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PANJARADAGI UCH ZARRACHALI SISTEMAGA MOS MODEL OPERATOR MUHIM SPEKTRINING JOYLASHUV O‘RNI

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REZYUME

Ushbu maqolada panjaradagi uchta zarrachalar sistemasiga mos model operator Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator sifatida qaralgan. Ikki o‘lchamli qo‘zg‘alishga ega chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma Fridrixs modelining spektral xossalaridan foydalanib, qaralayotgan model operatorning muhim spektri tadqiq qilingan. Muhim spektrning ikki va uch zarrachali tarmoqlari ajratilgan. Ikki zarrachali tarmoqlarning uch zarrachali tarmoqga nisbatan joylashuv o‘rni o‘rganilgan.

Kalit so‘zlar: Hilbert fazo, model operator, Fridrixs modeli, Fredgolm determinanti, muhim spektr.

1. Kirish.

Fizikaning ko‘plab sohalarida, xususan qattiq jismlar fizikasi [1], kvant maydon nazariyasi [2] kabi sohalarda panjaradagi ikki va uch zarrachali sistema Hamiltonianlariga mos model operatorlarning muhim spektri va xos qiymatlarining mavjudligi bilan bog‘liq masalalar uchrab turadi. Uch zarrachali sistemaga mos model operator muhim spektrining tuzilishini aniqlash [3, 4], xos qiymatlar sonining chekli yoki cheksiz bo‘lish shartlarini topish [5, 6, 7] alohida ahamiyat kasb etadi.

Ta’kidlash joizki, [3] maqolada kompakt qo‘zg‘alishli Fridrixs modeli spektri yordamida panjaradagi uchta zarrachalar sistemasiga mos model operator muhim spektrining joylashuv o‘rni aniqlangan. [4] maqolada esa qo‘zg‘alish operatori ikki o‘lchamli Fredgolm operatori (integral operator) bo‘lgan Fridrixs modeli uchun olingan natijalar panjaradagi lokal bo‘lmagan potensialga ega uch zarrachali model operator muhim spektrining ikki va uch zarrachali tarmoqlarining joylashuv o‘rni, tuzilishi va uni tashkil qiluvchi kesmalar sonini aniqlash imkonini bergan. [5] maqolada dispersiya funksiyasi bir nechta nuqtalarda aynimagan minimumga ega bo‘lgan holda panjaradagi uchta zarrachalar sistemasiga mos model operator uchun cheksiz sondagi xos qiymatlarning mavjudligi isbotlangan hamda xos qiymatlar soni uchun asimptotik formula topilgan.

Shuningdek, [6] maqolada uch o‘lchamli panjaradagi ikkita bir xil zarrachalar sistemasiga mos keluvchi ikkita chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma bo‘lgan bir o‘lchamli qo‘zg‘alishga ega Fridrixs modellari oilasi o‘rganilgan. Muhim spektrdan chapda yotuvchi xos qiymatlarning mavjudlik shartlari topilgan. Fridrixs modellari oilasi uchun bo‘sag‘aviy xos qiymat va nol energiyali rezonansning mavjudlik shartlari tahlil qilingan. Olingan natijalar panjaradagi uchta bir xil zarrachalar sistemasiga mos model operator xos qiymatlari sonining chekli yoki cheksiz ekanligini ko‘rsatishda hamda xos qiymatlar soni uchun asimptotik formula topishda qo‘llanilgan.

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Bundan tashqari, [7] maqolada [6] maqoladan farqli o‘laroq uch o‘lchamli panjaradagi ikkita har xil zarrachali sistemaga mos Fridrixs modellari oilasi tadqiq qilingan. Bu holda Fridrixs modeli uchun quyidagi natijalar olingan: muhim spektrdan chapda joylashgan xos qiymatlar soni aniqlangan; Fredgolm determinanti uchun asimptotik yoyilma olingan; bo‘sag‘aviy xos qiymat va virtual sathning mavjudlik shartlari topilgan. Olingan natijalar panjaradagi uchta har xil zarrachalar sistemasiga mos model operator xos qiymatlari sonining chekli yoki cheksiz ekanligini ko‘rsatishda hamda xos qiymatlar soni uchun asimptotik formula topishda qo‘llanilgan.

