Let  $f = u + iv$ .

**Theorem 1.** (see [3]). *The real part of the  $A(z)$ -analytic functions of  $f(z) \in O_A(D)$  satisfies equation*

$$\Delta_A u = \frac{\partial}{\partial z} \left( \frac{1}{1 - |A|^2} \left( (1 + |A|^2) \frac{\partial u}{\partial \bar{z}} - 2A \frac{\partial u}{\partial z} \right) \right) + \frac{\partial}{\partial \bar{z}} \left( \frac{1}{1 - |A|^2} \left( (1 + |A|^2) \frac{\partial u}{\partial z} - 2\bar{A} \frac{\partial u}{\partial \bar{z}} \right) \right) = 0 \tag{3}$$

in the domain of  $D$ .

In connection with Theorem 1, it is natural to define the  $A(z)$ -harmonic function as follows.

**Definition 2** (see [3]). *A double differentiable function  $u \in C^2(D), u : D \rightarrow R^1$  is called  $A(z)$ -harmonic in the  $D$  domain if the  $D$  domain if it satisfies the differential equation (3).*

The class of  $A(z)$ -harmonic functions in the domain of  $D$  is denoted as  $h_A(D)$ . Thus, the real part and hence the imaginary part, of the  $A(z)$ -harmonic function in the domain of  $D$ . The inverse theorem is also true for simply connected domains.

**Theorem 2.** (see [3]). *If the function is  $u(z) \in h_A(D)$ , where  $D$  is a simply connected domain, then  $f \in O_A(D) : u = \operatorname{Re} f$ .*

For  $A(z)$ -analytic and  $A(z)$ -harmonic functions, the following Dirichlet problem is naturally considered:

**Dirichlet problem.** *A bounded domain of  $G \subset D$  is given and a continuous function of  $\omega(\zeta)$  is set at the boundary of  $\partial G$ . It is required to find  $A(z)$ -harmonic in the domain of  $G$ , continuous on the closure of  $\bar{G}$  the function of  $u(z) \in h_A(G) \cap C(\bar{G}) : u|_{\partial G} = \omega$ .*

**Theorem 3.** (see [3]) (an analogue of the Poisson formula for  $A(z)$ -harmonic functions). *If the  $\omega(\zeta)$  function is continuous on the boundary of the lemniscate of  $L(a; r) \subset D$ , then the function*

$$u(z) = \frac{1}{2\pi r} \int_{|\psi(\zeta; a)|=r} \omega(\zeta) \frac{r^2 - |\psi(z; a)|^2}{|\psi(\zeta; z)|^2} |d\zeta + A(\zeta)d\bar{\zeta}| \quad (4)$$

is the solution of the Dirichlet problem in  $L(a; r)$ .

The  $f(\zeta; z) = \frac{\psi(a; \zeta) + \psi(a; z)}{\psi(z; \zeta)}$  function is an  $A(z)$ -analytic function for  $z \in L(a; r)$ , where  $\zeta \in \partial L(a; r)$ . Then

$$\begin{aligned} P(\zeta; z) &= \frac{1}{2\pi r} (f(\zeta; z) + \bar{f}(z; \zeta)) = \frac{1}{2\pi r} \left( \frac{\psi(a; \zeta) + \psi(a; z)}{\psi(z; \zeta)} + \frac{\bar{\psi}(a; \zeta) + \bar{\psi}(a; z)}{\bar{\psi}(z; \zeta)} \right) = \\ &= \frac{1}{2\pi r} \left( \frac{|\psi(a; \zeta)|^2 - |\psi(a; z)|^2}{|\psi(z; \zeta)|^2} \right) = \frac{1}{2\pi r} \left( \frac{r^2 - |\psi(a; z)|^2}{|\psi(z; \zeta)|^2} \right). \end{aligned}$$

Formula (4) is called an analogue of the Poisson formula for  $A(z)$ -harmonic functions.

The Hardy class  $H_A^p, p > 0$  for  $A(z)$ -analytic functions is given in [4]. Initially, we will introduce this class for also  $A(z)$ -harmonic functions in the case  $p > 1$ .

**Statement 1.**  *$u(z) \in h_A(L(a; r))$  is said to be in  $H_A^p$ , if the average integral*

$$\left( \frac{1}{2\pi\rho} \int_{|\psi(z; a)|=\rho} |u(z)| |dz + A(z)d\bar{z}| \right)^{\frac{1}{p}} \leq T \quad (5)$$

is bounded in lemniscate  $L(a; r)$ , where  $T > 0$ .

Now, we introduce an angular limit for  $A(z)$ -analytic functions.

**Notation 1.**  *$u(z)$  is called the (non-tangential or angular  $\triangleleft$ ) boundary value function for  $A(z)$ -harmonic function  $u(z)$ , we frequently write*

$$u(\zeta) = \lim_{\substack{z \rightarrow \zeta \\ \triangleleft}} u(z), \quad (6)$$

$L(a; R)$  lemniscate is almost everywhere.

