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A Mathematical Model of Corruption Dynamics with Prevention Parameters

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Abstract—A mathematical model is developed to analyze the dynamics of corruption by identifying significant parameters. The model allows us to simulate and observe changes in outcomes when these parameters are varied using Python. The model is proved to be both epidemiologically and mathematically well posed. We showed that all solutions of the model are positive and bounded with initial conditions in a certain meaningful set. The existence of unique corruption free and endemic equilibrium points are investigated and the basic reproduction number is computed. Then, we study the local asymptotic stability of these equilibrium points. The analysis shows that the system has a locally asymptotically stable corruption-free equilibrium point when the reproduction number is less than one and locally asymptotically stable endemic equilibrium point when the reproduction number is greater than one. The simulation result shows the agreement with the analytical results.

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1. INTRODUCTION

Corruption is derived from the Latin word “corruptus” which means “to disturb or harm” Corruption is defined as an illegal activity committed for personal gain and benefit through the abuse of power by private [1]. Moreover, International Transparency defines corruption as “the misuse of entrusted power for personal gain” [2]. Corruption can come from either the supply or demand side. It is a major issue in all countries, but particularly in developing countries. Indeed, the majority of countries have anti-corruption strategies in place, corruption still remains a societal epidemic [3].

Abdulrahman [4] developed a deterministic mathematical model of corruption transmission dynamics as a disease. The basic reproduction number, corruption-free and endemic equilibrium point was determined. Numerical simulation was carried out and further revealed that corruption can only be removed to a manageable level but not totally eliminated. Legesse and Shiferaw [5] proposed and analyzed a mathematical model for the spread of corruption dynamics. The model has similar with characteristics to Abdulrahman [4] except that an individual who loses immunity gained through council in jail does not directly join the corrupt class, but rather becomes susceptible because of human behavior, and also, they are concerned with the impact of the awareness created by anticorruption.

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2. MODEL DEFINITIONS

The corruption dynamics model is developed by modifying the work done by Athithan et al. [6, 7] by including compartment of immune individuals with intervention strategies through combination of mass education and religious teaching. Our corruption model divided the population into four classes/compartments namely: Susceptible individuals, Corrupt individuals, Recovered individuals and Honest individuals. Those who are susceptible to corruption are susceptible individuals $S(t)$, those who are performing corruption are corrupted individuals $C(t)$, those who group of individuals hesitant about engaging in corruption $R(t)$, and those who know the badness of corruption and do not perform it permanently are honest individuals $H(t)$ at time $t \geq 0$.

To ensure dimensional consistency, we normalize the time variable using the inverse of the natural death rate μ, i.e., we define a dimensionless time $t^* = t\mu$. This rescaling transforms the system into a non-dimensional form, where all variables and parameters are expressed in consistent units. As a result, the model equations become dimensionally balanced, and the time variable t^* is unitless. This rescaling simplifies the mathematical analysis and allows for clearer comparison of parameter influences.

In the following table, the definitions of the parameters involved in the mathematical model are illustrated [8–11].

The model's system of differential equations is formulated as follows

$$\begin{aligned}
\frac{dS}{dt} &= P\varepsilon + (1-\tau)\,R - \left(\frac{\kappa+\nu+\eta}{N}\right) SC - (\alpha+\varphi)\,S - \mu S,\\
\frac{dC}{dt} &= \left(\frac{\kappa+\nu+\eta}{N}\right) SC - \mu C - (\alpha+\varphi+\psi)\,C,\\
\frac{dR}{dt} &= (\alpha+\varphi+\psi)\,C - (\mu+1)\,R,\\
\frac{dH}{dt} &= P\,(1-\varepsilon) + (\alpha+\varphi)\,S + \tau R - \mu H.
\end{aligned} \tag{1}$$

with the following initial conditions

$$S(0) = S_0 \geq 0, \quad C(0) = E_0 \geq 0, \quad R\,(0) = R_0 \geq 0, \quad H(0) = H_0 \geq 0.$$

3. MODEL ANALYSIS

3.1. Positivity and Boundness of the Solution

Based on the steps outlined above, we will investigate the model: the existence of the solution will be established through the following theorem.

Theorem 1. *Let* $\Omega = \left\{(S,C,R,H) \in R_+^4 : S(0) > 0, C(0) > 0, R(0) > 0, H(0) > 0\right\}$. *Then, the solution set of system (1) is positive and bounded for all* $t \geq 0$.

