ABSTRACTS

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MATHEMATICAL ANALYSIS AND ITS APPLICATIONS IN MODERN MATHEMATICAL PHYSICS

PART I

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CONTENTS

SECTION I. MATHEMATICAL ANALYSIS

Non-tangential boundary values for $A(z)-$analytic functions

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Let $A(z)$ be an antianalytic function, i. e. $\frac{\partial A}{\partial z} = 0$ in the domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq C < 1$ for all $z \in D$. The function $f(z)$ is said to be $A(z)-$analytic in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z} \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)-$analytic functions defined in the domain D.

According to, the function

$$\psi\left(z;a\right) = z - a + \overline{\int\limits_{\gamma(a;z)} \overline{A(\tau)}d\tau}$$

is an $A(z)-$analytic function.

The following set is an open subset of arbitrary convex domain D :

$$L\left(a;r\right) = \left\{ |\psi\left(z;a\right)| = \left| z - a + \overline{\int\limits_{\gamma(a;z)} \overline{A(\tau)}d\tau} \right| < r \right\}.$$

For sufficiently small $r > 0$, this set compactly lies in D (we denote this fact by $L(a;r) \subset\subset D$) and contains the point a. This set $L(a;r)$ is called the $A(z)-$lemniscate centered at the point a. The lemniscate $L(a;r)$ is a simply - connected set (see [2]).

Hardy classes H^p were introduced by F. Riesz's. The Hardy class $H_A^p, p > 0$ for $A(z)$-analytic functions is given in [4]. Before we will introduce this class for $A(z)$-analytic functions in the case $p = 1$.

Definition 1. *$f(z) \in O_A(L(a;r))$ is said to be in H_A^1, if*

$$\frac{1}{2\pi\rho} \int\limits_{|\psi(z;a)|=\rho} |f(z)||dz + A(z)d\bar{z}| \tag{2}$$

is bounded in lemniscate $L(a;r)$, where $\rho < r, z \in L(a;r)$.

Let $f = u + iv$.

Theorem 1. (see [3]). *The real part of the $A(z)-$analytic functions of $f(z) \in O_A(D)$ satisfies equation*

$$\Delta_A u = \frac{\partial}{\partial z}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z}\right)\right) + \frac{\partial}{\partial \bar{z}}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial z} - 2\bar{A}\frac{\partial u}{\partial \bar{z}}\right)\right) = 0 \tag{3}$$

in the domain of D.

In connection with Theorem 1, it is natural to define the $A(z)-$harmonic function as follows.

Definition 2 (see [3]). *A double differentiable function $u \in C^2(D)$, $u : D \to R^1$ is called $A(z)-$harmonic in the D domain if the D domain if it satisfies the differential equation (3).*

The class of $A(z)-$harmonic functions in the domain of D is denoted as $h_A(D)$. Thus, the real part and hence the imaginary part, of the $A(z)-$harmonic function in the domain of D. The inverse theorem is also true for simply connected domains.

Theorem 2. (see [3]). *If the function is* $u(z) \in h_A(D)$, *where* D *is a simply connected domain, then* $f \in O_A(D) : u = Re\, f$.

For $A(z)-$analytic and $A(z)-$harmonic functions, the following Dirichlet problem is naturally considered:

Dirichlet problem. *A bounded domain of* $G \subset D$ *is given and a continuous function of* $\omega(\zeta)$ *is set at the boundary of* ∂G. *It is required to find* $A(z)-$*harmonic in the domain of* G, *continuous on the closure of* $\bar{G}$ *the function of* $u(z) \in h_A(G) \cap C(\bar{G}) : u|_{\partial G} = \omega$.

Theorem 3. (see [3]) (an analogue of the Poisson formula for $A(z)-$harmonic functions). *If the* $\omega(\zeta)$ *function is continuous on the boundary of the lemniscate of* $L(a;r) \subset D$, *then the function*

$$u(z) = \frac{1}{2\pi r} \int\limits_{|\psi(\zeta;a)|=r} \omega(\zeta) \frac{r^2 - |\psi(z;a)|^2}{|\psi(\zeta;z)|^2} |d\zeta + A(\zeta)d\bar{\zeta}| \tag{4}$$

is the solution of the Dirichlet problem in $L(a;r)$.

The $f(\zeta;z) = \frac{\psi(a;\zeta)+\psi(a;z)}{\psi(z;\zeta)}$ function is an $A(z)-$analytic function for $z \in L(a;r)$, where $\zeta \in \partial L(a;r)$. Then

$$P(\zeta;z) = \frac{1}{2\pi r}(f(\zeta;z) + \bar{f}(z;\zeta)) = \frac{1}{2\pi r}\left(\frac{\psi(a;\zeta)+\psi(a;z)}{\psi(a;\zeta)-\psi(a;z)} + \frac{\bar{\psi}(a;\zeta)+\bar{\psi}(a;z)}{\bar{\psi}(a;\zeta)-\bar{\psi}(a;z)}\right) =$$

$$= \frac{1}{2\pi r}\left(\frac{|\psi(a;\zeta)|^2 - |\psi(a;z)|^2}{|\psi(z;\zeta)|^2}\right) = \frac{1}{2\pi r}\left(\frac{r^2 - |\psi(a;z)|^2}{|\psi(z;\zeta)|^2}\right).$$

Formula (4) is called an analogue of the Poisson formula for $A(z)-$harmonic functions.

