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## ⋄ MUNDARIJA ⋄ CONTENTS ⋄ СОДЕРЖАНИЕ ⋄

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# RIESZ'S THEOREM FOR $A(z)$-ANALYTIC FUNCTIONS

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## RESUME

We consider $A(z)$-analytic functions in case when $A(z)$ is antianalytic function. In this paper, an analogue of theorem Riesz's is proved for $A(z)$-analytic functions from the Hardy class.

***Key words:*** $A(z)$-analytic function, Hardy class, $A(z)$-lemniscate, Taylor series, Blaschke product, Riesz's theorem for $A(z)$-analytic functions.

## Introduction

$A(z)$**-analytic functions.** Let $A(z)$ be antianalytic function, i.e. $\frac{\partial A}{\partial z} = 0$ in the domain $D \subset \mathbb{C}$ and there is a constant $c < 1$ such that $|A(z)| \leq c$ for all $z \in D$. The function $f(z)$ is said to be $A(z)$-*analytic* in the domain $D$ if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z}. \tag{1}$$

We denote by $O_A(D)$ the class of all $A(z)$-analytic functions defined in the domain $D$. Since an antianalytic function is infitely smooth, then $O_A(D) \subset C^\infty(D)$ (see [2]). In this case, the following takes place:

**Theorem 1.** (see [3], analogue of Cauchy integral theorem). *If $f \in O_A(D) \cap C\left(\bar{D}\right)$, where $D \subset \mathbb{C}$ is a domain with smooth $\partial D$, then*

$$\int\limits_{\partial D} f(z)(dz + A(z)d\bar{z}) = 0\cdot$$

Now we assume that the domain $D \subset \mathbb{C}$ is convex, and $\xi \in D$ is a fixed point in it. Since the function $\overline{A}(z)$ is analytic, the integral

$$I(z) = \overline{\int\limits_{\gamma(\xi,z)} \overline{A(\tau)}d\tau}$$

is independent of the path of integration; it coincides with the antiderivative $I'(z) = \overline{A}(z)$. Consider the function

$$K(z,\xi) = \frac{1}{2\pi i}\frac{1}{z - \xi + I(z)},$$

where $\gamma(\xi, z)$ is a smooth curve which connects the points $\xi, z \in D$ [5].

**Theorem 2.** (see [5]). *$K(z,\xi)$ is an $A(z)$-analytic function outside of the point $z=\xi$, i.e. $K(z,\xi) \in O_A(D\backslash\{\xi\})$. Moreover, at $z=\xi$ the function $K(z,\xi)$ has a simple pole.*

**Remark 1.** (see [5]). *If a simply connected domain $D \subset \mathbb{C}$ is not convex, then the function*

$$\psi(\xi, z) = z - \xi + I(z),$$

*although well defined in $D$, may have other isolated zeros except $\xi : \psi(\xi, z) = 0$ for except $z \in P\backslash\{\xi, \xi_1, \xi_2, ...\}$. Consequently, $\psi \in O_A(D)$, $\psi(\xi, z) \neq 0$ when $z \notin P$ and $K(z, \xi)$ is an $A(z)$-analytic function only in $D\backslash P$, it has poles at the points of $P$. Due to this fact we consider the class of $A(z)$-analytic functions only in convex domains.*

According to [5], Theorem 1.2, the function $\psi(\xi, z)$ is an $A(z)$-analytic function.

The following set is an open subset of $D$:

$$L(a,r) = \{z \in D : |\psi(a,z)| < r\}.$$

For suffiently small $r > 0$, this set compactly lies in $D$ (we denote it by $L(a,r) \subset\subset D$) and contains the point $a$. The set $L(a,r)$ is called an $A(z)$-*lemniscate* centered at the point $a$. The lemniscate $L(a,r)$ is a simply-connected set (see [5]).