Aytish lozimki, [8] maqolada cheksiz separabel Hilbert fazosida aniqlangan o‘z-o‘ziga qo‘shma kompakt operatorlar tenzor yig‘indisining muhim va diskret spektrlari o‘rganilgan. [9] maqolada uch zarrachali model operator muhim spektrining tuzilishi tadqiq qilingan. Manfiy xos qiymatlarning mavjudligi isbotlangan va manfiy xos qiymatlar soni uchun baholash olingan. [10] maqolada uch zarrachali model Shryodinger operatori uchun cheksiz sondagi xos qiymatlarning mavjudligi masalasi o‘rganilgan. Model operator muhim spektridan chapda yotuvchi cheksiz sondagi xos qiymatlar mavjud bo‘lishining zaruriy va yetarlilik shartlari topilgan.

Mazkur maqolada panjaradagi uchta zarrachalar sistemasiga mos model operator Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator sifatida o‘rganilgan. Uning muhim spektrining ikki va uch zarrachali tarmoqlari ajratilgan. Bu tarmoqlarning joylashuv o‘rni ikki o‘lchamli qo‘zg‘alishga ega chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma Fridrixs modelining spektral xossalari orqali tadqiq qilingan.

2. Panjaradagi ikkita va uchta zarrachalar sistemalariga mos model operatorlar.

Dastlab zarur tushunchalarni kiritamiz. $\mathbb{T}^{\mathrm{d}} := (-\pi;\pi]^{\mathrm{d}}$ orqali d o‘lchamli torni, $L_2(\mathbb{T}^{\mathrm{d}})$ orqali $\mathbb{T}^{\mathrm{d}}$ torda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Hilbert fazosini, $L_2^{\mathrm{s}}((\mathbb{T}^{\mathrm{d}})^2)$ orqali esa $(\mathbb{T}^{\mathrm{d}})^2$ to‘plamda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) simmetrik funksiyalarning Hilbert fazosini belgilaymiz.

$L_2^{\mathrm{s}}((\mathbb{T}^{\mathrm{d}})^2)$ Hilbert fazosida ta’sir qiluvchi va

$$H_{\mu,\lambda} := H_{0,0} - \mu(V_{11} + V_{12}) + \lambda(V_{21} + V_{22}), \quad \mu, \lambda > 0 \tag{1}$$

tenglik orqali aniqlanuvchi model operatorni qaraymiz. Bunda $\mu, \lambda > 0$ ta’sirlashish parametrlari, $H_{0,0}$ operator $L_2^{\mathrm{s}}((\mathbb{T}^{\mathrm{d}})^2)$ Hilbert fazosidagi ko‘paytirish operatori bo‘lib,

$$(H_{0,0}f)(p,q) = (u(p) + u(q))\, f(p,q)$$

tenglik bilan aniqlangan.

$V_{ij},\ i,j = 1,2$ operatorlar esa $L_2((\mathbb{T}^{\mathrm{d}})^2)$ Hilbert fazosidagi lokal bo‘lmagan potensial operatorlar bo‘lib, quyidagicha aniqlangan:

$$(V_{i1}f)(p,q) = v_i(p) \int_{\mathbb{T}^{\mathrm{d}}} v_i(t) f(t,q)dt, \quad (V_{i2}f)(p,q) = v_i(q) \int_{\mathbb{T}^{\mathrm{d}}} v_i(t) f(p,t)dt.$$

Bu yerda $u(\cdot)$ va $v_i(\cdot)$, $i = 1,2$ funksiyalar $\mathbb{T}^{\mathrm{d}}$ torda aniqlangan haqiqiy qiymatli uzluksiz funksiyalar.

