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# АМАЛИЙ МАТЕМАТИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ЗАМОНАВИЙ МУАММОЛАРИ

ХАЛҚАРО ИЛМИЙ-АМАЛИЙ АНЖУМАН

## МАТЕРИАЛЛАРИ

2022 йил, 11-12 май

БУХОРО – 2022



**ЎЗБЕКИСТОН РЕСПУБЛИКАСИ  
ОЛИЙ ВА ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ  
ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ФАНЛАР АКАДЕМИЯСИ  
В.И. РОМАНОВСКИЙ НОМИДАГИ МАТЕМАТИКА ИНСТИТУТИ  
ЎЗБЕКИСТОН МИЛЛИЙ УНИВЕРСИТЕТИ  
ТОШКЕНТ ДАВЛАТ ТРАНСПОРТ УНИВЕРСИТЕТИ  
БУХОРО ДАВЛАТ УНИВЕРСИТЕТИ**

*Бухоро фарзанди, Беруний номидаги Давлат мукофоти лауреати, кўплаб ёш изланувчиларнинг ўз йўлини топиб олишида раҳнамолик қилган етук олим, физика-математика фанлари доктори Файбулла Назруллаевич Салиховнинг 90 йиллик юбилейларига бағишланади*

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Шадиметов Холмат

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Дурдиев Дурдимурод

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Худойберганаов Мирзоали

ЎзМУ, ф.-м.ф.д.

Эшанкулов Ҳамза

БухДУ, факультет декани, т.ф.ф.д. (PhD)

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ривожлантириш илмий-тадқиқот институти, (PhD)

Шадманов И.У.

Математика Инститuti Бухоро бўлини маси, (PhD)

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БухДУ, катта ўқитувчи

Хазратов Ф.Х.

БухДУ, катта ўқитувчи

Эргашев А.А.

БухДУ, катта ўқитувчи

Авезов А.А

БухДУ, катта ўқитувчи

Assume  $\{\mathbf{x}^{(n)} \in S^{m-1} : n = 0, 1, 2, \dots\}$  is the trajectory of the initial point  $\mathbf{x} \in S^{m-1}$ , where  $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$  for all  $n = 0, 1, 2, \dots$ , with  $\mathbf{x}^{(0)} = \mathbf{x}$ .

**Definition 1.** A point  $\mathbf{x} \in S^{m-1}$  is called a fixed point of a QSO  $V$  if  $V(\mathbf{x}) = \mathbf{x}$ .

**Definition 2.** A QSO  $V$  is called regular if for any initial point  $\mathbf{x} \in S^{m-1}$ , the limit

$$\lim_{n \rightarrow \infty} V(\mathbf{x}^{(n)})$$

exists.

**Definition 3.** A continuous function  $\phi : \text{int } S^{m-1} \rightarrow R$  for an operator  $V$  if the limit  $\lim_{n \rightarrow \infty} \phi(V^n(\mathbf{x}))$

exists and finite for all  $\mathbf{x} \in S^{m-1}$ .

**Definition 4.**[2] A fixed point  $\mathbf{x}^*$  is called hyperbolic if its Jacobian  $D_{\mathbf{x}}V(\mathbf{x}^*)$  has no eigenvalues on the unit circle.

**Definition 5.**[2] A hyperbolic fixed point  $\mathbf{x}^*$  is called:

- i) attracting if all the eigenvalues of the Jacobian  $D_{\mathbf{x}}V(\mathbf{x}^*)$  are less than 1 in absolute value;
- ii) repelling if all the eigenvalues of the Jacobian  $D_{\mathbf{x}}V(\mathbf{x}^*)$  are greater than 1 in absolute value;
- iii) a saddle otherwise.

Consider the following two strictly non-Volterra QSOs on the two-dimensional simplex

$$V : \begin{cases} x'_1 = \frac{1}{3}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + 2x_1x_2, \\ x'_2 = \frac{1}{3}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + 2x_2x_3, \\ x'_3 = \frac{1}{3}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + 2x_3x_1. \end{cases} \quad (3)$$

**Lemma 1.** The center  $\mathbf{x}$  is a unique and attracting point of the QSO (3).

**Lemma 2.** The function  $\phi(\mathbf{x}) = |x_1 - x_2| \cdot |x_2 - x_3| \cdot |x_3 - x_1|$  is a Lyapunov function for the operator (3).

**Lemma 3.**  $\lim_{n \rightarrow \infty} V^n(\mathbf{x}^{(0)}) = \mathbf{c}$  for any initial point  $\mathbf{x}^{(0)} \in S^2$ .

**Theorem.** a) The QSO (3) has a unique fixed point  $\mathbf{c} = (1/3, 1/3, 1/3)$ ;

b) The fixed point  $\mathbf{c}$  is an attracting point;

c) For any  $\mathbf{x}^{(0)} \in S^2$ , the trajectory  $\{\mathbf{x}^{(n)}\}$  tends to the fixed point  $\mathbf{c}$ ;

d) The QSO (3) is a regular transformation.

#### REFERENCES

1. Bernstein S 1942 *The Annals of Math. Stat.* **13** 53.
2. Devaney R L 2003 *An introduction to chaotic dynamical systems*, (New York: Westview Press).
3. Ganikhodzhaev R N 1992 *Sb. Math.* **183** 489.
4. Jamilov U and Ladra M 2020 *Qual. Theory Dyn. Syst.* **19** 95.
5. Lyubich Yu I 1992 *Mathematical Structures in Population Genetics* (Berlin: Sprenger).
6. Mamurov B J, Rozikov U A and Xudayarov S S 2020 *Markov Pros. Relat. Fields* **26** 915.