Adding the model equations to verify that the solution of the model system is bounded, we get

$$\begin{aligned}
\frac{dN(t)}{dt} &= P - \mu\,(S + C + R + H)\,,\\
N(t) &= S(t) + C(t) + R(t) + H(t)
\end{aligned}$$

simplifying that $\frac{dN(t)}{dt} = P - \mu N$, we evaluate the equality

$$\omega = \left\{(S,C,R,I) \in R_+^4 : N(t), \text{ if } N(0) \leq P/\mu\right\}.$$

3.2. Corruption Free Equilibrium Point (CFE)

In order to have CFE, we set the right-hand side of the system of differential equations (1) equal to zero. In this case, we will reach a corruption-free equilibrium state [9]. Thus, $S \neq 0, H \neq 0, C = R = 0$,

$$CFE = \{S,C,R,I\} = \left(\frac{\varepsilon P}{\alpha+\varphi+\mu}, 0, 0, \frac{P(1-\varepsilon)\,(\alpha+\varphi+\mu) + (\alpha+\varphi)\,\varepsilon P}{\mu\,(\alpha+\varphi+\mu)}\right). \tag{2}$$

3.3. Basic Reproduction Number R_0

The basic reproduction number R_0 measures the expected number of secondary infections that result from one newly infected individual introduced into a susceptible population. The calculation was provided R_0 of the model using the next-generation matrix method as described in [10]. The first step to get R_0 is rewriting the model equations starting with second infective from the system of differential equations (1)

$$\frac{dC}{dt} = \left(\frac{\kappa+\nu+\eta}{N}\right) SC - \mu C - (\alpha+\varphi+\psi)\, C.$$

We define the first term as f, the remaining term as ν,

$$f = \left(\frac{\kappa+\nu+\eta}{S+C+R+H}\right) SC, \quad \nu = (\mu+\alpha+\varphi+\psi)\, C,$$

since $N(t) = S(t)+C(t)+R(t)+H(t)$. If partial derivative respect to C,

$$F = \frac{\partial f}{\partial C} = \frac{(\kappa+\nu+\eta)}{S+C+R+H} S$$

and

$$V = \frac{\partial v}{\partial C} = \alpha+\varphi+\psi+\mu$$

from Corruption free equilibrium (CFE) $S \neq 0$, $H \neq 0$, $C = R = 0$,

$$S = \frac{\varepsilon P}{\alpha+\varphi+\mu} \tag{3}$$

and

$$H = \frac{P(1-\varepsilon)\,(\alpha+\varphi+\mu) + (\alpha+\varphi)\,\varepsilon P}{\mu\,(\alpha+\varphi+\mu)}.$$

Therefore,

$$F = \frac{(\kappa+\nu+\eta)}{S+H} S = \frac{\varepsilon\mu\,(\kappa+\nu+\eta)}{\varepsilon\mu + (1-\varepsilon)\,(\alpha+\varphi) + \mu\,(1-\varepsilon) + (\alpha+\varphi)\,\varepsilon} = \frac{\varepsilon\mu\,(\kappa+\nu+\eta)}{\alpha+\varphi+\mu}.$$

$$V^{-1} = \frac{1}{\alpha+\varphi+\psi+\mu}.$$

Thus,

$$R_0 = \frac{\varepsilon\mu\,(\kappa+\nu+\eta)}{(\alpha+\varphi+\psi+\mu)(\alpha+\varphi+\mu)}. \tag{4}$$

3.4. Local Stability of Corruption Free Equilibrium (CFE) Point

To check for local stability of equilibrium points we take into consideration all model equations and find the Jacobian matrix that will be used to evaluate whether the equilibrium point is stable or not depending on the sign of the eigenvalues [11]. Therefore, we linearize the model system (1) by computing the Jacobian matrix in the system with respect to the state variable S, C, R, and H. If all eigenvalues are negative, then the equilibrium points are stable, otherwise it is unstable. Here we use Jacobian matrix to determine local stability of (CFE) which is obtained by first considering the given four equations as functions f, g, h, and z as follows:

$$f\,(S,C,R,I) = P\varepsilon + (1-\tau)\,R - \left(\frac{\kappa+\nu+\eta}{N}\right) SC - (\alpha+\varphi)\,S - \mu S = 0,$$

