O'ZBEKISTON RESPUBLIKASI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI
ANDIJON DAVLAT UNIVERSITETI

ZAMONAVIY MATEMATIKANING NAZARIY ASOSLARI VA AMALIY MASALALARI

Respublika ilmiy-amaliy anjumani materiallari to'plami

II

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Andijon, 28 mart 2022 yil

МИНИСТЕРСТВОВЫСШЕГОИСРЕДНЕГОСПЕЦИАЛЬНОГООБРАЗОВАНИЯ
РЕСПУБЛИКИ УЗБЕКИСТАН
АНДИЖАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

Сборник материалов Республиканской научно-практической конференции

ТЕОРЕТИЧЕСКИЕ ОСНОВЫ И ПРИКЛАДНЫЕ ЗАДАЧИ СОВРЕМЕННОЙ МАТЕМАТИКИ

II

Андижан, 28 марта 2022 года

MINISTRY OF HIGHER AND SECONDARY SPECIAL EDUCATION
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ANDIJAN STATE UNIVERSITY

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THEORETICAL FOUNDATIONS AND APPLIED PROBLEMS OF MODERN MATHEMATICS

II

Andijan, March 28, 2022

Zamonaviy matematikaning nazariy asoslari va amaliy masalalari. Respublika ilmiy-amaliy anjumani materiallari to‘plami. II qism. Andijon, 2022 yil. 324 bet.

Ushbu to‘plam O’zbekiston Respublikasi Vazirlar Mahkamasining 2022 yil 7 martdagi №101-F sonli farmoyishi bilan tasdiqlangan “2022 yilda Xalqaro va Respublika miqyosida o’tkaziladigan ilmiy va ilmiy-texnik tadbirlar rejasi”ga ko’ra 2022 yil 28 mart kuni Andijon davlat universitetida o’tkazilgan “Zamonaviy matematikaning nazariy asoslari va amaliy masalalari” mavzusida Respublika miqyosidagi ilmiy-amaliy anjumaniga kelib tushgan tezislar matnlaridan tashkil topgan.

To’plamga kiritilgan tezislar mazmuni, ilmiyligi va dalillarning haqqoniyligi uchun mualliflar mas’uldirlar.

Mas’ul muharrir: Umrzaqov Nodirbek

Muharrirlar: Nishonov Tulanmirza
Zaynobiddinov Ibrohimjon
Atabayev Odiljon

Anjuman materiallari to’plami Andijon davlat universiteti Ilmiy kengashining 2022 yil 17 fevraldagi 8- yig’ilishi qarori bilan nashrga tavsiya etilgan.

$$\left(V_\mu f\right)(p) = \frac{\lambda}{2\pi}\sin p\int_T \sin q\, f(q)\, dq, \qquad f\in L^{2,o}(T).$$

V operatorning rangi ko'pi bilan birga teng bo'lganligi uchun, muhim spektr turg'unligi haqidagi Veyl teoremasiga ko'ra

$$\sigma_{ess}\left(H_\mu\right) = \sigma(H_0) = \left[0,\frac{25}{8}\right].$$

Teorema: $a)\ \lambda > \frac{20}{25-7\sqrt{5}}$ bo'lsin. U holda H_λ operator muhim spektrdan chapda yagona z_λ xos qiymatga ega va unga mos xos funksiya

$$f(p) = \frac{C\lambda\sin p}{2-\cos p-\cos 2p - z_\lambda}, \quad C - normallashtiruvchi\ o'zgarmas$$

ko'rinishda.

$b)\ \lambda < \frac{20}{25-7\sqrt{5}}$ bo'lsin. U holda H_λ operator muhim spektrdan chapda xos qiymatga ega emas.