(1) tenglik yordamida ta’sir qiluvchi $H_{\mu,\lambda}$ model operator $L_2^{\mathrm{s}}((\mathbb{T}^{\mathrm{d}})^2)$ Hilbert fazosida chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma bo‘ladi.

Mazkur maqolaning asosiy natijalarini bayon qilish maqsadida $H_{\mu,\lambda}$ model operator bilan bir qatorda $L_2(\mathbb{T}^{\mathrm{d}})$ Hilbert fazosida

$$h_{\mu,\lambda} := h_{0,0} - \mu k_1 + \lambda k_2$$

kabi ta’sir qiluvchi operatorni qaraymiz. Bu yerda $h_{0,0}$ operator ko‘paytirish operatori bo‘lib, $L_2(\mathbb{T}^{\mathrm{d}})$ Hilbert fazosida

$$(h_{0,0}g)(p) = u(p)g(p)$$

tenglik yordamida aniqlangan. k_i, $i = 1, 2$ potensial operatorlari bo‘lib, $L_2(\mathbb{T}^{\mathrm{d}})$ Hilbert fazosida

$$(k_i g)(p) = v_i(p) \int_{\mathbb{T}^{\mathrm{d}}} v_i(t) f(t) dt, \ i = 1, 2$$

kabi aniqlangan.

Ko‘rinib turibdiki, k_1 va k_2 integral operatorlar bir o‘lchamlidir. Shu sababli, $h_{\mu,\lambda}$ operatorga ikki o‘lchamli qo‘zg‘alishga ega Fridrixs modeli deb ataladi. Bu operatorning $L_2(\mathbb{T}^{\mathrm{d}})$ Hilbert fazosidagi chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma operator ekanligini oson ko‘rsatish mumkin.

Chekli o‘lchamli qo‘zg‘alishlarga muhim spektrning o‘zgarmasligi haqidagi mashhur Veyl teoremasiga ko‘ra $h_{\mu,\lambda}$ Fridrixs modelining muhim spektri $h_{0,0}$ operatorning muhim spektri bilan ustma-ust tushadi. Bizga yaxshi ma’lumki, $h_{0,0}$ ko‘paytirish operatori sof muhim spektrga ega va

$$\sigma_{\mathrm{ess}}(h_{0,0}) = [E_1; E_2]$$

tenglik o‘rinlidir. Bu yerda E_1 va E_2 sonlari

$$E_1 = \min_{p \in \mathbb{T}^{\mathrm{d}}} u(p), \quad E_2 = \max_{p \in \mathbb{T}^{\mathrm{d}}} u(p)$$

tengliklar yordamida aniqlanadi. Demak, $h_{\mu,\lambda}$ Fridrixs modelining muhim spektri uchun

$$\sigma_{\mathrm{ess}}(h_{\mu,\lambda}) = [E_1; E_2]$$

tenglikni hosil qilamiz.

Har bir $\mu, \lambda > 0$ sonlari uchun $\mathbb{C} \setminus [E_1; E_2]$ sohada regulyar bo‘lgan

$$\Delta_{\mu,\lambda}(z) := \Delta_\mu^{(1)}(z)\Delta_\lambda^{(2)}(z) + \mu\lambda I_{12}^2(z)$$

funksiyani qaraymiz, bunda

$$\Delta_\mu^{(1)}(z) := 1 - \mu I_{11}(z), \quad \Delta_\lambda^{(2)}(z) := 1 + \lambda I_{22}(z),$$

$$I_{\alpha\beta}(z) := \int_{\mathbb{T}^{\mathrm{d}}} \frac{v_\alpha(t) v_\beta(t)}{u(t) - z} dt, \quad \alpha, \beta = 1, 2.$$

Odatda $\Delta_{\mu,\lambda}(\cdot)$ funksiyaga $h_{\mu,\lambda}$ Fridrixs modeliga mos Fredgolm determinanti deyiladi hamda bu funksiya $h_{\mu,\lambda}$ Fridrixs modelining diskret spektrini tadqiq qilishda muhim ahamiyat kasb etadi.

