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CONSTRUCTION OF A QUENCHING $A(Z)-$ANALYTIC FUNCTION

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We consider $A(z)-$analytic functions in case when $A(z)$ is anti-analytic function. In this article for the Hardy class of H_A^1 functions integral formula of Cauchy are given. Here is constructed a quenching $A(z)-$analytic function.

Keywords: $A(z)-$analytic function

Hardy class for $A(z)-$analytic function, Cauchy formula for class H_A^1, Green function in $A(z)-$lemniscate, quenching $A(z)-$analytic function, $A(z)-$harmonic measure.

Let $A(z)$ be an antianalytic function, i. e. $\frac{\partial A}{\partial z} = 0$ in the convex domain $D \subset \mathbb{C}$; moreover, let $|A(z)| \leq C < 1$ for all $z \in D$. The function $f(z)$ is said to be $A(z)-$analytic in the domain D if for any $z \in D$, the following equality holds:

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z} \qquad (1)$$

We denote by $O_A(D)$ the class of all $A(z)-$analytic functions defined in the domain D.

According to, the function

$$\psi\left(z;a\right) = z - a + \overline{\int\limits_{\gamma(a;z)} \overline{A(\tau)}d\tau}$$

is an $A(z)-$analytic function.

The following set is an open subset of arbitrary convex domain $D:$

$$L\left(a;r\right) = \left\{ \left|\psi\left(z;a\right)\right| = \left|z - a + \overline{\int\limits_{\gamma(a;z)} \overline{A(\tau)}d\tau}\right| < r \right\}.$$

For sufficiently small $r > 0$, this set compactly lies in D (we denote this fact by $L(a;r) \subset\subset D$) and contains the point a. This set $L(a;r)$ is called the

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$A(z)-$lemniscate centered at the point a. The lemniscate $L(a;r)$ is a simply - connected set (see [3]).

Now we assume that the domain $D \subset \mathbb{C}$ is convex, and $\zeta \in D$ is a fixed point in it. Consider the function

$$K(z;\zeta) = \frac{1}{2\pi i} \frac{1}{z - \zeta + \overline{\int\limits_{\gamma(\zeta;z)} \overline{A(\tau)} d\tau}}, \qquad (2)$$

where $\gamma(\zeta;z)$ is a smooth curve which points of $\zeta; z \in D$. Since the domain is simply connected and the function $\overline{A}(z)$ is holomorphic, the integral

$$I(z) = \overline{\int\limits_{\gamma(a;z)} \overline{A(\tau)} d\tau}$$

does not depend on a path of integration; it coincides with a primitive, i.e. $I'(z) = \overline{A}(z)$.

Let $f = u + iv$.

Theorem 1. (see [4]). *The real part of the $A(z)-$analytic functions of $f(z) \in O_A(D)$ satisfies equation*

$$\Delta_A u = \frac{\partial}{\partial z}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial \bar z} - 2A\frac{\partial u}{\partial z}\right)\right) +$$
$$+\frac{\partial}{\partial \bar z}\left(\frac{1}{1-|A|^2}\left((1+|A|^2)\frac{\partial u}{\partial z} - 2\bar A\frac{\partial u}{\partial \bar z}\right)\right) = 0 \qquad (3)$$

in the domain of D.

In connection with Theorem 1, it is natural to define the $A(z)$-harmonic function as follows.

Definition 1 (see [4]). *A double differentiable function $u \in C^2(D)$, $u : D \to R^1$ is called $A(z)$-harmonic in the D domain if the D domain if it satisfies the differential equation (3).*

The class of $A(z)$ - harmonic functions in the domain of D is denoted as $h_A(D)$. Thus, the real part and hence the imaginary part, of the $A(z)$-harmonic function in the domain of D. The inverse theorem is also true for simply connected domains.