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PANJARADAGI UCH ZARRACHALI SISTEMAGA MOS MODEL OPERATORNING XOS FUNKSIYALARI UCHUN FADDEYEV TENGLAMASI

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Buxoro davlat universiteti

$d\in N$ natural soni uchun $T^d :\equiv (-\pi;\pi]^d$ orqali d o'lchamli torni, $L_2^s((T^d)^2)$ orqali $(T^d)^2$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) simmetrik funksiyalarning Hilbert fazosini belgilaymiz. $L_2^s((T^d)^2)$ Hilbert fazosida ta'sir qiluvchi

$$H_{\mu,\lambda} := H_0 - \mu(V_{11}+V_{12}) + \lambda(V_{21}+V_{22}) \tag{1}$$

tenglik orqali aniqlanuvchi operatorni qaraymiz. Bunda $\mu,\lambda > 0$ ta'sirlashish parametrlari, H_0 operator $u(\cdot,\cdot)$ funksiyaga ko'paytirish operatori:

$$(H_0 f)(x,y) = u(x,y)f(x,y),$$

$V_{ij},\ i,j=1,2-$ operatorlar esa lokal bo'lmagan potensial operatorlari:

$$(V_{i1}f)(x,y) = \upsilon_i(x)\int_{T^d}\upsilon_i(t)f(t,y)dt, \quad (V_{i2}f)(x,y) = \upsilon_i(y)\int_{T^d}\upsilon_i(t)f(x,t)dt;$$

$u(\cdot,\cdot)-$ $(T^d)^2$ aniqlangan haqiqiy qiymatli uzluksiz, simmetrik funksiya, $\upsilon_i(\cdot), i=1,2$ lar esa T^d da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar.

Funksional analiz elementlaridan foydalanib, (1) tenglik yordamida ta'sir qiluvchi $H_{\mu,\lambda}$ operatorning $L_2^s((T^d)^2)$ Hilbert fazosidagi chiziqli, chegaralangan va o'z-o'ziga qo'shma ekanligini ko'rsatish mumkin.

Mazkur ishning asosiy natijalarini bayon qilish maqsadida $H_{\mu,\lambda}$ operator bilan bir qatorda $L_2(T^d)$ Hilbert fazosida

$$h_{\mu,\lambda}(x) := h_{0,0}(x) - \mu\upsilon_1 + \lambda\upsilon_2,\ \mu,\lambda > 0,\ x \in T^d$$

kabi ta'sir qiluvchi va Fridrixs modellari oilasi deb ataluvchi operatorni qaraymiz. Bu yerda

$$(h_{0,0}(x)f)(y) = u(x,y)f(y),$$

$$(\upsilon_i f)(y) = \upsilon_i(y)\int_{T^d} \upsilon_i(t)f(t)dt\,.$$

Kirtilgan $h_{\mu,\lambda}(x)$ operator $L_2(T^d)$ Hilbert fazosidagi chiziqli, chegaralangan va o'z-o'ziga qo'shma ekanligini oson ko'rsatish mumkin.

Chekli o'lchamli qo'zg'alishlarda muhim spektrning o'zgarmasligi haqidagi Veyl teoremasiga ko'ra

$$\sigma_{ess}(h_{\mu,\lambda}(x)) = [m(x); M(x)]$$

tenglik o'rinlidir, bu yerda $m(x)$ va $M(x)$ sonlari

$$m(x) := \min_{y\in T^d} u(x,y), \quad M(x) := \max_{y\in T^d} u(x,y).$$

formulalar orqali aniqlanadi.

Har bir fiksirlangan $\mu,\lambda > 0$ va $x \in T^d$ lar uchun $C \setminus [m(x), M(x)]$ sohada regulyar bo'lgan

$$\Delta_{\mu,\lambda}(z) := \Delta_\mu^{(1)}(z)\Delta_\lambda^{(2)}(z) + \mu\lambda\,(I_{12}(z))^2$$

funksiyani qaraymiz. Odatda $\Delta_{\mu,\lambda}(z)$ funksiya $h_{\mu,\lambda}(x)$ operatroga mos Fredgolm detrminanti deyiladi.

Quyidagi belgilashlarni kiritamiz:

$$m := \min_{x,y\in T^d} u(x,y), \quad M := \max_{x,y\in T^d} u(x,y),$$

$$\sum_{\mu,\lambda} := \bigcup_{x\in T^d} \sigma_{disc}(h_{\mu,\lambda}(x)) \bigcup [m; M].$$

