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- 1) $P_{ij,k} = 0$ for every triple $(i, j, k) \in \mathbb{N}^3$ with $\min\{i, j\} > k$;
- 2) $\sum_{l=i}^k P_{ij,l} \leq \frac{1}{2}$ for every triple $(i, j, k) \in \mathbb{N}^3$ with $j > i$.

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Riesz's Theorem For $A(z)$ -Analytic Functions

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Let $A(z)$ be an antianalytic function in the convex domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq c$ for all $z \in D$, where $c < 1$. The function $f(z)$ is said to be $A(z)$ -analytic in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z) \frac{\partial f}{\partial z} \quad (1)$$

We denote by $O_A(D)$ the class of all $A(z)$ -analytic functions defined in the domain D .

According to, the function

$$\psi(a, z) = z - a + \overline{\int_{\gamma(a, z)} A(\tau) d\tau}$$

is an $A(z)$ -analytic function.

The following set is an open subset of arbitrary convex domain D :

$$L(a, r) = \{|\psi(a, z)| < r\}.$$

For sufficiently small $r > 0$, this set compactly lies in D (we denote this fact by $L(a, r) \subset\subset D$) and contains the point a . This set $L(a, r)$ is called the $A(z)$ -lemniscate centered at the point a . The lemniscate $L(a, r)$ is a simply - connected set (see [2]).

At first, we introduce the Hardy class for $A(z)$ -analytic functions:

Definition 1 (see [3]). *The Hardy class $H^p, p > 0$, for $A(z)$ -analytic functions is the set of all functions $f(z)$ such that its averages*

$$\frac{1}{2\pi\rho} \int_{|\psi(a, z)|=\rho} |f(z)|^p |dz + A(z)d\bar{z}| \quad (2)$$

are uniformly bounded for $\rho < r$, i.e. $\sup_{\rho < r} \left\{ \frac{1}{2\pi\rho} \int_{|\psi(a,z)|=\rho} |f(z)|^p |dz + A(z)d\bar{z}| \right\} < \infty$.

The Hardy class for $A(z)$ -analytic functions in the domain $L(a, r)$ is denoted as $H_A^p(L(a, r))$.

Now, we define the concepts of angular and radial limits of $A(z)$ -analytic functions in lemniscate $L(a, r) \subset\subset D$. The radial limits of the function $f(z)$ at some point $\zeta \in \partial L(a, r)$ is denoted as $f^*(\zeta)$, and the angular limits is denoted as $f_{\triangleleft}^*(\zeta)$.

In the classical case of the disk $U = \{|w| < 1\} \subset \mathbb{C}_w$, the limit by the radius $\tau_\zeta = \{w = t\zeta\}, 0 \leq t \leq 1, \zeta \in \partial U$ of the function $\varphi(w)$,

$$\varphi^*(\zeta) = \lim_{w \rightarrow \zeta, w \in \tau_\zeta} \varphi(w)$$

is called the *radial limit*, and the limit by the angle $\triangleleft \subset U$, leaving the point $\zeta \in \triangleleft$, is called the *angular limit*,

$$\varphi_{\triangleleft}^*(\zeta) = \lim_{w \rightarrow \zeta, w \in \triangleleft_\zeta} \varphi(w).$$

Since lemniscate $L(a, r)$ is a simply connected domain with a real analytic boundary, then according to Riemann's theorem there exists a conformal map $\chi(z) : U \rightarrow L(a, r)$, which is also conformal in some neighborhood of closure $\bar{U}$. Let χ maps the boundary point $\lambda \in \partial U$ to the boundary point $\zeta \in \partial L(a, r)$. Then the curve γ_ζ has the property that it connects points a, ζ and is perpendicular to all lines of level $\partial L(a, \rho) = \{|\psi(a, z)| = \rho\}, 0 < \rho \leq r$. In the theory of $A(z)$ -analytic functions, the curve $\gamma_\zeta = \chi(\tau_\lambda)$ plays the role of the radial direction, and the image of the angle $\chi(\triangleleft)$ plays the role of the angular set at the point $\zeta \in \partial L(a, r)$. We will denote this angle by $\triangleleft = \triangleleft_\zeta$. The limit $f^*(\zeta) = \lim_{z \rightarrow \zeta, z \in \gamma_\zeta} f(z)$ is called the radial limit, and $f_{\triangleleft}^*(\zeta) = \lim_{z \rightarrow \zeta, z \in \triangleleft_\zeta} f(z)$ is the angular limit of the function $f(z)$ at the point $\zeta \in \partial L(a, r)$ (see [3]).

Next we will consider the Fatou's theorem for this class:

Theorem 1 (see [3], the Fatou's theorem for the class of functions H_A^1). *If $f(z) \in H_A^1(L(a, r))$, then the angular limit*

$$f_{\triangleleft}^*(\zeta) = \lim_{z \rightarrow \zeta, z \in \triangleleft_\zeta} f(z)$$

exists and is finite for almost all $\zeta \in \partial L(a, r)$, except, perhaps, the points of some set E of measure zero.

The following statements follow from Theorem 4:

Theorem 2 (see [3]). *If $f(z) \in H_A^1(L(a, r))$, then $f_{\triangleleft}^*(\zeta) \in L_A^1(\partial L(a, r))$. As $\rho \rightarrow r$*

$$\int_{|\psi(a,z)|=\rho} f(z) |dz + A(z)d\bar{z}| \longrightarrow \int_{|\psi(a,\zeta)|=r} f_{\triangleleft}^*(\zeta) |d\zeta + A(\zeta)d\bar{\zeta}| \quad (3)$$

and

$$\int_{|\psi(a,z)|=\rho} |f(z) - f_{\triangleleft}^*(\zeta)| |dz + A(z)d\bar{z}| \longrightarrow 0. \quad (4)$$

Theorem 3 (Riesz's theorem for $A(z)$ -analytic functions). *If $f(z) \in H^p(L(a, r))$, then*

$$\int_{|\psi(a, z)|=\rho} |f(z) - f_{\triangleleft}^*(\zeta)| |dz + A(z)d\bar{z}| \rightarrow 0$$

at $\rho \rightarrow r$, where $f_{\triangleleft}^(\zeta)$ - there is almost everywhere an angular value of the function $f(z)$.*

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On The Sharp Estimates For Convolution Operators Related To Some Non-Convex Hypersurfaces

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0.1 Introduction

It is well known that solution operator of the Cauchy problem for homogeneous constant coefficient strictly hyperbolic equation, up to a regularizing operator and after scaling of time variable can be written as a sum of convolution operators of the type (see [6], [7]):

$$M_k = F^{-1}[e^{i\varphi(\xi)}a_k]F, \quad (1)$$

where F is the Fourier transform operator, $\varphi \in C^\infty(\mathbb{R}^\nu \setminus \{0\})$ is homogeneous of order one function, $a_k \in C^\infty(\mathbb{R}_\xi^\nu)$ is a homogeneous function of order $-k$ for large ξ .

We demonstrate motivation of the main problem in a simple example. Consider the example of strictly hyperbolic equation of order 4 in the space $\mathbb{R} \times \mathbb{R}^3$, for which the corresponding surface Σ can be written as the graph of a function having A_∞ type singularities (see [2] for definition of A type singularities):

$$\frac{\partial^4 u}{\partial t^4} - 2 \frac{\partial^2(\Delta + \partial_3^2)u}{\partial t^2} + \frac{1}{2}(\Delta^2 u + \partial_3^4 u) = 0, \quad (2)$$

where Δ is the standard Laplace operator in $\mathbb{R}^2$. It is the axisymmetric partial differential equation.