Quyida $h_{\mu,\lambda}$ Fridrixs modeli xos qiymatlari va $\Delta_{\mu,\lambda}(\cdot)$ funksiya nollari orasidagi bog‘lanishni ifodalovchi lemmani keltiramiz.

1-lemma. $z \in \mathbb{C} \setminus [E_1; E_2]$ soni $h_{\mu,\lambda}$ Fridrixs modelining xos qiymati bo‘lishi uchun $\Delta_{\mu,\lambda}(z) = 0$ bo‘lishi zarur va yetarlidir.

supp$v(\cdot)$ orqali $v(\cdot)$ funksiya tashuvchisini, mes(Ω) orqali $\Omega \subset \mathbb{T}^{\mathrm{d}}$ to‘plamning Lebeg o‘lchovini belgilaymiz.

2-lemma. A) $h_{\mu,\lambda}$ Fridrixs modeli E_1 dan chapda va E_2 dan o‘ngda joylashgan ko‘pi bilan bittadan sodda xos qiymatga ega.

B) Faraz qilaylik,

$$\mathrm{mes}(\mathrm{supp}\{v_1(\cdot)\} \cap \mathrm{supp}\{v_1(\cdot)\}) = 0 \tag{2}$$

shart bajarilsin. U holda $z \in (-\infty; E_1)$ $(z \in (E_2; +\infty))$ soni $h_{\mu,\lambda}$ Fridrixs modelining xos qiymati bo‘lishi uchun $\Delta_\mu^{(1)}(z) = 0$ $\left(\Delta_\lambda^{(2)}(z) = 0\right)$ tenglik o‘rinli bo‘lishi zarur va yetarlidir.

Yuqoridagi 1-lemma va 2-lemmalarning isbotlari [11] ishda keltirilgan.

3. Panjaradagi uchta zarrachalar sistemasiga mos model operator muhim spektrining tuzilishi.

Keyingi izlanishlarda (2) shart hamisha bajarilishini talab qilamiz.

$I_{\alpha\alpha}(\cdot)$, $\alpha = 1, 2$ funksiya $(-\infty; E_1)$ va $(E_2; +\infty)$ oraliqlarda monoton o‘suvchi bo‘lganligi uchun Lebeg integrali ostida limitga o‘tish haqidagi teoremaga ko‘ra

$$I_{11}(E_1) = \lim_{z \to E_1 - 0} I_{11}(z), \quad I_{22}(E_2) = \lim_{z \to E_2 + 0} I_{22}(z)$$

chekli yoki cheksiz limitlar mavjud bo‘ladi.

Ushbu

$$|I_{\alpha\alpha}(E_\alpha)| < +\infty, \quad \alpha = 1, 2$$

shartlar bajarilganda quyidagicha

$$\mu_0 := (I_{11}(E_1))^{-1}, \quad \lambda_0 := -(I_{22}(E_2))^{-1}$$

belgilash kiritamiz.

$h_{\mu,\lambda}$ Fridrixs modelining muhim spektridan tashqarida xos qiymatlari mavjud bo‘lgan holda ularni mos ravishda $E_1(\mu)$ va $E_2(\lambda)$ orqali belgilaymiz. Bunda $E_1(\mu) < E_1$ va $E_2(\lambda) > E_2$.

1-teorema. Faraz qilaylik, $|I_{\alpha\alpha}(E_\alpha)| = +\infty, \alpha = 1, 2$ bo‘lsin.