**Theorem 3.** (see [4], Cauchy's integral formula). *Let $D \subset \mathbb{C}$ be a convex domain and $G \subset\subset D$ be an arbitrary subdomain with a smooth or piecewise smooth $\partial G$. Then for any function $f(z) \in O_A(G) \bigcap C(\bar{G})$, the following formula holds:*

$$f(z) = \int_{\partial G} f(\xi) K(z,\xi) \left(d\xi + A(\xi) d\bar{\xi}\right), \quad z \in G. \tag{2}$$

Note that from formula (2) it follows that if $f(z) \in O_A(L(a,r)) \bigcap C\left(\bar{L}(a,r)\right)$, where $L(a,r) \subset\subset D$ is a fixed $A(z)$-lemniscate, then in $L(a,r)$ the function $f(z)$ is expanded in a Taylor series:

$$f(z) = \sum_{k=0}^{\infty} c_k \psi^k(a,z), \tag{3}$$

where $c_k = \frac{1}{2\pi i} \int\limits_{|\psi(a,\xi)|=\rho} \frac{f(\xi)}{(\psi(a,\xi))^{k+1}} \left(d\xi + A(\xi) d\bar{\xi}\right), 0 < \rho < r, k = 0, 1, 2, \ldots$ .

**Angular limits and Hardy classes for $A(z)$-analytic functions.** Let $L(a,r) \subset\subset D$ and $f(z) \in O_A(L(a,r))$. We define the concepts of angular and radial limits of $A(z)$-subharmonic and $A(z)$-analytic functions in lemniscate $L(a,r)$. The radial limits of the function $f(z)$ at some point $\zeta \in \partial L(a,r)$ are denoted as $f^*(\zeta)$ and the angular limits are denoted as $f^*_{\sphericalangle}(\zeta)$ (see [6]).

In the classical case of the disk $U = \{w \in \mathbb{C} : |w| < 1\} \subset \mathbb{C}_w$, the limit by the radius $\tau_\zeta = \{w = t\zeta\}$, $0 \le t \le 1$, $\zeta \in \partial U$ of the function $g(w)$,

$$g^*(\zeta) = \lim_{w \to \zeta, w \in \tau_\zeta} g(w)$$

is called the radial limit, and the limit by the angle $\sphericalangle \subset U$, ending at the point $\zeta \in \sphericalangle$, is called the angular limit,

$$g^*_{\sphericalangle}(\zeta) = \lim_{w \to \zeta, w \in \sphericalangle_\zeta} g(w).$$

Since lemniscate $L(a, r)$ is a simply connected domain with a real analytic boundary, then according to Riemann's theorem there exists a conformal map $\chi(z) : U \to L(a, r)$, which is also conformal in some neighborhood of closure $\bar{U}$. Let $\chi$ maps the boundary point $\lambda \in \partial U$ to the boundary point $\zeta \in \partial L(a, r)$. Then the curve $\gamma_\zeta = \chi(\tau_\lambda)$ has the property that it connects points $a, \zeta$ and is perpendicular to all lines of level $\partial L(a, \rho) = \{|\psi(a, z)| = \rho\}, 0 < \rho \le r$. In the theory of $A(z)$–analytic functions, the curve $\gamma_\zeta = \chi(\tau_\lambda)$ plays the role of the radial direction, and the image of the angle $\chi(\sphericalangle)$ plays the role of the angular set at the point $\zeta \in \partial L(a, r)$. We will denote this angle by $\sphericalangle = \sphericalangle_\zeta$. The limit $f^*(\zeta) = \lim\limits_{z\to\zeta, z\in\gamma_\zeta} f(z)$ is called the radial limit, and $f^*_\sphericalangle(\zeta) = \lim\limits_{z\to\zeta, z\in\sphericalangle_\zeta} f(z)$ is the angular limit of the function $f(z)$ at the point $\zeta \in \partial L(a, r)$ (see [6]).

Now we will show the smoothness of the boundary of lemniscate $L(a, r)$. For this, we take automorphism $\chi^{-1}(z) : \overline{L}(a, r) \to \overline{U}$ by Riemann's theorem. Let there be some neighborhood $V = \{\psi(a, \zeta) = re^{i\theta}, |\theta| < \varepsilon\}$ for $\forall\varepsilon > 0$. Also has $\chi^{-1}(V) \subset \partial U$ and $\chi^{-1}(\zeta_0) = \lambda_0 \in \partial U$. Further, there is a diffeomorphism $\pi = -i \ln \chi^{-1}(\zeta) : V \to [-1; 1]$. This diffeomorphism represents all boundary points of the differentiability of the function $f^*(\zeta)$ and $f^*_\sphericalangle(\zeta)$ (see [8]).