$$g\,(S,C,R,I) = \left(\frac{\kappa+\nu+\eta}{N}\right) SC - \mu C - (\alpha+\varphi+\psi)\,C,$$

$$h\,(S,C,R,I)=(\alpha+\varphi+\psi)\,C-(\mu+1)\,R,$$
$$z\,(S,C,R,I)=P\,(1-\varepsilon)+(\alpha+\varphi)\,S+\tau R-\mu H, \tag{5}$$

where $N(t)=S(t)+C(t)+R(t)+H(t)$. Then, its Jacobian Matrix J_{CFE} at corruption free equilibrium (CFE) is given by

$$J_{CFE}=\begin{pmatrix}\frac{\partial f}{\partial S} & \frac{\partial f}{\partial C} & \frac{\partial f}{\partial R} & \frac{\partial f}{\partial H}\\ \frac{\partial g}{\partial S} & \frac{\partial g}{\partial C} & \frac{\partial g}{\partial R} & \frac{\partial g}{\partial H}\\ \frac{\partial h}{\partial S} & \frac{\partial h}{\partial C} & \frac{\partial h}{\partial R} & \frac{\partial h}{\partial H}\\ \frac{\partial z}{\partial S} & \frac{\partial z}{\partial C} & \frac{\partial z}{\partial R} & \frac{\partial z}{\partial H}\end{pmatrix}=\begin{pmatrix}-(\mu+\alpha+\varphi) & -(\kappa+\nu+\eta) & (1-\tau) & 0\\ 0 & (\kappa+\nu+\eta)-(\alpha+\varphi+\psi+\mu) & 0 & 0\\ 0 & (\alpha+\varphi+\psi) & -(\mu+1) & 0\\ (\alpha+\varphi) & 0 & \tau & -\mu\end{pmatrix}.$$

The first eigenvalue $\lambda_1=-\mu<0$, therefore, the reduced Jacobian matrix at CFE is given by

$$J^1_{CFE}=\begin{pmatrix}-(\mu+\alpha+\varphi) & -(\kappa+\nu+\eta) & (1-\tau)\\ 0 & (\kappa+\nu+\eta)-(\alpha+\varphi+\psi+\mu) & 0\\ 0 & (\alpha+\varphi+\psi) & -(\mu+1)\end{pmatrix}.$$

Also, for the second eigenvalue is $\lambda_2=-(\mu+\alpha+\varphi)<0$ and reduced Jacobian matrix at CFE is given below

$$J^2_{CFE}=\begin{pmatrix}(\kappa+\nu+\eta)-(\alpha+\varphi+\psi+\mu) & 0\\ (\alpha+\varphi+\psi) & -(\mu+1)\end{pmatrix}.$$

Again, for the third eigenvalue is $\lambda_3=-(\mu+1)<0$ and, hence, the fourth eigenvalue is $\lambda_4=(\kappa+\nu+\eta)-(\alpha+\varphi+\psi+\mu)$. By considering λ_4, we find that the CFE will be asymptotically stable only if $\lambda_4<0$, which $(\kappa+\nu+\eta)<(\alpha+\varphi+\psi+\mu)$ must hold true for stability of CFE point [12].

3.5. Global Stability of CFE Point

Theorem 2. *The corruption free equilibrium point of model (1) is global asymptotically stable if $R_0<1$.*

There are several methods to solve for global stability using a Lyapunov function, but for the purposes of this paper, we will apply LaSalle's Invariance Principle [13]. If the Lyapunov function $\dot V(x)$ is negative definite, then the global asymptotic stability of the origin follows directly from Lyapunov's second theorem. The invariance principle provides a criterion for asymptotic stability in cases, where $\dot V(x)$ is only negative semidefinite [14]. Consider the following Lyapunov function $V=\frac{1}{2}C^2$ by differentiating equation with respect to t gives

$$\frac{dV}{dt}=\frac{\partial V}{\partial C}\frac{\partial C}{\partial t}. \tag{6}$$

If $V=C^2/2$, then $\frac{\partial V}{\partial C}=C$ and from the system of equation (1) above

$$\frac{\partial C}{\partial t}=\left(\frac{\kappa+\nu+\eta}{N}\right)SC-\mu C-(\alpha+\varphi+\psi)\,C,$$
$$\frac{dV}{dt}=\frac{\partial V}{\partial C}\frac{\partial C}{\partial t}=C\left(\left(\frac{\kappa+\nu+\eta}{N}\right)SC-\mu C-(\alpha+\varphi+\psi)\,C\right)$$
$$=C^2\left((\kappa+\nu+\eta)\,\frac{S}{N}-(\mu+\alpha+\varphi+\psi)\right).$$