U holda $\mu, \lambda > 0$ parametrlarning ixtiyoriy qiymatida $H_{\mu,\lambda}$ model operator ikkita $2E_1(\mu)$ va $2E_2(\lambda)$ oddiy xos qiymatlarga ega bo‘lib,

$$\sigma_{\mathrm{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2];$$

$$\sigma_{\mathrm{pp}}(H_{\mu,\lambda}) = \{2E_1(\mu); E_1(\mu) + E_2(\lambda); 2E_2(\lambda)\}$$

tengliklar o‘rinli.

2-teorema. Faraz qilaylik, $|I_{\alpha\alpha}(E_\alpha)| < +\infty, \alpha = 1, 2$ bo‘lsin.

A) Agar $0 < \mu \leq \mu_0$ va $0 < \lambda \leq \lambda_0$ bo‘lsa, u holda

$$\sigma_{\rm ess}(H_{\mu,\lambda}) = [2E_1; 2E_2]; \quad \sigma_{\rm pp}(H_{\mu,\lambda}) = \emptyset$$

tengliklar o‘rinli;

B) Agar $\mu > \mu_0$ va $0 < \lambda \leq \lambda_0$ bo‘lsa, u holda $H_{\mu,\lambda}$ model operator bitta $2E_1(\mu)$ oddiy xos qiymatga ega bo‘lib,

$$\sigma_{\rm ess}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2];$$

$$\sigma_{\rm pp}(H_{\mu,\lambda}) = \{2E_1(\mu)\}$$

tengliklar o‘rinli;

C) Agar $0 < \mu \leq \mu_0$ va $\lambda > \lambda_0$ bo‘lsa, u holda $H_{\mu,\lambda}$ model operator bitta $2E_2(\lambda)$ oddiy xos qiymatga ega bo‘lib,

$$\sigma_{\rm ess}(H_{\mu,\lambda}) = [2E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2];$$

$$\sigma_{\rm pp}(H_{\mu,\lambda}) = \{2E_2(\lambda)\}$$

tengliklar o‘rinli;

D) Agar $\mu > \mu_0$ va $\lambda > \lambda_0$ bo‘lsa, u holda $H_{\mu,\lambda}$ model operator ikkita $2E_1(\mu)$ va $2E_2(\lambda)$ oddiy xos qiymatlarga ega bo‘lib,

$$\sigma_{\rm ess}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2];$$

$$\sigma_{\rm pp}(H_{\mu,\lambda}) = \{2E_1(\mu); E_1(\mu) + E_2(\lambda); 2E_2(\lambda)\}$$

tengliklar o‘rinli.

Maqolada keltirilgan 1-teorema va 2-teoremalar isbotlari [12,13] ishlarda keltirilgan.

Ushbu belgilashlarni kiritamiz:

$$\mu_1 := (I_{11}(2E_1 - E_2))^{-1}, \quad \lambda_1 := -(I_{22}(2E_2 - E_1))^{-1}.$$

3-teorema. Faraz qilaylik, $|I_{\alpha\alpha}(E_\alpha)| = +\infty, \alpha = 1, 2$ bo‘lsin.

A) Agar $0 < \mu \leq \mu_1$ va $0 < \lambda \leq \lambda_1$ bo‘lsa, u holda

$$\sigma_{\rm ess}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli;

B) Agar $\mu > \mu_1$ va $0 < \lambda \leq \lambda_1$ bo‘lsa, u holda

$$\sigma_{\rm ess}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli bo‘lib, $E_1(\mu) + E_2 < 2E_1$ bo‘ladi;

C) Agar $0 < \mu \leq \mu_1$ va $\lambda > \lambda_1$ bo‘lsa, u holda

$$\sigma_{\rm ess}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2]$$

tengliklar o‘rinli bo‘lib, $2E_2 < E_2(\lambda) + E_1$ bo‘ladi;

D) Agar $\mu > \mu_1$ va $\lambda > \lambda_1$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli bo‘lib, $E_1(\mu) + E_2 < 2E_1$ va $2E_2 < E_2(\lambda) + E_1$ bo‘ladi.

