MATEMATIK ANALIZ VA UNING ZAMONAVIY MATEMATIK FIZIKAGA TATBIQLARI

SAMARQAND 2025

Let $k = 2$. Finding translation-invariant splitting Gibbs measures (TISGMs) reduces to solving the equation

$$\theta y^3 - \eta y^2 + (\theta^2 - \theta + 1)y - 2\eta\theta = 0. \tag{1}$$

One can show that there exists a unique critical value $\theta_c \approx 0.302$ that satisfies the equation such that

- If $\theta > \theta_c$, Eq. (1) has a unique positive solution, denoted by y_1.
- If $\theta = \theta_c$, Eq. (1) has two positive solutions, denoted by $y_2 < y_1$.
- If $\theta < \theta_c$, Eq. (1) has three positive solutions, denoted by $y_3 < y_2 < y_1$.

The TISGMs associated with the solutions y_i are denoted by μ_i for $i = 1, 2, 3$.
We have
Theorem 1. For the four-state SOS model on the Cayley tree of order two the following assertions hold:

- There are values $\hat{\theta}_1(\approx 0.172)$, $\hat{\theta}_2(\approx 0.4323)$, $\hat{\theta}_3(\approx 2.2698)$ and $\hat{\theta}_4(\approx 3.5731)$ such that the measure μ_1 is extreme if $\theta \in (\hat{\theta}_2, \hat{\theta}_3)$ and is non-extreme if $\theta \in (0, \hat{\theta}_1) \cup (\hat{\theta}_4, +\infty)$.
- The measures μ_2 and μ_3 are non-extreme (where they exist).

LITERATURE

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A Modified AA-Proximal Point Type Iterative Algorithm for Convex Optimization and Fixed Point Problems

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In order to identify common solutions of convex optimization and fixed point problems in uniformly convex Banach spaces, we present a proximal point type version of the AA-iterative technique in this work.By using the accelerated structure of the AA-iteration, the suggested approach generalizes the well-known proximal point algorithm and achieves a faster rate of convergence. A numerical comparison demonstrates the effectiveness of the suggested method, and some convergence theorems are established under common assumptions.

Nonlinear analysis relies heavily on convex optimization and fixed point theory. For approximating fixed points of nonlinear mappings, iterative methods like Picard, Mann, Ishikawa, and the recently created AA-iteration have been employed extensively. Nevertheless, the function to be reduced is not always smooth in convex optimization situations. The proximal point method's resilience and strong convergence features make it an effective tool in these situations. To improve the rate of convergence, we modify the AA-iterative algorithm by incorporating the proximal operator, resulting in a *proximal-AA iteration.*

Let H be a real Hilbert space and let $f : H \to (-\infty, +\infty]$ be a proper, convex, and lower semi-continuous function. The *proximal operator* associated with f is defined as

$$\operatorname{prox}_{\lambda f}(x) = \arg\min_{y\in H} \left\{ f(y) + \frac{1}{2\lambda}\|y-x\|^2 \right\}, \quad \lambda > 0.$$

It satisfies the relation

$$x = \operatorname{prox}_{\lambda f}(x) \iff 0 \in \partial f(x),$$

which shows that the fixed points of $\operatorname{prox}_{\lambda f}$ correspond to minimizers of f.

Inspired by the AA-iteration, we define the following *proximal-AA iterative algorithm*:

$$\begin{cases} x_{n+1} = \operatorname{prox}_{\lambda f}(y_n), \\ y_n = \operatorname{prox}_{\lambda f}\big((1-\sigma_n)\operatorname{prox}_{\lambda f}(h_n) + \sigma_n \operatorname{prox}_{\lambda f}(z_n)\big), \\ z_n = \operatorname{prox}_{\lambda f}\big((1-\lambda_n)h_n + \lambda_n \operatorname{prox}_{\lambda f}(h_n)\big), \\ h_n = (1-\xi_n)x_n + \xi_n \operatorname{prox}_{\lambda f}(x_n), \end{cases} \tag{84}$$

where $\{\sigma_n\}$, $\{\lambda_n\}$, and $\{\xi_n\}$ are sequences in $(0,1)$, and $x_1 \in H$ is arbitrary.
The iterative sequence $\{x_n\}$ generated by (84) is called the *proximal-AA sequence*.

Assume that f is convex, proper, and lower semi-continuous. If $\lambda > 0$ and $\operatorname{prox}_{\lambda f}$ is nonexpansive, then the sequence $\{x_n\}$ defined by (84) satisfies:

$$\|x_{n+1} - x^*\| \le \|x_n - x^*\|, \quad \forall x^* \in \operatorname{argmin} f.$$

Hence, $\{x_n\}$ converges weakly to a minimizer of f. If, in addition, f is strongly convex, then the convergence is strong.
Theorem. Let $f : H \to (-\infty, +\infty]$ be convex and lower semi-continuous. Then the sequence $\{x_n\}$ generated by (84) converges strongly to the unique minimizer x^* of f.

LITERATURE

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ОЦЕНКИ ДЛЯ ГРАНИЦЫ САМОСОПРЯЖЕННЫХ ПОЛУОГРАНИЧЕННЫХ 3×3 ОПЕРАТОРНЫХ МАТРИЦ

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Матрицы, элементы которых являются линейными операторами в банаховых или гильбертовых пространствах, называются блочно-операторными матрицами [1]. Одним из основных классов таких матриц являются гамильтонианы системы с несохраняющимся ограниченным числом частиц на непрерывном пространстве или на решетке.