

Non-Stationary Vibration of a Viscoelastic Cylindrical Shell with a Viscous Fluid

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Abstract: As part of this study, we consider the problem of no stationary interaction of a viscoelastic cylindrical shell of limited length with a viscous fluid. To illustrate the relationship between the forces and displacements of the shell, the Boltzmann-Volterra heredity integral was used. In this case, general solutions of the linearized Navier-Stokes equations for a viscous fluid are applied. We apply the Laplace transform to the equations in time, and the Fourier transform in coordinates on the constrained interval. It has been established that with increasing time, the influence of the compressibility of the liquid manifests itself as an increase in displacement.

Keywords: shell, viscous fluid, heredity integral, axisymmetric problem, Laplace transforms.

1. Introduction.

The study of viscoelastic body's dynamic reactivity to non-stationary influences is very important right now, and there's a lot of interest in it. It should be noted that numerical methods are often used to calculate bodies interacting with a medium. Along with, analytical methods make it possible to reveal many features of dynamic deformation that cannot be obtained numerically. Works [1, 2] are devoted to the study of the problem of interaction of a viscoelastic shell with a viscous compressible fluid. In [3-5], an axisymmetric problem is considered, and in [5, 6] the general (non-axisymmetric) problem of wave propagation in an isotropic homogeneous shell filled with a viscous fluid is studied. In [7], a solution was obtained for the plane case of no stationary interaction of a cylindrical shell with an ideal fluid. We look at the non-stationary interaction of a viscoelastic cylindrical shell of limited length with a viscous fluid in this work. The Boltzmann-Volterra heredity integral was employed to describe the relationship between the shell's forces and displacements [8,9]. General solutions of the linearized Stokes-Navier equations for a viscous fluid are used in this situation [10, 11]. The shell motion equation is described using the Kirchhoff-Love assumptions.

2. Methods.

2.1. Problem Statements and Solution Method

Consider the radius, length, and thickness of a viscoelastic cylindrical shell. The motion equations for a shell that satisfy the Kirchhoff-Love hypothesis are given in the form

$$\begin{aligned} & \frac{1}{R^2} \frac{\partial u_r}{\partial \theta} + \left(\frac{1-\nu_1}{2} \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right) u_0 + \frac{1-\nu_1}{2R} \frac{\partial^2 u_z}{\partial z \partial \theta} - \\ & - \int_0^t R_p(t-\tau) \left(\frac{1}{R^2} \frac{\partial u_r(r,\tau)}{\partial \theta} + \left(\frac{1-\nu_1}{2} \frac{\partial^2}{\partial z^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} \right) u_\theta(r,\tau) + \frac{1-\nu_1}{2R} \frac{\partial^2 u_z(r,\tau)}{\partial z \partial \theta} \right) d\tau = \quad (1) \\ & = \frac{1-\nu_1^2}{E_0 h} \left(\rho_1 h \frac{\partial^2 u_r}{\partial t^2} + q_\theta|_{r=R} + p_{r\theta}|_{r=R} \right); \\ & \frac{\nu_1}{R^2} \frac{\partial u_r}{\partial z} + \frac{1-\nu_1}{2R} \frac{\partial^2 u_\theta}{\partial z \partial \theta} + \frac{1}{R^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \left(\frac{\partial^2}{\partial z^2} + \frac{1-\nu_1}{2R^2} \frac{\partial^2}{\partial \theta^2} \right) u_z - \\ & - \int_0^t R_p(t-\tau) \left(\frac{\nu_1}{R^2} \frac{\partial u_r(r,\tau)}{\partial z} + \frac{1-\nu_1}{2R} \frac{\partial^2 u_\theta(r,\tau)}{\partial z \partial \theta} + \frac{1}{R^2} \frac{\partial^2 u_\theta(r,\tau)}{\partial \theta^2} + \left(\frac{\partial^2}{\partial z^2} + \frac{1-\nu_1}{2R^2} \frac{\partial^2}{\partial \theta^2} \right) u_z(r,\tau) \right) d\tau = \\ & = \frac{1-\nu_1^2}{E_0 h} \left(\rho_1 h \frac{\partial^2 u_z}{\partial t^2} + q_z|_{r=R} + p_{rz}|_{r=R} \right); \end{aligned}$$

Here u_z, u_r, u_θ – components of the vector of viscoelastic displacements of points of the middle surface of the shell; E_0 and ν_1 – instants modulus of elasticity and Poisson's ratio; p_z, p_r, p_θ – given non-stationary effects on an absolutely rigid surface; $p_{rz}, p_{rr}, p_{r\theta}$ – components of the stress tensor of a viscous fluid, taking into account compressibility;

$$\begin{aligned} p_{rr} &= -p + \lambda_1 \left(\frac{\partial v_r}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} \right) + 2\mu \frac{\partial v_r}{\partial r}; \\ p_{r\theta} &= \mu \left(\frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right); \quad p_{rz} = \mu \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right); \end{aligned}$$

v_z, v_r, v_θ, p – parameters of the velocity field resulting from elastic shell deformations; μ – viscosity coefficient; $\lambda_1 = -\frac{2}{3}\mu$ – second viscosity coefficient.