From

$$R_0=\frac{\varepsilon\mu\,(\kappa+\nu+\eta)}{(\alpha+\varphi+\psi+\mu)(\alpha+\varphi+\mu)}$$

we have

$$(\alpha+\varphi+\psi+\mu)=\frac{\varepsilon\mu\,(\kappa+\nu+\eta)}{R_0(\alpha+\varphi+\mu)}.$$

Therefore, for equation (6), substituting $\frac{\partial V}{\partial C}$ and $\frac{\partial C}{\partial t}$, we get

$$\frac{dV}{dt}=C^2\,(\kappa+\nu+\eta)\left(1-\frac{\varepsilon\mu}{R_0(\alpha+\varphi+\mu)}\right).$$

From at CFE we have $S=\frac{\varepsilon\prod}{\alpha+\varphi+\mu}$ which implies $\frac{\varepsilon}{\alpha+\varphi+\mu}=\frac{S}{\prod}$. Then,

$$\frac{dV}{dt}=C^2\,(\kappa+\nu+\eta)\left(1-\frac{S\mu}{R_0\prod}\right).$$

If $\frac{\varepsilon\prod}{\alpha+\varphi+\mu}<\prod$, then $\frac{dV}{dt}=C^2\,(\kappa+\nu+\eta)\,(1-1/R_0)$, hence, $\frac{dV}{dt}\leq 0$ when $R_0\leq 1$.

3.6. The Corruption Endemic Equilibrium (CEE) Point

The endemic equilibrium point denoted by $E^*=(S^*,C^*,R^*,I^*)$ is the steady state solution, where corruption persists in the population. It can be obtained by equating each equation in (1) to zero:

$$\begin{gathered}P\varepsilon+(1-\tau)\,R-\left(\frac{\kappa+\nu+\eta}{N}\right)SC-(\alpha+\varphi)\,S-\mu S=0,\\ \left(\frac{\kappa+\nu+\eta}{N}\right)SC-\mu C-(\alpha+\varphi+\psi)\,C=0,\\ (\alpha+\varphi+\psi)\,C-(\mu+1)\,R=0,\\ P\,(1-\varepsilon)+(\alpha+\varphi)\,S+\tau R-\mu H=0.\end{gathered}\tag{7}$$

Then, we obtain

$$\begin{gathered}S^*=\frac{\mu+(\alpha+\varphi+\psi)}{B},\quad C^*=\frac{(\mu+1)\,R}{(\alpha+\varphi+\psi)},\\ R^*=\frac{M\,((M+\mu)\,(\alpha+\beta)+\mu\,(M+\mu))}{B\,(M-M\gamma-(M+\mu)\,(\mu+1))}-\varepsilon P,\\ H^*=\frac{1}{\mu}\left(P\,(1-\varepsilon)+(\alpha+\varphi)\,\frac{\mu+(\alpha+\varphi+\psi)}{B}\right.\\ \left.+\tau\left(\frac{M\,((M+\mu)\,(\alpha+\varphi)+\mu\,(M+\mu))}{B\,(M-M\gamma-(M+\mu)\,(\mu+1))}-\varepsilon P\right)\right),\end{gathered}\tag{8}$$

where $B=\frac{\kappa+\nu+\eta}{N}$ and $M=(\alpha+\varphi+\psi)$.

3.7. Global Stability of CEE Point

Theorem 3. *If $R_0>1$, therefore, CEE E^* of system of equation (1) is globally asymptotically stable.*

Proof. Proof by considering Lyapunov function

$$\begin{gathered}U=a_1\left(S-S^*-S^*\ln\frac{S}{S^*}\right)+a_2\left(C-C^*-C^*\ln\frac{C}{C^*}\right)\\ +a_3\left(R-R^*-R^*\ln\frac{R}{R^*}\right)+a_4\left(H-H^*-H^*\ln\frac{H}{H^*}\right).\end{gathered}$$