Har bir fiksirlangan $\mu,\lambda > 0$ va $z \in C \setminus [m; M]$ sonlari uchun $L_2^{(2)}(T^d)$ fazoda

$$A_{\mu,\lambda}(z) := \begin{pmatrix} A_{11}(\mu,z) & A_{12}(\lambda,z) \\ A_{21}(\mu,z) & A_{22}(\lambda,z) \end{pmatrix}, \quad K_{\mu,\lambda}(z) := \begin{pmatrix} K_{11}(\mu,z) & K_{12}(\lambda,z) \\ K_{21}(\mu,z) & K_{22}(\lambda,z) \end{pmatrix}$$

kabi aniqlanuvchi 2-tartibli operatorli matritsalarni qaraymiz. Ularning matritsaviy elementlari

$$(A_{11}(\mu,z)\varphi_1)(x) = \varphi_1(x) - \mu\varphi_1(x)\int_{T^d} \frac{\upsilon_1^2(t)}{u(x,t)-z}dt\,;$$

$$(A_{12}(\mu,z)\varphi_2)(x) = \lambda\varphi_2(x)\int_{T^d} \frac{\upsilon_1(t)\upsilon_2(t)}{u(x,t)-z}dt\ ;$$

$$(A_{21}(\mu,z)\varphi_1)(x) = -\mu\ \varphi_1(x)\int_{T^d} \frac{\upsilon_1(t)\upsilon_2(t)}{u(x,t)-z}dt\ ;$$

$$(A_{22}(\mu,z)\varphi_2)(x) = \varphi_2(x) + \lambda\,\varphi_1(x)\int_{T^d} \frac{\upsilon_2^2(t)}{u(x,t)-z}dt\,;$$

$$(K_{11}(\mu,z)\varphi_1)(x) = \mu\,\upsilon_1(x)\int_{T^d} \frac{\upsilon_1(t)\varphi_1(t)}{u(x,t)-z}dt\,;$$

$$(K_{12}(\mu,z)\varphi_2)(x) = -\lambda\,\upsilon_2(x)\int_{T^d} \frac{\upsilon_1(t)\varphi_2(t)}{u(x,t)-z}dt\ ;$$

$$(K_{21}(\mu,z)\varphi_1)(x) = \mu\,\upsilon_1(x)\int_{T^d} \frac{\upsilon_2(t)\varphi_1(t)}{u(x,t)-z}dt\,;$$

$$(K_{22}(\mu,z)\varphi_2)(x) = -\lambda\,\upsilon_2(x)\int_{T^d} \frac{\upsilon_2(t)\varphi_2(t)}{u(x,t)-z}dt\ ;$$

tengliklar orqali ta'sir qiladi.

1-teorema. Agar $\mu,\lambda > 0$ va $z \in C \setminus \sum_{\mu,\lambda}$ sonlari uchun $A_{\mu,\lambda}(z)$ teskarilanuvchan operator bo‘lib, uning $A^{-1}_{\mu,\lambda}(z)$ teskari operatori

$$A^{-1}_{\mu,\lambda}(z) = \frac{1}{\Delta_{\mu,\lambda}(z)}\begin{pmatrix} A_{22}(\lambda,z) & -A_{12}(\lambda,z) \\ -A_{21}(\mu,z) & A_{11}(\mu,z) \end{pmatrix}$$

ko‘rinishga ega.

Endi har bir fiksirlangan $\mu,\lambda > 0$ va $z \in C \setminus \sum_{\mu,\lambda}$ sonlari uchun $L_2^{(2)}(T^d)$ Hilbert fazosida

$$T_{\mu,\lambda}(z) = A^{-1}_{\mu,\lambda}(z)K_{\mu,\lambda}(z)$$

operatorni qaraymiz.

Quyidagi teorema $H_{\mu,\lambda}$ va $T_{\mu,\lambda}(z)$ operatorlarning xos qiymatlari orasidagi bog‘lanishni ifodalaydi.

2-teorema. $z \in C \setminus \sum_{\mu,\lambda}$ soni $H_{\mu,\lambda}$ operatorning xos qiymati bo'lishi uchun 1 soni $T_{\mu,\lambda}(z)$ operatorning xos qiymati bo‘lishi zarur va yetarli. Bundan tashqari z va 1 sonlarining karraligi ustma-ust tushadi.

Odatda $\varphi = T_{\mu,\lambda}(z)\varphi$ operatorli tenglamaga $H_{\mu,\lambda}$ model operator xos funksiyalariga mos Faddeyev tenglamasi deyiladi. Bu tenglama yordamida $H_{\mu,\lambda}$ operator muhim spektrini tavsiflash mumkin.

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