Next we introduce the Hardy class for $A(z)$-analytic functions:

**Definition 1.** (see [6]). *The Hardy class $H^p, p > 0$, for $A(z)$-analytic functions is the set of all functions $f(z)$ such that its averages*

$$\frac{1}{2\pi\rho} \int\limits_{|\psi(a,z)|=\rho} |f(z)|^p \left|dz + A(z)d\bar{z}\right| \tag{4}$$

*are uniformly bounded for $\rho < r$, i.e.* $\sup\limits_{\rho<r} \left\{ \frac{1}{2\pi\rho} \int\limits_{|\psi(a,z)|=\rho} |f(z)|^p \left|dz + A(z)d\bar{z}\right| \right\} < \infty$.

The Hardy class for $A(z)$-analytic functions in the domain $L(a, r)$ is denoted as $H^p_A(L(a, r))$. The norms in them are defined by the formula (see [6]):

$$\| f \|_{H^p_A} = \sup_{|\psi(a,z)|<r} \left( \frac{1}{2\pi\rho} \int\limits_{|\psi(a,z)|=\rho} |f(z)|^p \left|dz + A(z)d\bar{z}\right| \right)^{\frac{1}{p}} < \infty.$$

Further, from the inequality $b^q < b^p + 1, \quad 0 < q < p, \quad b \ge 0$ we conclude that $f \in H^p_A$ follows $f \in H^q_A$, i.e. $H^p_A \subset H^q_A$ for all $p$ and $q$. Let us define a class of bounded functions

$$H^\infty_A(L(a, r)) = \left\{ f(z) \in O_A(L(a, r)) : \sup_{|\psi(a,z)|<r} \{|f(z)|\} < \infty \right\}.$$

The norm in $H^\infty_A(L(a, r))$ is defined as $\| f(z) \|_{H^\infty_A} = \sup\limits_{z\in L(a,r)} \{|f(z)|\}$ (see [6]).

**The Fatou's theorems and Cauchy's integral formula for Hardy class $H^1_A$.**

Now, we will consider the Fatou's theorem for the class of functions $H^1_A$ :

**Theorem 4.** (see [6], the Fatou's theorem for the class of functions $H^1_A$). *If $f(z) \in H^1_A(L(a, r))$, then the angular limit*

$$f^*_\sphericalangle(\zeta) = \lim_{z\to\zeta, z\in\sphericalangle_\zeta} f(z)$$

*exists and is finite for almost all $\zeta \in \partial L(a,r)$, except, perhaps, the points of some set $E$ of measure zero.*

The following statements follow from Theorem 4:

**Theorem 5.** (see [6]). *If $f(z) \in H_A^1(L(a,r))$, then $f^*(\zeta) \in L_A^1(\partial L(a,r))$. As $\rho \to r$*

$$\int\limits_{|\psi(a,z)|=\rho} f(z)\left|dz + A(z)d\bar{z}\right| \longrightarrow \int\limits_{|\psi(a,\zeta)|=r} f^*(\zeta)\left|d\zeta + A(\zeta)d\bar{\zeta}\right| \tag{5}$$

*and*

$$\int\limits_{|\psi(a,z)|=\rho} |f(z) - f^*(\zeta)|\left|dz + A(z)d\bar{z}\right| \longrightarrow 0\cdot \tag{6}$$

According to Cauchy integral formula (2) for lemniscates $L(a,r)$

$$f(z) = \frac{1}{2\pi i} \int\limits_{|\psi(a,\xi)|=\rho} f(\xi)K(\xi,z)\left(d\xi + A(\xi)d\bar{\xi}\right),$$

we conclude that

$$f(z) = \frac{1}{2\pi i} \int\limits_{|\psi(a,\zeta)|=r} f^*(\zeta)K(\zeta,z)\left(d\zeta + A(\zeta)d\bar{\zeta}\right)\cdot \tag{7}$$

This is the Cauchy integral formula for functions of $H_A^1$.