Now we take the derivates along the solutions of the model equations

$$U' = a_1\left(1-\frac{S^*}{S}\right)S' + a_2\left(1-\frac{C^*}{C}\right)C' + a_3\left(1-\frac{R^*}{R}\right)R' + a_4\left(1-\frac{H^*}{H}\right)H'. \tag{9}$$

From (1),

$$S' = P\varepsilon + (1-\tau)\,R - \left(\frac{\kappa+\nu+\eta}{N}\right)SC - (\alpha+\varphi)\,S - \mu S,$$

$$C' = \left(\frac{\kappa+\nu+\eta}{N}\right)SC - \mu C - (\alpha+\varphi+\psi)\,C,$$

$$R' = (\alpha+\varphi+\psi)\,C - (\mu+1)\,R, \quad H' = \prod(1-\varepsilon) + (\alpha+\varphi)\,S + \tau R - \mu H.$$

After substituting, we take

$$\begin{aligned} U' = a_1\left(1-\frac{S^*}{S}\right)\left(P\varepsilon + (1-\tau)\,R - \left(\frac{\kappa+\nu+\eta}{N}\right)SC - (\alpha+\varphi)\,S - \mu S\right) \\ + a_2\left(1-\frac{C^*}{C}\right)\left(\left(\frac{\kappa+\nu+\eta}{N}\right)SC - \mu C - (\alpha+\varphi+\psi)\,C\right) \\ + a_3\left(1-\frac{R^*}{R}\right)\left((\alpha+\varphi+\psi)\,C + (\mu+1)\,R\right) + a_4\left(1-\frac{H^*}{H}\right)\left(P\,(1-\varepsilon) + (\alpha+\varphi)\,S + \tau R - \mu H\right). \end{aligned}$$

At CEE, taking into consideration of E^* and $a_i > 0$ for are continuous and differentiable in ω and $U(\omega^*) = 0$ for $\omega = (S^*, C^*, R^*, H^*)$. Global stability for endemic equilibrium holds if $U' \leq 0$, therefore, at endemic equilibrium we have [12–15]

$$\begin{aligned} U' = a_1\left(1-\frac{S^*}{S}\right)\left(\left(\frac{\kappa+\nu+\eta}{N}\right)S^*C^* + (\alpha+\varphi)\,S^* + \mu S^* - \left(\frac{\kappa+\nu}{N}\right)SC - (\alpha+\varphi)\,S - \mu S\right) \\ + a_2\left(1-\frac{C^*}{C}\right)\left(\mu C^* + (\alpha+\varphi+\psi)\,C^* - \mu C - (\alpha+\varphi+\psi)\,C\right) \\ + a_3\left(1-\frac{R^*}{R}\right)\left((\mu+1)\,R^* - (\mu+1)\,R\right) + a_4\left(1-\frac{H^*}{H}\right)\left(\mu H^* - \mu H\right). \end{aligned}$$

By simplifying, we have

$$\begin{aligned} U' = a_1\left(1-\frac{S^*}{S}\right)\left(\left(\frac{\kappa+\nu+\eta}{N}\right)(S^*C^* - SC) + (\alpha+\varphi)\,(S^* - S) + \mu\,(S^* - S)\right) \\ + a_2\left(1-\frac{C^*}{C}\right)\left(\mu\,(C^* - C) + (\alpha+\varphi+\psi)\,(C^* - C)\right) \\ + a_3\left(1-\frac{R^*}{R}\right)\left((\mu+1)\,(R^* - R)\right) + a_4\left(1-\frac{H^*}{H}\right)\left(\mu\,(H^* - H)\right). \end{aligned}$$

Now

$$\begin{aligned} U' = a_1\left(\frac{S-S^*}{S}\right)\left(-\left(\frac{\kappa+\nu+\eta}{N}\right)SC\left(\frac{SC - S^*C^*}{SC}\right) - (\alpha+\varphi)\,S\left(\frac{S-S^*}{S}\right) - \mu\left(\frac{S-S^*}{S}\right)\right) \\ + a_2\left(\frac{C-C^*}{C}\right)\left(-\mu C\left(\frac{C-C^*}{C}\right) - (\alpha+\varphi+\psi)\,C\left(\frac{C-C^*}{C}\right)\right) \\ + a_3\left(\frac{R-R^*}{R}\right)\left(-(\mu+1)\,R\left(\frac{R-R^*}{R}\right)\right) + a_4\left(\frac{H-H^*}{H}\right)\left(-\mu H\left(\frac{H-H^*}{H}\right)\right). \end{aligned}$$

