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II TOM

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Национальный университет Узбекистана имени Мирзо Улугбека
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СОВРЕМЕННЫЕ МЕТОДЫ МАТЕМАТИЧЕСКОЙ ФИЗИКИ И ИХ ПРИЛОЖЕНИЯ

Республиканская научная конференция посвященная 80-летию со дня рождения академика Ш.А. Алимова
Ташкент, 22–24 апреля, 2025 год

ТЕЗИСЫ ДОКЛАДОВ

ТОМ II

СЕКЦИЯ 5. ТЕОРИЯ ФУНКЦИЙ
SECTION 5. FUNCTION THEORY

1) $P_{ij,k} = 0$ for every triple $(i, j, k) \in \mathbb{N}^3$ with $\min\{i, j\} > k$;

2) $\sum\limits_{l=i}^{k} P_{ij,l} \le \frac{1}{2}$ for every triple $(i, j, k) \in \mathbb{N}^3$ with $j > i$.

References

1. Mukhamedov F.M., Embong A. On b-bistochastic quadratic stochastic operators. // Journal of Inequalities and Applications (2015) 2015:226.

2. Mukhamedov F.M., Embong A. b-bistochastic quadratic stochastic operators and their properties. // Journal of Physics: Conference series, **697** 012010 (2016).

Riesz's Theorem For $A(z)$-Analytic Functions

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Let $A(z)$ be an antianalytic function in the convex domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \le c$ for all $z \in D$, where $c < 1$. The function $f(z)$ is said to be $A(z)$–*analytic* in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z) \frac{\partial f}{\partial z} \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)$–analytic functions defined in the domain D.

According to, the function

$$\psi(a, z) = z - a + \overline{\int\limits_{\gamma(a,z)} \overline{A(\tau)} d\tau}$$

is an $A(z)$–analytic function.

The following set is an open subset of arbitrary convex domain D :

$$L(a, r) = \{|\psi(a, z)| < r\}\,.$$

For sufficiently small $r > 0$, this set compactly lies in D (we denote this fact by $L(a, r) \subset\subset D$) and contains the point a. This set $L(a, r)$ is called the $A(z)$–lemniscate centered at the point a. The lemniscate $L(a, r)$ is a simply - connected set (see [2]).

At first, we introduce the Hardy class for $A(z)$-analytic functions:

Definition 1 (see [3]). *The Hardy class $H^p, p > 0$, for $A(z)$-analytic functions is the set of all functions $f(z)$ such that its averages*

$$\frac{1}{2\pi\rho} \int\limits_{|\psi(a,z)|=\rho} |f(z)|^p \left|dz + A(z) d\bar{z}\right| \tag{2}$$

are uniformly bounded for $\rho < r$, *i.e.* $\sup\limits_{\rho<r}\left\{\frac{1}{2\pi\rho}\int\limits_{|\psi(a,z)|=\rho}|f(z)|^p\left|dz + A(z)d\bar{z}\right|\right\} < \infty\cdot$

The Hardy class for $A(z)$-analytic functions in the domain $L(a,r)$ is denoted as $H_A^p(L(a,r))$.

Now, we define the concepts of angular and radial limits of $A(z)$–analytic functions in lemniscate $L(a,r) \subset\subset D$. The radial limits of the function $f(z)$ at some point $\zeta \in \partial L(a,r)$ is denoted as $f^*(\zeta)$, and the angular limits is denoted as $f_\sphericalangle^*(\zeta)$.

In the classical case of the disk $U = \{|w| < 1\} \subset \mathbb{C}_w$, the limit by the radius $\tau_\zeta = \{w = t\zeta\}, 0 \le t \le 1, \zeta \in \partial U$ of the function $\varphi(w)$,

$$\varphi^*(\zeta) = \lim_{w\to\zeta, w\in\tau_\zeta}\varphi(w)$$

is called the *radial limit*, and the limit by the angle $\sphericalangle \subset U$, leaving the point $\zeta \in \sphericalangle$, is called the *angular limit*,

$$\varphi_\sphericalangle^*(\zeta) = \lim_{w\to\zeta, w\in\sphericalangle_\zeta}\varphi(w)\cdot$$