Theorem 2. (see [4]). *If the function is $u(z) \in h_A(D)$, where D is a simply connected domain, then $f \in O_A(D) : u = Re\, f$.*

The Hardy class $H_A^p, p > 0$ for $A(z)$-analytic functions is given in [5]. Before we will introduce this class for $A(z)$-analytic functions in the case $p = 1$.

Definition 2. *$f(z) \in O_A(L(a;r))$ is said to be in H_A^1, if*

$$\frac{1}{2\pi\rho} \int\limits_{|\psi(z;a)|=\rho} |f(z)||dz + A(z)d\bar{z}| \tag{4}$$

is bounded in lemniscate $L(a;r)$, where $\rho < r, z \in L(a;r)$.

Definition 3. *H_A^∞ denotes the lemniscate of functions $A(z)-$analytic and bounded in $L(a;r)$. If $f \in H_A^\infty$ we put*

$$||f||_{H_A^\infty} = \sup_{|\psi(z;a)|=r} |f(z)|. \tag{5}$$

Now we give the Cauchy integral formula for the class of functions H_A^1.

Theorem 3. *For the functions $f(z)$ from the Hardy class $H_A^1(L(a;R))$ the Cauchy formula*

$$f(z) = \int\limits_{\partial L(a;R)} K(\zeta;z)f(\zeta)(d\zeta + A(\zeta)d\bar{\zeta}), \quad z \in L(a;R), \tag{6}$$

is valid.

On the boundary $\partial L(a;r)$, consider the measurable set M of the positive Lebesgue measure. The task of restoring $f(z)$ in lemniscate $L(a;r)$ by the boundary values is not the entire boundary of $\partial L(a;r)$, as in (6), but only by $M \subset \partial L(a;r)$. For this purpose, it is necessary to construct an auxiliary function $\varphi(z) \in H_A^\infty(L(a;r))$ that satisfies two conditions:

1) $|\varphi(\zeta)| = 1$ almost everywhere on $\partial L(a;r)\backslash M$,

2) $|\varphi(z)| > 1$ at $L(a;r)$.

This can be done by solving a suitable Dirichlet problem, for example, by considering the function

$$u(x;y) = \frac{1}{2\pi} \int\limits_M \frac{\partial G}{\partial n} ds, \tag{7}$$

where s is the arc $\partial L(a;r)$, and G is the Green function for the lemniscate $L(a;r)$. Green's function was constructed in [6] for $L(a;r)$ lemniscates. In this case, $L(a;r)$ for lemniscate, this function represents as:

$$G(z;\zeta) = \frac{1}{2\pi} \ln \left| \frac{r(\psi(z;a) - \psi(\zeta;a))}{r^2 - \psi(z;a)\bar{\psi}(\zeta;a)} \right|.$$

This analogue of the Green's function gives a solution to the Dirichlet problem for $A(z)$-harmonic function [4] through the relation (7) for $L(a;r)$ lemniscate, such as the Poisson kernel. Formula (7) represents $A(z)$-harmonic and bounded in $L(a;r)$ function $u(x;y) = u(x+iy) = u(z)$ such that

$$u(z) = \begin{cases} 1, & \text{almost everywhere on } M, \\ 0, & \text{almost everywhere on } \partial L(a;r) \backslash M, \end{cases}$$

i. e. $A(z)-$harmonic measure M with respect to the lemniscate $L(a;r)$. In this case, regularization

$$u(z) = \varpi_A^*(z, M, L(a;r)) = \overline{\lim_{w\to z}} \varpi_A(w, M, L(a;r))$$

is represented by an $A(z)-$harmonic measure of the set M, where

$$\varpi_A(z, M, L(a;r)) = \sup_{u\in U_A} u(z)$$

and such a class is a function

$$U_A = U(M, L(a;r)) = u \in sh_A(L(a;r)) : u|_M \leq -1, u|_{L(a;r)} \leq 0.$$

Let v be a $A(z)-$harmonic function conjugate to u, then $\varphi(z) = \exp(u+iv)$ satisfies the specified conditions 1) and 2).

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