By opening brackets, we take

$$U' = -a_1\,(\alpha+\varphi)\,S\left(\frac{S-S^*}{S}\right)^2 - a_1\mu S\left(\frac{S-S^*}{S}\right)^2 - a_2\mu C\left(\frac{C-C^*}{C}\right)^2$$

Table 1. Definition and value of parameters

Parameters	Description	Value
α	Rate of change of corruption due to mass education	0.0095 [assumed]
φ	Rate of change of corruption due to religious teaching	0.25
ψ	Legal culture and legal consciousness	0.1
κ	The parameter of corruption due to greed-driven	0.000597
ν	The parameter of corruption due to need-driven	0.135
η	Nepotism	0.23 [assumed]
ε	Percentage of individuals who are not born immune	0.416328
P	Recruitment number	400
μ	Natural death rate	0.004942
τ	Rate at which reformed individuals become honest (enlightened)	0.06 [assumed]

$$- a_2\left(\alpha+\varphi+\psi\right)C\left(\frac{C-C^*}{C}\right)^2 - a_3\left(\mu+1\right)R\left(\frac{R-R^*}{R}\right)^2$$

$$- a_4\mu I\left(\frac{H-H^*}{H}\right)^2 - a_1\left(\frac{\kappa+\nu+\eta}{N}\right)SC\left(\frac{SC-S^*C^*}{SC}\right)\left(\frac{S-S^*}{S}\right),$$

$$U' = -\left[a_1\left(\alpha+\varphi\right)S\left(\frac{S-S^*}{S}\right)^2 + a_1\mu S\left(\frac{S-S^*}{S}\right)^2\right.$$

$$+ a_2\mu C\left(\frac{C-C^*}{C}\right)^2 + a_2\left(\alpha+\varphi+\psi\right)C\left(\frac{C-C^*}{C}\right)^2 + a_3\left(\mu+1\right)R\left(\frac{R-R^*}{R}\right)^2$$

$$\left. + a_4\mu H\left(\frac{H-H^*}{H}\right)^2\right] + F\left(\theta\right),$$

where $F\left(\theta\right) = -a_1\left(\frac{\kappa+\nu+\eta}{N}\right)SC\left(\frac{SC-S^*C^*}{SC}\right)\left(\frac{S-S^*}{S}\right)$. □

Note. All parameters are greater or equal to zero and all state variables S, C, R, and $H \geq 0$. Therefore,

$$\left(\frac{S-S^*}{S}\right)^2 \geq 0, \quad \left(\frac{C-C^*}{C}\right)^2 \geq 0, \quad \left(\frac{R-R^*}{R}\right)^2 \geq 0, \quad \left(\frac{H-H^*}{H}\right)^2 \geq 0.$$

From La Salle Invariance Principle $F\left(\theta\right) \leq 0$ and, hence, $U' \leq 0$ as required, therefore, corruption endemic equilibrium is stable if $R_0 > 1$.

3.8. The Sensitivity Analysis

In order to see the relative effect of each parameter to the corruption transmission, we performed sensitivity analysis. To go through, we followed the approach defined by [12] as done in [13, 14] which used the definition of normalized forward sensitivity index defined as a variable R_0, that depends differentiable on a parameter l, defined as

$$T_l^{R_0} = \frac{\partial R_0}{\partial l}\frac{l}{R_0}$$

for l represents all the basic parameters to [15–18]. From the derivative of R_0 with respect to model parameter it is demonstrated that there are some values of the derivative of R_0 which are less than

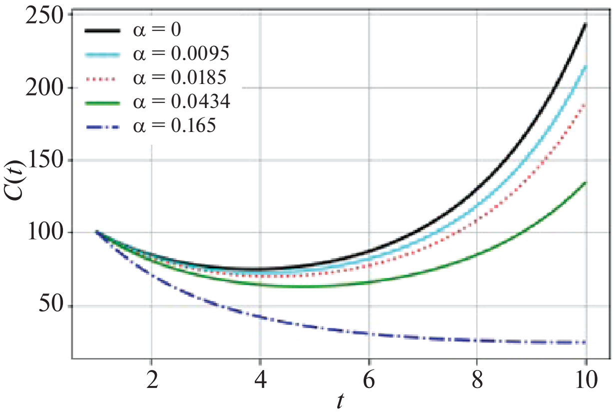

Fig. 1. Change in the number of corrupt individuals with variation rate of mass education.