Now we give the following theorem.

**Theorem 4.** *Let  $u(z) \in H^p(L(a; R)), p > 1$  and let  $u(z)$  be  $A(z)$ -harmonic function this lemniscate  $L(a; R)$ . Then, for almost all  $\zeta \in \partial L(a; R)$ ,  $u(z)$  tends to a finite limit, say  $u(\zeta)$ , as  $z \xrightarrow{\triangleleft} \zeta$*

$$u(z) = \frac{1}{2\pi r} \int_{|\psi(\zeta; a)|=r} u(\zeta) P(\zeta; z) |d\zeta + A(\zeta)d\bar{\zeta}|, \quad (7)$$

where  $z \in L(a; R)$ .

In future, whenever we have a function  $u(z) \in h_A(L(a; R))$ , satisfying the hypothesis of the above theorem (for class  $H_A^p, p > 1$ ), we assume it to be automatically extended a. e. to boundary lemniscate  $\partial L(a; R)$  in the manner described.

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**Estimates for convolution operators**

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In this paper we consider the convolution operator  $M_k$  with oscillatory kernel given by :

$$M_k = F^{-1}[e^{i\varphi(\xi)} a_k]F,$$

where  $F$  is the Fourier transform operator,  $\varphi \in C^\infty(\mathbb{R}^n \setminus \{0\})$ ,  $\varphi > 0$ , besides  $\varphi$  is a homogeneous function of order one,  $a_k \in C^\infty(\mathbb{R}^n)$  is a classical symbol of PDO of order  $-k$ .

**Problem.** *Let  $1 \leq p \leq 2$  be a fixed number. We consider the problem: find a number  $k(p)$  such that the onepamop  $M_k : L^p(\mathbb{R}^n) \rightarrow L^{p'}(\mathbb{R}^n)$  is bounded for  $k > k(p)$ , where  $p'$  is a conjugate exponent, e.g.  $1/p + 1/p' = 1$ .*

Note that if  $a(\xi) = |\xi|^{-k}$  with  $0 < k < n$  and  $\varphi \equiv 0$  then the problem is reduced to Hardy-Littlewood-Sobolev's inequality. Then if  $k = 2n(1/p - 1/2)$  then the operator is bounded from  $L^p(\mathbb{R}^n)$  to  $L^{p'}(\mathbb{R}^n)$ . Moreover, if  $a$  is a symbol of PDO and  $\varphi \equiv 0$  then we dealt with  $L^p \mapsto L^{p'}$  boundedness problem for the pseudo-differential operators. Note that, the oscillation factor gain better estimate for the order  $k$  of the symbol  $a$ .

It turns out that the number  $k(p)$  depends on the geometric properties of the following smooth hypersurface:

$$\Sigma = \{\xi \in \mathbb{R}^n : \varphi(\xi) = 1\}.$$

Further, we use notation:

$$k(p, \Sigma) := \inf_{k>0} \{k > 0 : M_k : L^p(\mathbb{R}^n) \rightarrow L^{p'}(\mathbb{R}^n) \text{ is bounded}\}.$$

M. Sugimoto [2] consider the problem for the case when  $\Sigma \subset \mathbb{R}^3$  is a smooth hypersurface having at least one non-vanishing principal curvature at every point and obtained an upper bound for the number  $k(p, \Sigma)$ . We consider more general surfaces in  $\mathbb{R}^3$  for which both principal curvatures can vanish and obtain an upper bound for  $k(p, \Sigma)$  in terms of a height of smooth functions improving the results proved by M. Sugimoto for the case  $n = 3$ . Moreover, we obtain the exact value of  $k(p, \Sigma)$  for some partial classes of surfaces.

Since  $\Sigma$  is a compact hypersurface, following M. Sugimoto it is enough to consider the local version of the problem. More, precisely we will assume that the amplitude function  $a_k(\xi)$  is concentrated in a sufficiently small conic neighborhood  $\Gamma$  of a fixed point  $v \in S^2$  (where  $S^2$  is a unit sphere centered at the origin of the space  $\mathbb{R}^3$ ) and  $\varphi(\xi) \in C^\infty(\Gamma)$ .

Also, for the sake of being definite we will assume  $v = (0, 0, 1)$  and  $\varphi(0, 0, 1) = 1$ . Thus, in a sufficiently small neighborhood of the point  $v$  the hypersurface  $\Sigma$  is given as the graph of a smooth function:

$$\Sigma \cap \Gamma = \{\xi \in \Gamma : \varphi(\xi) = 1\} = \{(\xi_1, \xi_2), \xi_3 = 1 + \phi(\xi_1, \xi_2), (\xi_1, \xi_2) \in U\},$$