Since lemniscate $L(a,r)$ is a simply connected domain with a real analytic boundary, then according to Riemann's theorem there exists a conformal map $\chi(z) : U \to L(a,r)$, which is also conformal in some neighborhood of closure $\bar{U}$. Let χ maps the boundary point $\lambda \in \partial U$ to the boundary point $\zeta \in \partial L(a,r)$. Then the curve γ_ζ has the property that it connects points a, ζ and is perpendicular to all lines of level $\partial L(a,\rho) = \{|\psi(a,z)| = \rho\}, 0 < \rho \le r$. In the theory of $A(z)$–analytic functions, the curve $\gamma_\zeta = \chi(\tau_\lambda)$ plays the role of the radial direction, and the image of the angle $\chi(\sphericalangle)$ plays the role of the angular set at the point $\zeta \in \partial L(a,r)$. We will denote this angle by $\sphericalangle = \sphericalangle_\zeta$. The limit $f^*(\zeta) = \lim\limits_{z\to\zeta, z\in\gamma_\zeta} f(z)$ is called the radial limit, and $f_\sphericalangle^*(\zeta) = \lim\limits_{z\to\zeta, z\in\sphericalangle_\zeta} f(z)$ is the angular limit of the function $f(z)$ at the point $\zeta \in \partial L(a,r)$ (see [3]).

Next we will consider the Fatou's theorem for this class:

Theorem 1 (see [3], the Fatou's theorem for the class of functions H_A^1). *If* $f(z) \in H_A^1(L(a,r))$, *then the angular limit*

$$f_\sphericalangle^*(\zeta) = \lim_{z\to\zeta, z\in\sphericalangle_\zeta} f(z)$$

exists and is finite for almost all $\zeta \in \partial L(a,r)$, *except, perhaps, the points of some set* E *of measure zero.*

The following statements follow from Theorem 4:

Theorem 2 (see [3]). *If* $f(z) \in H_A^1(L(a,r))$, *then* $f_\sphericalangle^*(\zeta) \in L_A^1(\partial L(a,r))$. *As* $\rho \to r$

$$\int\limits_{|\psi(a,z)|=\rho} f(z)\left|dz + A(z)d\bar{z}\right| \longrightarrow \int\limits_{|\psi(a,\zeta)|=r} f_\sphericalangle^*(\zeta)\left|d\zeta + A(\zeta)d\bar{\zeta}\right| \tag{3}$$

and

$$\int\limits_{|\psi(a,z)|=\rho} |f(z) - f_\sphericalangle^*(\zeta)|\left|dz + A(z)d\bar{z}\right| \longrightarrow 0\cdot \tag{4}$$

Theorem 3 (Riesz's theorem for $A(z)$-analytic functions). *If $f(z) \in H^p(L(a,r))$, then*

$$\int\limits_{|\psi(a,z)|=\rho} |f(z) - f^*_{\triangleleft}(\zeta)|\big|dz + A(z)d\bar{z}\big| \longrightarrow 0$$

at $\rho \to r$, where $f^*_{\triangleleft}(\zeta)$ - there is almost everywhere an angular value of the function $f(z)$.

Литература

1. Koosis P. Introduction to H^p Spaces. Moscow.: Mir, 1984 (in Russian).
2. Sadullaev A., Zhabborov N. M. On a class of A-analytic functions. Journal of Siberian Federal University. Mathematics-Physics. 2016, V.9, №.3, p.374–383.
3. Zhabborov N. M., Khursanov Sh. Y., Husenov B. E. Existence of boundary values of Hardy class functions H^1_A. Bulleten of NUUz. 2022, V.5., №.2, p. 374–383.

On The Sharp Estimates For Convolution Operators Related To Some Non-Convex Hypersurfaces

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0.1 Introduction

It is well known that solution operator of the Cauchy problem for homogeneous constant coefficient strictly hyperbolic equation, up to a regularizing operator and after scaling of time variable can be written as a sum of convolution operators of the type (see [6], [7]):

$$M_k = F^{-1}[e^{i\varphi(\xi)}a_k]F, \tag{1}$$

where F is the Fourier transform operator, $\varphi \in C^\infty(\mathbb{R}^\nu\setminus\{0\})$ is homogeneous of order one function, $a_k \in C^\infty(\mathbb{R}^\nu_\xi)$ is a homogeneous function of order $-k$ for large ξ.

We demonstrate motivation of the main problem in a simple example. Consider the example of strictly hyperbolic equation of order 4 in the space $\mathbb{R} \times \mathbb{R}^3$, for which the corresponding surface Σ can be written as the graph of a function having A_∞ type singularities (see [2] for definition of A type singularities):

$$\frac{\partial^4 u}{\partial t^4} - 2\frac{\partial^2(\Delta + \partial_3^2)u}{\partial t^2} + \frac{1}{2}\left(\Delta^2 u + \partial_3^4 u\right) = 0, \tag{2}$$

where Δ is the standard Laplace operator in $\mathbb{R}^2$. It is the axisymmetric partial differential equation.