Isboti. Faraz qilaylik, $|I_{\alpha\alpha}(E_\alpha)| = +\infty, \alpha = 1, 2$ bo‘lsin. U holda 1-teoremaga ko‘ra $H_{\mu,\lambda}$ model operatorning muhim spektri uchun

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli.

A) Agar $0 < \mu \le \mu_1$ bo‘lsa, u holda $\Delta^{(1)}_\mu(2E_1 - E_2) \ge \Delta^{(1)}_{\mu_1}(2E_1 - E_2) = 0$ bo‘ladi, ya’ni $\Delta^{(1)}_\mu(2E_1 - E_2) \ge 0$. $E_1(\mu) < E_1$ soni $h_{\mu,\lambda}$ Fridrixs modelini xos qiymati bo‘lgani uchun 2-lemmaning B) tasdig‘iga ko‘ra $\Delta^{(1)}_\mu(E_1(\mu)) = 0$ bo‘ladi. Bu munosabatlardan esa $\Delta^{(1)}_\mu(2E_1 - E_2) \ge \Delta^{(1)}_\mu(E_1(\mu))$ tengsizlikni hosil qilamiz. $\Delta^{(1)}_\mu(\cdot)$ funksiya aniqlanishiga ko‘ra $(-\infty; E_1)$ oraliqda uzluksiz va kamayuvchi bo‘lganligi uchun $E_1(\mu) + E_2 \ge 2E_1$ tengsizlik o‘rinli bo‘ladi.

Agar $0 < \lambda \le \lambda_1$ bo‘lsa, u holda $\Delta^{(2)}_\lambda(2E_2 - E_1) \ge \Delta^{(2)}_{\lambda_1}(2E_2 - E_1) = 0$ bo‘ladi, ya’ni $\Delta^{(2)}_\lambda(2E_2 - E_1) \ge 0$. $E_2(\lambda) > E_2$ soni $h_{\mu,\lambda}$ Fridrixs modelini xos qiymati bo‘lgani uchun 2-lemmaning B) tasdig‘iga ko‘ra $\Delta^{(2)}_\lambda(E_2(\lambda)) = 0$ bo‘ladi. Bu munosabatlardan esa $\Delta^{(2)}_\lambda(2E_2 - E_1) \ge \Delta^{(2)}_\lambda(E_2(\lambda))$ tengsizlikni hosil qilamiz. $\Delta^{(2)}_\lambda(\cdot)$ funksiya aniqlanishiga ko‘ra $(E_2; +\infty)$ oraliqda uzluksiz va o‘suvchi bo‘lganligi uchun $2E_2 \ge E_2(\lambda) + E_1$ tengsizlik o‘rinli bo‘ladi. Yuqoridagi munosabatlar

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli ekanligini bildiradi. Bu esa teoremaning A) tasdig‘ini isbotlaydi.

D) Faraz qilaylik, $\mu > \mu_1$ bo‘lsin. U holda $\Delta^{(1)}_\mu(2E_1 - E_2) < \Delta^{(1)}_{\mu_1}(2E_1 - E_2) = 0$ munosabat o‘rinlidir, ya’ni $\Delta^{(1)}_\mu(2E_1 - E_2) < 0$. $E_1(\mu) < E_1$ soni $h_{\mu,\lambda}$ Fridrixs modelini xos qiymati bo‘lgani uchun 2-lemmaning B) tasdig‘iga ko‘ra $\Delta^{(1)}_\mu(E_1(\mu)) = 0$ bo‘ladi. Bu munosabatlardan esa $\Delta^{(1)}_\mu(2E_1 - E_2) < \Delta^{(1)}_\mu(E_1(\mu))$ tengsizlikni hosil qilamiz. $\Delta^{(1)}_\mu(\cdot)$ funksiya $(-\infty; E_1)$oraliqda uzluksiz va kamayuvchi bo‘lganligi uchun $E_1(\mu) + E_2 < 2E_1$ tengsizlik bajariladi.