Let the function $f(z) \in O_A(L(a,r))$ and $a_1, a_2, a_3, ...$ be the zeros of the function in this domain, $r_n = |\psi(a,a_n)|$. If

$$M = \sup_{0<\rho<r}\left\{\frac{1}{2\pi\rho}\int\limits_{\partial L(a,\rho)} |f(z)|\,|dz + A(z)d\bar{z}|\right\}$$

then

$$\sum_{n=1}^{\infty}(r - |\psi(a,a_n)|)$$

is bounded and the Blaschke product

$$B(z) = \sum_{n=1}^{\infty} r \cdot \frac{|\psi(a,a_n)|}{\psi(a,a_n)}\frac{\psi(a,a_n) - \psi(a,z)}{r^2 - \bar{\psi}(a,a_n)\psi(a,z)}$$

is $A(z)$-analytic in $L(a,r)$, $f(z) = B(z)G(z)$, where the function $G(z)$ is $A(z)$-analytic and has no zeros at $|\psi(a,z)| < r$ (see [7]).

Also, from formula (7) it follows that if $f(z) \in H_A^1(L(a,r))$, then there exists a Blaschke product $B(z)$ and a function $G(z) \in H_A^1(L(a,r))$ that has no zeros in $L(a,r)$, such that $f(z) = B(z)G(z)$.

Therefore, for almost all $\zeta \in \partial L(a,r)$, the limit function $B_{\triangleleft}^*(\zeta)$ at $z \to \zeta_{\triangleleft}$ exists from Theorem 4.

## Main result

**Theorem 6.** (Riesz's theorem for $A(z)$-analytic functions). *If $f(z) \in H^p(L(a,r))$, then*

$$\int\limits_{|\psi(a,z)|=\rho} |f(z) - f^*(\zeta)|\big|dz + A(z)d\bar{z}\big| \longrightarrow 0$$

*at $\rho \to r$, where $f^*(\zeta)$ - there is almost everywhere an angular value of the function $f(z)$.*

**Proof.** If $B(z)$ is the Blaschke product constructed from the zeros of the function $f(z)$, then $f(z) = B(z)G(z)$, where $G(z) \in H^p$ and $G(z)$ have no zeros in $L(a,r)$. For $\rho < r$ and $0 < p < 1$

$$\int\limits_{|\psi(a,z)|=\rho} \big|f(z) - f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big| \leq \int\limits_{|\psi(a,z)|=\rho} \big|B(z)\big|^p\big|f(z) - f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big| +$$

$$+ \int\limits_{|\psi(a,z)|=\rho} \big|B(z) - B^*(\zeta)\big|^p\big|f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big| \leq$$

$$\leq \int\limits_{|\psi(a,z)|=\rho} \big|f(z) - f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big| + \int\limits_{|\psi(a,z)|=\rho} \big|B(z) - B^*(\zeta)\big|^p\big|f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big|\cdot$$

For $p \geq 1$, similar inequalities hold, with the only difference being that the $p^{\text{nd}}$ degree roots are extracted from the integrals.

Now, from the fact that $|B(z)| \leq 1, B(z) \to B^*(\zeta)$ is almost everywhere for $\rho \to r$ and $|G^*(\zeta)|^p \in L^1_A$, it follows by Lebesgue's theorem on majorized convergence that

$$\int\limits_{|\psi(a,z)|=\rho} \big|B(z) - B^*(\zeta)\big|^p\big|f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big| \to 0 \ \text{ is for } \ \rho \to r\cdot$$

So we see that it is enough to show the following fact:

$$\int\limits_{|\psi(a,z)|=\rho} \big|f(z) - f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big| \to 0 \ \text{ at } \ \rho \to r$$

for functions $G(z) \in H^p$ that have no zeros in $L(a,r)$.

If $p \geq 1$, then this is simple, since according to the previous subsection

$$G(z) = \frac{1}{2\pi\rho} \int\limits_{|\psi(a,\zeta)|=r} P(z,\zeta)G^*(\zeta)\big|dz + A(z)d\bar{z}\big|,$$

because it is obviously $G(z) \in H^1_A(L(a,r))$, where $P(z,\zeta) = \frac{r^2 - |\psi(a,z)|^2}{|\psi(\zeta,z)|^2}$ is the Poisson kernel. By Theorem 5 the function is $G^*(\zeta) \in L^p_A(L(a,r))$ if $G(z) \in H^p_A(L(a,r))$; therefore, since the kernel $P(z,\zeta)$ is an approximate unit,

$$\int\limits_{|\psi(a,z)|=\rho} \big|f(z) - f^*(\zeta)\big|^p\big|dz + A(z)d\bar{z}\big| \to 0,$$

if $\rho \to r$.