### EXISTENCE OF THE EIGENVALUES OF A TENSOR SUM OF THE FRIEDRICHS MODELS WITH RANK 2 PERTURBATION

<sup>1</sup>Bahronov B.I., <sup>1,2</sup>Rasulov T.H.

<sup>1</sup>Bukhara State University

<sup>2</sup>Bukhara branch of the Institute of Mathematics

Let  $T^1$  be the one-dimensional torus. In the Hilbert space  $L_2^s(T^2)$  of square-integrable symmetric (complex) functions defined on  $T^2$ , we consider the model operator:

$$H_{\mu, \lambda} := H_{0,0} - \mu(V_{11} + V_{12}) + \lambda(V_{21} + V_{22}), \quad \mu, \lambda > 0, \quad (1)$$

where  $H_{0,0}$  is the multiplication operator:

$$(H_{0,0}f)(x, y) = (u(x) + u(y))f(x, y),$$

and  $V_{ij}$ ,  $i, j = 1, 2$  are non-local interaction operators:

$$(V_{i1}f)(x, y) = v_i(x) \int_{T^1} v_i(t) f(t, y) dt, \quad (V_{i2}f)(x, y) = v_i(y) \int_{T^1} v_i(t) f(x, t) dt.$$

Here,  $f \in L_2^s(T^2)$ , the functions  $u(\cdot)$  and  $v_i(\cdot)$ ,  $i = 1, 2$  are real-valued continuous functions on  $T^1$ .

Under these assumptions, the operator  $H_{\mu, \lambda}$  is bounded and self-adjoint.

To study the spectral properties of the model operator  $H_{\mu, \lambda}$ , we introduce a Friedrichs model  $h_{\mu, \lambda}$  with rank 2 perturbation, acting on  $L_2(T^1)$  by the rule:

$$h_{\mu, \lambda} := h_{0,0} - \mu k_1 + \lambda k_2,$$

where the operators  $h_{0,0}$  and  $k_i(\cdot)$ ,  $i = 1, 2$  are defined as

$$(h_{0,0}g)(x) = u(x)g(x), \quad (k_i g)(x) = v_i(x) \int_{T^1} v_i(t) g(t) dt, \quad i = 1, 2.$$

From the definitions of  $H_{\mu, \lambda}$  and  $h_{\mu, \lambda}$  we obtain the representation

$$H_{\mu, \lambda} = h_{\mu, \lambda} \otimes I + I \otimes h_{\mu, \lambda},$$

where  $I$  is an identity operator on  $L_2(T^1)$ .

Therefore, by theorem on the spectrum of the tensor sum of two operators the equality

$$\sigma(H_{\mu, \lambda}) = \sigma(h_{\mu, \lambda}) + \sigma(h_{\mu, \lambda})$$

holds.

Let  $\text{supp}\{v_\alpha(\cdot)\}$  be the support of the function  $v_\alpha(\cdot)$  and  $\text{mes}(\Omega)$  be the Lebesgue measure of the measurable set  $\Omega \subset T^1$  and

$$m := \min_{x \in T^1} u(x), \quad M := \max_{x \in T^1} u(x).$$

Assume that the function  $u(\cdot)$  has the non-degenerate global minimum at the points  $x_1, x_2, \dots, x_m \in T^1$  and the non-degenerate global maximum at the points  $y_1, y_2, \dots, y_n \in T^1$ .

Main result of the note is the following theorem.

**Theorem.** Suppose that

$$\text{mes}(\text{supp}\{v_1(\cdot)\} \cap \text{supp}\{v_2(\cdot)\}) = 0$$

and  $v_1(x_i) \neq 0$ ,  $v_2(y_j) \neq 0$  for some  $i \in \{1, \dots, m\}$ ,  $j \in \{1, \dots, n\}$ .

(a) For all values of  $\mu, \lambda > 0$  the operator  $h_{\mu, \lambda}$  has a two simple eigenvalues  $E_\mu^{(1)} < m$  and  $E_\lambda^{(2)} > M$ .

(b) For any  $\mu, \lambda > 0$  the numbers  $2E_\mu^{(1)}$  and  $2E_\lambda^{(2)}$  are simple eigenvalues of  $H_{\mu, \lambda}$ . Moreover

$$\sigma_{\text{ess}}(H_{\mu, \lambda}) = [E_\mu^{(1)} + m; E_\mu^{(1)} + M] \cup [2m; 2M] \cup [E_\lambda^{(2)} + m; E_\lambda^{(2)} + M]$$

$$\sigma_{\text{pp}}(H_{\mu, \lambda}) = \{2E_\mu^{(1)}; E_\mu^{(1)} + E_\lambda^{(2)}; 2E_\lambda^{(2)}\}.$$

(c) For any fixed  $a$  and  $b > M$ , there are two numbers  $\mu_0 = \mu_0(a) > 0$  and  $\lambda_0 = \lambda_0(b) > 0$ , respectively, such that the numbers  $2a$ ,  $a + b$  and  $2b$  are eigenvalues of  $H_{\mu_0, \lambda_0}$ .

(d) For any  $c \in [2m; 2M]$  there exist two numbers  $\mu_1 = \mu_1(c) > 0$  and  $\lambda_1 = \lambda_1(c) > 0$  such that the number  $c$  is an eigenvalue of  $H_{\mu_1, \lambda_1}$ .

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