Agar $\lambda > \lambda_1$ bo‘lsa, u holda $\Delta^{(2)}_\lambda(2E_2 - E_1) < \Delta^{(2)}_{\lambda_1}(2E_2 - E_1) = 0$ bo‘ladi, ya’ni $\Delta^{(2)}_\lambda(2E_2 - E_1) \ge 0$. $E_2(\lambda) > E_2$ soni $h_{\mu,\lambda}$ Fridrixs modelini xos qiymati bo‘lgani uchun 2-lemmaning B) tasdig‘iga ko‘ra $\Delta^{(2)}_\lambda(E_2(\lambda)) = 0$ bo‘ladi. Bu munosabatlardan esa $\Delta^{(2)}_\lambda(2E_2 - E_1) < \Delta^{(2)}_\lambda(E_2(\lambda))$ tengsizlikni hosil qilamiz. $\Delta^{(2)}_\lambda(\cdot)$ funksiya $(E_2; +\infty)$ oraliqda uzluksiz va o‘suvchi bo‘lganligi uchun $2E_2 < E_2(\lambda) + E_1$ tengsizlik o‘rinli bo‘ladi va shu orqali teoremaning D) tasdig‘i isbotlanadi. Teoremaning B) va C) tasdiqlari ham yuqoridagi kabi isbotlanadi.

1-eslatma. Ushbu $\mu_0 < \mu_1$ va $\lambda_0 < \lambda_1$ tengsizliklar o‘rinli.

4-teorema. Faraz qilaylik, $|I_{\alpha\alpha}(E_\alpha)| < +\infty, \alpha = 1, 2$ bo‘lsin.

A_1) Agar $0 < \mu \le \mu_0$ va $0 < \lambda \le \lambda_0$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [2E_1; 2E_2]$$

tenglik o‘rinli;

A_2) Agar $\mu_0 < \mu \leq \mu_1$ va $0 < \lambda \leq \lambda_0$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; 2E_2]$$

tenglik o‘rinli;

A_3) Agar $\mu > \mu_1$ va $0 < \lambda \leq \lambda_0$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2]$$

kabi bo‘lib, $E_1(\mu) + E_2 < 2E_1$ tengsizlik o‘rinli;

B_1) Agar $0 < \mu \leq \mu_0$ va $\lambda_0 < \lambda \leq \lambda_1$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [2E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli;

B_2) Agar $\mu_0 < \mu \leq \mu_1$ va $\lambda_0 < \lambda \leq \lambda_1$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli;

B_3) Agar $\mu > \mu_1$ va $\lambda_0 < \lambda \leq \lambda_1$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; E_2(\lambda) + E_2]$$

kabi bo‘lib, $E_1(\mu) + E_2 < 2E_1$ tengsizlik o‘rinli;

C_1) Agar $0 < \mu \leq \mu_0$ va $\lambda > \lambda_1$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [2E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2]$$

kabi bo‘lib, $2E_2 < E_2(\lambda) + E_1$ tengsizlik o‘rinli;

C_2) Agar $\mu_0 < \mu \leq \mu_1$ va $\lambda > \lambda_1$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2]$$

kabi bo‘lib, $2E_2 < E_2(\lambda) + E_1$ tengsizlik o‘rinli;

C_3) Agar $\mu > \mu_1$ va $\lambda > \lambda_1$ bo‘lsa, u holda

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2] \cup [E_2(\lambda) + E_1; E_2(\lambda) + E_2]$$

tenglik o‘rinli bo‘lib, $E_1(\mu) + E_2 < 2E_1$ va $2E_2 < E_2(\lambda) + E_1$ bo‘ladi.

Isbot. Faraz qilaylik, $|I_{\alpha\alpha}(E_\alpha)| < +\infty$, $\alpha = 1, 2$ shart bajarilsin.