Let's assume $p \geq \frac{1}{2}$. Using Zygmund's clever trick, let us set $h(z) = \sqrt{G(z)}$; the square root in this case is well defined, since the function $G(z)$ never vanishes $L(a,r)$. Then $h(z) \in H_A^{2p}(L(a,r))$ and $2p \geq 1$. We have

$$\int\limits_{|\psi(a,z)|=\rho} \left|G(z) - G^*(\zeta)\right|^p \left|dz + A(z)d\bar{z}\right| = \int\limits_{|\psi(a,z)|=\rho} \left|\left(h(z) - h^*(\zeta)\right)\left(h(z) + h^*(\zeta)\right)\right|^p \left|dz + A(z)d\bar{z}\right|,$$

which, according to the Schwarz inequality, does not exceed

$$\sqrt{\int\limits_{|\psi(a,z)|=\rho} \left|h(z) - h^*(\zeta)\right|^{2p} \left|dz + A(z)d\bar{z}\right|} \cdot \sqrt{\int\limits_{|\psi(a,z)|=\rho} \left|h(z) + h^*(\zeta)\right|^{2p} \left|dz + A(z)d\bar{z}\right|} \leq$$

$$\leq C' \sqrt{\int\limits_{|\psi(a,z)|=\rho} \left|h(z) - h^*(\zeta)\right|^{2p} \left|dz + A(z)d\bar{z}\right|},$$

where $C'$ does not depend on $\rho$, since $h(z) \in H_A^{2p}(L(a,r))$. But due to the fact that $2p \geq 1$, and what was proved above, the last expression tends to zero at $\rho \to r$. Therefore, it is proved that the required result is valid for $p \geq \frac{1}{2}$.

If now $p \geq \frac{1}{4}$, then, again introducing the auxiliary function $h = \sqrt{G}$, we will reason as above, using the result just proven. $\rhd$

First of all, we note that the proof in case $p = 2$ is elementary. Indeed, if the function $f(z) = \sum\limits_{n=0}^{\infty} c_n \psi^n(a,z)$ belongs to the space $H_A^2$, then

$$\int\limits_{|\psi(a,z)|=\rho} |f(z)|^2 \left|dz + A(z)d\bar{z}\right| = \pi\rho \sum_{n=0}^{\infty} |c_n|^2 \rho^{2n},$$

due to absolute convergence and orthogonality, therefore $\sum\limits_{n=0}^{\infty} |c_n|^2$ is bounded.

According to the Riesz-Fischer theorem, there exists a function $f^*(\zeta) \in L_A^2(\partial L(a,r))$ with a Fourier series $\sum\limits_{n=0}^{\infty} c_n \psi^n(a,z) = \sum\limits_{n=0}^{\infty} c_n' e^{in\theta}$, where $\psi(a,z) = \rho e^{i\theta}$. Now, by direct calculation it is easy to check that for $\rho < r$

$$f(z) = \frac{1}{2\pi r} \int\limits_{|\psi(a,\zeta)|=r} P(z,\zeta) f^*(\zeta) \left|dz + A(z)d\bar{z}\right|$$

(using series expansion), so that in fact

$$f(z) \to f^*(\zeta) \ \text{ almost everywhere at } \ z \to \zeta_{\sphericalangle}$$

and

$$\int\limits_{|\psi(a,z)|=\rho} \left|f(z) - f^*(\zeta)\right|^2 \left|dz + A(z)d\bar{z}\right| \to 0$$

at $\rho \to r$.