0 and other values are greater than 0. Analytically it indicates that all values of R_0 which are less than 0 are the important factors for controlling corruption in the country. Therefore, increasing these parameters becomes the most control strategy of corruption in the country. Considering the most negative sensitivity indices which are α, φ, and ψ. Table 2 below shows the sensitive parameter values and its graph, respectively.

Table 2 is illustrated that parameters α, φ, and ψ are most negative sensitive to the fight of corruption in the country this indicates that the more increase in these parameters leads to more control in the corruption of the country [19]. Moreover, the parameters φ and ψ are the most positive influential of corruption in the country, therefore, for achieving a decrease in the level of corruption the attention should be paid on these parameters, as they continue to rise, the reproduction number also increases sharply, which can lead to a high prevalence of corruption in the country. Therefore, φ and ψ must be fully controlled.

4. NUMERICAL SIMULATION

4.1. Influence of mass education against corruption dynamics. Figure 1 demonstrates a significant decline in corruption levels due to the implementation of public education programs across various social structures-namely, society, neighborhoods, and families. This reduction is further attributed to increased values of the education-related parameter that reflects moral awareness and conscientious behavior. Notably, as time approaches $t = 10$ months, the corruption rate continues to decrease in response to these interventions.

4.2. Public education is the influence of education, religious teaching and the influence in combating corruption through legal culture. Figure 2 illustrates that mass education, religious

Table 2. Sensitivity index table

Parameter symbol	Sensitivity indices
α	-0.0596
φ	-1.56
μ	1
η	0.7156
ψ	-1.1379
ε	1.000
κ	0.0018
ν	0.42

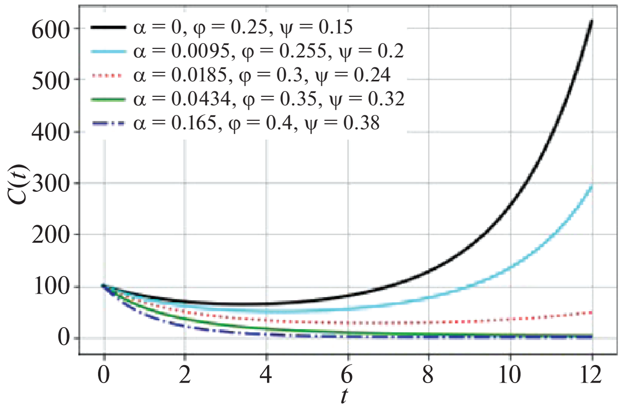

Fig. 2. Mass education is a change in the number of corrupt individuals influenced by the influence of upbringing, religious teaching and legal culture.

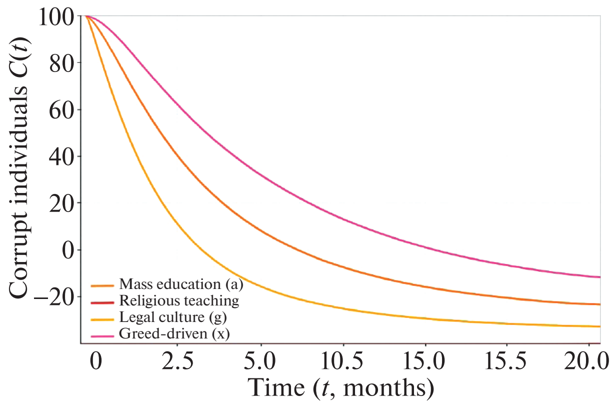

Fig. 3. Corruption level over time under the influence of key parameters α, φ, ψ, and κ. Vertical lines indicate time snapshots at $t = 5$ and $t = 10$.

teaching, and legal culture are the most effective parameters in combating corruption. Their combined increase significantly reduces corruption levels over time.

4.3. Corruption level under the influence of key parameters. The simulation results and sensitivity analysis indicate that increasing the values of the parameters α (mass education), φ (religious teaching), and ψ (legal culture and legal consciousness) significantly contributes to reducing the corruption level over time. These parameters appear with negative sensitivity indices, implying that their increase leads to a decrease in the basic reproduction number R_0, thus, making the corruption-free equilibrium more stable. In contrast, the parameter κ (greed-driven corruption) has a positive influence on corruption prevalence, meaning that higher values of κ result in slower reductions or even increases in corruption levels.