The Hardy class $H_A^p, p > 0$ for $A(z)$-analytic functions is given in [4]. Initially, we will introduce this class for also $A(z)$-harmonic functions in the case $p > 1$.

Statement 1. $u(z) \in h_A(L(a;r))$ *is said to be in* H_A^p, *if the average integral*

$$\left(\frac{1}{2\pi\rho} \int\limits_{|\psi(z;a)|=\rho} |u(z)||dz + A(z)d\bar{z}|\right)^{\frac{1}{p}} \le T \tag{5}$$

is bounded in lemniscate $L(a;r)$, *where* $T > 0$.

Now, we introduce an angular limit for $A(z)-$analytic functions.

Notation 1. $u(z)$ *is called the (non-tangential or angular* $\sphericalangle$*) boundary value function for* $A(z)-$*harmonic function* $u(z)$, *we frequently write*

$$u(\zeta) = \lim_{\substack{z \to \zeta \\ \sphericalangle}} u(z), \tag{6}$$

$L(a;R)$ *lemniscate is almost everywhere.*

Now we give the following theorem.

Theorem 4. *Let* $u(z) \in H^p(L(a;R)), p > 1$ *and let* $u(z)$ *be* $A(z)-$*harmonic function this lemniscate* $L(a;R)$. *Then, for almost all* $\zeta \in \partial L(a;R)$, $u(z)$ *tends to a finite limit, say* $u(\zeta)$, *as* $\underset{\sphericalangle}{z \to \zeta}$

$$u(z) = \frac{1}{2\pi r} \int\limits_{|\psi(\zeta;a)|=r} u(\zeta) P(\zeta;z) |d\zeta + A(\zeta)d\bar{\zeta}|, \tag{7}$$

where $z \in L(a;R)$.

In future, whenever we have a function $u(z) \in h_A(L(a;R))$, satisfying the hypothesis of the above theorem (for class $H_A^p, p > 1$), we assume it to be automatically extended a. e. to boundary lemniscate $\partial L(a;R)$ in the manner described.

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Estimates for convolution operators

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In this paper we consider the convolution operator M_k with oscillatory kernel given by :

$$M_k = F^{-1}[e^{i\varphi(\xi)}a_k]F,$$

where F is the Fourier transform operator, $\varphi \in C^{\infty}(\mathbb{R}^n\setminus\{0\}), \varphi > 0$, besides φ is a homogeneous function of order one, $a_k \in C^{\infty}(\mathbb{R}^n)$ is a classical symbol of PDO of order $-k$.

Problem. *Let $1 \leq p \leq 2$ be a fixed number. We consider the problem: find a number $k(p)$ such that the onepamop $M_k : L^p(\mathbb{R}^n) \to L^{p'}(\mathbb{R}^n)$ is bounded for $k > k(p)$, where p' is a conjugate exponent, e.g. $1/p + 1/p' = 1$.*

Note that if $a(\xi) = |\xi|^{-k}$ with $0 < k < n$ and $\varphi \equiv 0$ then the problem is reduced to Hardy-Littlewood-Sobolev's inequality. Then if $k = 2n(1/p - 1/2)$ then the operator is bounded from $L^p(\mathbb{R}^n)$ to $L^{p'}(\mathbb{R}^n)$. Moreover, if a is a symbol of PDO and $\varphi \equiv 0$ then we dealt with $L^p \mapsto L^{p'}$ boundedness problem for the pseudo-differential operators. Note that, the oscillation factor gain better estimate for the order k of the symbol a.

It turns out that the number $k(p)$ depends on the geometric properties of the following smooth hypersurface:

$$\Sigma = \{\xi \in \mathbb{R}^n : \varphi(\xi) = 1\}.$$

Further, we use notation:

$$k(p, \Sigma) := \inf_{k>0}\{k > 0 : M_k : L^p(\mathbb{R}^n) \to L^{p'}(\mathbb{R}^n) \quad \text{is bounded}\}.$$

M. Sugimoto [2] consider the problem for the case when $\Sigma \subset \mathbb{R}^3$ is a smooth hypersurface having at least one non-vanishing principal curvature at every point and obtained an upper bound for the number $k(p, \Sigma)$. We consider more general surfaces in $\mathbb{R}^3$ for which both principal curvatures can vanish and obtain an upper bound for $k(p, \Sigma)$ in terms of a height of smooth functions improving the results proved by M. Sugimoto for the case $n = 3$. Moreover, we obtain the exact value of $k(p, \Sigma)$ for some partial classes of surfaces.

Since Σ is a compact hypersurface, following M. Sugimoto it is enough to consider the local version of the problem. More, precisely we will assume that the amplitude function $a_k(\xi)$ is concentrated in a sufficiently small conic neighborhood Γ of a fixed point $v \in S^2$ (where S^2 is a unit sphere centered at the origin of the space $\mathbb{R}^3$) and $\varphi(\xi) \in C^{\infty}(\Gamma)$.

Also, for the sake of being definite we will assume $v = (0, 0, 1)$ and $\varphi(0, 0, 1) = 1$. Thus, in a sufficiently small neighborhood of the point v the hypersurface Σ is given as the graph of a smooth function:

$$\Sigma \cap \Gamma = \{\xi \in \Gamma : \varphi(\xi) = 1\} = \{(\xi_1, \xi_2), \xi_3 = 1 + \phi(\xi_1, \xi_2), (\xi_1, \xi_2) \in U\},$$