A_1) Agar $0 < \mu \leq \mu_0$ va $0 < \lambda \leq \lambda_0$ bo‘lsa, u holda 2-teoremaning A) tasdig‘iga ko‘ra $H_{\mu,\lambda}$ model operatorning muhim spektri uchun

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [2E_1; 2E_2]$$

tenglik o‘rinli bo‘ladi.

A_2) Agar $\mu_0 < \mu \leq \mu_1$ va $0 < \lambda \leq \lambda_0$ bo‘lsa, u holda 2-teoremaning B) tasdig‘iga ko‘ra $H_{\mu,\lambda}$ model operatorning muhim spektri uchun

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; E_1(\mu) + E_2] \cup [2E_1; 2E_2]$$

tenglik o‘rinli bo‘ladi hamda $\Delta_\mu^{(1)}(2E_1 - E_2) \geq \Delta_{\mu_1}^{(1)}(2E_1 - E_2) = 0$ munosabat o‘rinli, ya’ni $\Delta_\mu^{(1)}(2E_1 - E_2) \geq 0$. $E_1(\mu) < E_1$ soni $h_{\mu,\lambda}$ Fridrixs modelining xos qiymati bo‘lgani uchun 2-lemmaning B) tasdig‘iga ko‘ra $\Delta_\mu^{(1)}(E_1(\mu)) = 0$ bo‘ladi. Bu munosabatlardan esa $\Delta_\mu^{(1)}(2E_1 - E_2) \geq \Delta_\mu^{(1)}(E_1(\mu))$ tengsizlikni hosil qilamiz. $\Delta_\mu^{(1)}(\cdot)$ funksiyaning aniqlanishiga ko‘ra bu funksiya $(-\infty; E_1)$ oraliqda uzluksiz va kamayuvchi hamda $E_1(\mu) + E_2 \geq 2E_1$ tengsizlik o‘rinli bo‘ladi. Yuqoridagi munosabatlardan

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = [E_1(\mu) + E_1; 2E_2]$$

tenglik kelib chiqadi. Bu esa teoremaning A_2 tasdig‘ini isbotlaydi. Teoremaning qolgan tasdiqlari ham A_1 va A_2 tasdiqlar kabi isbotlanadi.

Xulosa. Maqolada qattiq jismlar fizikasi, kvant maydon nazariyasi va boshqa ko’plab sohalarda uchraydigan panjaradagi uch zarrachali sistema Hamiltonianiga mos $H_{\mu,\lambda}$ model operator qaralgan, bu yerda $\mu, \lambda > 0$ - ta’sirlashish parametrlari. $H_{\mu,\lambda}$ model operator muhim spektrining joylashuv o‘rni ikki o‘lchamli qo‘zg‘alishga ega $h_{\mu,\lambda}$ Fridrixs modeli spektral xossalaridan foydalanib o‘rganilgan.

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РЕЗЮМЕ

В данной статье модельный оператор, соответствующий системе трех частиц на решетке, рассматривается как линейный, ограниченный и самосопряженный оператор в гильбертовом пространстве. Используя спектральные свойства линейной, ограниченной и самосопряженной модели Фридрихса с двумерным возмущением, исследован существенный спектр рассматриваемого модельного оператора. Выделены двухчастичные и трехчастичные ветви существенного спектра. Изучено расположение двухчастичных ветвей относительно трехчастичной ветви.

Ключевые слова: Гильбертово пространство, модельный оператор, модель Фридрихса, определитель Фредгольма, существенный спектр.

RESUME

In this paper, the model operator corresponding to the three particle systems in the lattice is considered as a linear, bounded and self-adjoint operator in Hilbert space. Using the spectral properties of the linear, bounded and self-adjoint Friedrichs model with two-dimensional perturbation, the essential spectrum of the considered model operator was investigated. Two-particle and three-particle branches of the essential spectrum are singled out. The position of two-particle branches with respect to a three-particle branch is studied.

Key words: Hilbert space, model operator, Friedrichs model, Fredholm determinant, essential spectrum.