5. CONCLUSIONS

In this paper, a mathematical model for the transmission dynamics of corruption in a population was formulated. The basic reproduction number R_0 was computed, and the stability of equilibrium points was investigated. Through Lyapunov's theory, the corruption-free equilibrium point is globally asymptotically stable whenever $R_0 < 1$ was proven. The global stability of the unique endemic equilibrium whenever $R_0 > 1$ was demonstrated. Using the definition of normalized forward sensitivity, the sensitivity parameters were determined. Whereas if parameters α, φ and ψ which represent mass education, religious teaching and legal culture, legal consciousness respectively such that when these parameters increases while other parameters remain constant the basic reproduction number will be less than 1 as a result there will be a control of corruption in the community. For these results it suggests that there is a need to invest more efforts in mass education to the population and increase more emphasize to religious leaders to teach their followers effectively about corruption as it is against their faith and doctrines.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

REFERENCES

1. M. Johnston, "The search for definitions: The vitality of politics and the issue of corruption," Int. Soc. Sci. J. A **48**, 321–335 (1996).
2. W. O. Kermack and A. G. McKendrick, "Containing papers of a mathematical and physical character," Proc. R. Soc. London, Ser. A **115** (772), 700–721 (1927).
3. O. Danford, M. Kimathi, and S. Mirau, "Mathematical modelling and analysis of corruption dynamics with control measures in Tanzania," J. Math. Inform., No. 19, 57–79 (2020).
4. S. Abdulrahman, "Stability analysis of the transmission dynamics and control of corruption," Pacif. J. Sci. Technol. **15**, 99–113 (2014).
5. L. Lemecha and Sh. Feyissa, "Mathematical modeling and analysis of corruption dynamics," Ethiop. J. Sci. Sustainable Develop. **5** (2), 13–26 (2018).
6. S. Athithan, M. Ghosh, and X. Z. Li, "Mathematical modeling and optimal control of corruption dynamics," Asian-Eur. J. Math. **11** (6), 41–93 (2018).
7. S. R. Waykar, "Mathematical modelling: A comparatively mathematical study model base between corruption and development," IOSR J. Math. **6** (2), 54–62 (2013).
8. H. T. Alemneh, "Mathematical modeling, analysis, and optimal control of corruption dynamics," J. Appl. Math. **2020**, 5109841 (2020).
9. S. R. Wayker, "Mathematical modeling: A study of corruption in the society of India," Int. J. Sci. Eng. Res. **4**, 2303–2318 (2013).
10. S. Rose-Ackerman, "The economics of corruption," J. Publ. Econ. **4**, 187–203 (1975).
11. J. Li, D. Blakeley, and R. J. Smith, "The failure of R_0," Comput. Math. Methods Med. **2011**, 527610 (2011).
12. N. I. Nikolaeva, *Differential Equations. Elements of the Theory of Stability,* Part 5 of *Lecture Notes* (OmGTU, Omsk, 2011) [in Russian].
13. S. M. Blower and H. Dowlatabadi, "Sensitivity and uncertainty analysis of complex models of disease transmission: An hiv model, as an example," Int. Stat. Rev., No. 62, 229–243 (1994).
14. M. A. Mikucki, "Sensitivity analysis of the basic reproduction number and other quantities for infectious disease models," Thesis (Colorado State Univ. Fort Collins, Colorado Spring, 2012).
15. H. S. Rodrigues, T. T. Monteiro, and F. M. Torres, "Sensitivity analysis in a dengue epidemiological model," Conf. Papers Math. **2013**, 721406 (2013).
16. F. M. Omar, M. A. Sohaly, and H. El-Metwally, "Lyapunov functions and global stability analysis for epidemic model with n-infectious," Indian J. Phys. **98**, 1913 (2024). https://doi.org/10.1007/s12648-023-02895-6
17. A. Floor, "LaSalle's invariance principle on measure chains," Honors Projects No. 7 (Illinois Wesleyan Univ., 2000).
18. S. R. Waykar, "A study of corruption for issuing Aadharr Card in India by using mathematical modeling," Int. Ref. J. Eng. Sci. **7** (2), 57–64 (2018).
19. A. K. Fantaye and Z. K. Birhanu, "Mathematical model and analysis of corruption dynamics with optimal control," J. Appl. Math. **1**, 1–16 (2022).