Using this result we can prove the theorem of F. and M. Riesz as follows. Let's take any measure $\mu(\zeta)$ by $\partial L(a,r)$, such that

$$\int\limits_{|\psi(a,\zeta)|=r} \psi^n(a,\zeta)|d\mu(\zeta)| = 0, \quad n \in \mathbb{N},$$

and put

$$f(z) = \frac{1}{2\pi r} \int\limits_{|\psi(a,\zeta)|=r} P(z,\zeta)\big|d\mu(\zeta)\big|$$

for $0 \leq \rho < r$. As is easy to see,

$$f(z) = \sum_{n=0}^{\infty} c_n \psi^n(a,z) = \sum_{n=0}^{\infty} c'_n e^{in\theta},$$

where $c_n = \frac{1}{2\pi r} \int\limits_{|\psi(a,z)|=\rho} \psi^n(a,z)\,(dz + A(z)d\bar{z}) = \frac{1}{2\pi} \int\limits_0^{2\pi} e^{-in\theta} d\mu(\theta) = c'_n$; therefore, the function is $f(z) \in O_A(L(a,r))$ and according to the previous subsection, $f(z) \in H^1_A(L(a,r))$. As at the beginning of the proof of the theorem, we can represent the function $G(z)$ as $f(z) = B(z)G(z)$, where $B(z)$ is the Blaschke product, and $h(z) \in H^1_A(L(a,r))$, and the function $h(z)$ has no zeros in $L(a,r)$. Therefore, there exists an $A(z)$-analytic function in $L(a,r)$ such that $h(z) = f^2(z)$; it is almost obvious that $f(z) \in H^2_A(L(a,r))$. Now we use a special case of the theorem just proven: let's put $G^*(\zeta) = B^*(\zeta)(f^*(\zeta))^2$; then $G^*(\zeta) \in L^1_A(\partial L(a,r))$. We have

$$\int\limits_{|\psi(a,z)|=\rho} \big|f(z) - f^*(\zeta)\big|\big|dz + A(z)d\bar{z}\big| \leq \int\limits_{|\psi(a,z)|=\rho} \big|B(z) - B^*(\zeta)\big|\big|f^*(\zeta)\big|^2\big|dz + A(z)d\bar{z}\big| +$$

$$+ \int\limits_{|\psi(a,z)|=\rho} \big|B(z)\big|\big|f^2(z) - (f^*(\zeta))^2\big|\big|dz + A(z)d\bar{z}\big|.$$

At $\rho \to r$, the first integral on the right-hand side tends to zero by Lebesgue's theorem on dominated convergence. The second does not exceed

$$\int\limits_{|\psi(a,z)|=\rho} \big|f^2(z) - (f^*(\zeta))^2\big|\big|dz + A(z)d\bar{z}\big|,$$

and this last value tends to zero at $\rho \to r$ (we reason in the same way as when using Zigmund's method, taking into account the already proven fact that $\int\limits_{|\psi(a,z)|=\rho} \big|f(z) - f^*(\zeta)\big|^2\big|dz + A(z)d\bar{z}\big| \to 0$ at $\rho \to r$). Finally,

$$\int\limits_{|\psi(a,z)|=\rho} \big|G(z) - G^*(\zeta)\big|^2\big|dz + A(z)d\bar{z}\big| \to 0, \quad \text{at} \quad \rho \to r.$$

But according to the previous subsection at $\rho \to r$

$$G(z)\,(dz + A(z)d\bar{z}) \to d\mu(z).$$

Consequently, $d\mu(\zeta) = G^*(\zeta)\left(d\zeta + A(\zeta)d\bar{\zeta}\right)$ and the measure $\mu$ are absolutely continuous, which proves the theorem of F. and M. Riesz.

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## REZYUME


Ushbu maqolada $A(z)$ funksiyani antianalitik bo'lgan holda $A(z)$-analitik funksiyalarni qaraymiz. Ushbu maqolada biz Xardi sinfidagi $A(z)$-analitik funksiyalar uchun Riesz teoremasining analogini isbotlaymiz.



***Kalit so'zlar:*** $A(z)$-analitik funksiya, Xardi sinfi, $A(z)$-lemniskata, Teylor qatori, Blyashke ko'paytmasi, $A(z)$-analitik funksiyalar uchun Riss teoremasi.


## РЕЗЮМЕ


Мы рассматриваются $A(z)$-аналитические функции в случае, когда $A(z)$ - антианалитическая функция. В данной работе доказывается аналог теоремы Рисса для $A(z)$-аналитических функций из класса Харди.



***Ключевые слова:*** $A(z)$-аналитическая функция, класс Харди, $A(z)$-лемниската, ряд Тейлора, произведение Бляшке, теорема Рисса для $A(z)$-аналитических функций.