Taking into account the assumption that for a thin shell, the radial stresses are equal to zero, the generalized Hooke's law can be written as:

$$\sigma_z = \frac{\tilde{E}_n(1-R^*)}{1-\tilde{\nu}_n^2} (\varepsilon_z + \tilde{\nu}_n \varepsilon_\theta); \quad \sigma_\theta = \frac{\tilde{E}_n(1-R^*)}{1-\tilde{\nu}_n^2} (\tilde{\nu}_n \varepsilon_z + \varepsilon_\theta); \quad \sigma_{z\theta} = \sigma_{\theta z} = \frac{\tilde{E}_n(1-R^*)}{1+\tilde{\nu}_n} \varepsilon_{z\theta},$$

Where R^* - integral operator with relaxation kernel $\Gamma^*(t)$ acting on a function φ :

$$\tilde{E}_n = E_{0n} (1 - R_n^*); \quad \tilde{\nu}_n = \nu_{0n} + \frac{1 - 2\nu_{0n}}{2} R_n^* ;$$

$$R_n^* f(t) = m_n \int_{-\infty}^t \mathcal{D}_{-1/2}^{(n)}(-\beta_n, t-\tau) f(\tau) d\tau \quad (a)$$

or

$$\tilde{E}_n f(t) = E_{0n} \left[f(t) - \int_0^t R_{En}(t-\tau) f(\tau) d\tau \right] \quad (b) \quad (2)$$

τ - the time preceding the moment of observation; $\varphi(t)$ - arbitrary function of time; $R_{En}(t-\tau)$ - relaxation core; E_n - instant modulus of elasticity; ν_n - Poisson's ratio; m, β_n - material parameters.

As the kernel of the integral operator, we will use the fractional-exponential Rabotnov function [5]

$$m_n \mathfrak{E}_{-1/2}^{(n)}(-\beta, t) = \frac{m_n}{t^{1/2}} \sum_{j=0}^{\infty} \Gamma[(j+1)/2] (-\beta_n)^j t^{j/2}$$

где $\Gamma(j) = \int_0^{\infty} \exp(-y) y^{j-1} dy$ - gamma function.

The kinematic conditions are satisfied on the surface of the deformable shell

$$v_z = \frac{\partial u_r}{\partial t}, v_r = \frac{\partial u_r}{\partial t}, v_\theta = \frac{\partial u_\theta}{\partial t} \quad (r = R) \quad (3)$$

We also assume that the ends of the shell have a pivotally movable support and there are no deformations at the initial moment of time [6].

In [1,2,3], a general solution of the Navier - Stokes equations for a viscous fluid was obtained. According to [3], we obtain

$$\vec{v} = \frac{\partial}{\partial t} \left[\vec{\nabla} \psi + \vec{\nabla} \times \vec{e}_3 \chi_1 + \vec{\nabla} \times (\vec{\nabla} \times \vec{e}_3 \chi_2) \right];$$

$$p = p_0 \left(\frac{\lambda_1 + 2\mu}{p_0} \Delta - \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \psi; \quad (4)$$

$$\frac{\partial p}{\partial t} = \frac{p_0}{a_0^2} \left(\frac{\lambda_1 + 2\mu}{p_0} \Delta - \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \psi;$$

Here the potentials ψ, χ_1, χ_2 satisfy the equations

$$\left[\Pi \Delta - \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} \right] \psi = 0, \Pi = \left(1 + \frac{\lambda_1 + 2\mu}{p_0} \right); \quad (5)$$

$$\frac{\partial \chi_1}{\partial t} - \nu \Delta \chi_1 = 0, \frac{\partial \chi_2}{\partial t} - \nu \Delta \chi_2 = 0.$$

From (4), following the works [3,4,12], we obtain the representation of the components of the velocity vector through the potentials

$$v_r = \frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial r} + \frac{1}{r} \frac{\partial \chi_1}{\partial \theta} + \frac{\partial^2 \chi_2}{\partial r \partial z} \right); v_\theta = \frac{\partial}{\partial t} \left(\frac{1}{r} \frac{\partial \psi}{\partial \theta} - \frac{\partial \chi_1}{\partial r} + \frac{1}{r} \frac{\partial^2 \chi_2}{\partial \theta \partial z} \right);$$

ISSN 2792-4025 (online), Published under Volume: 2 Issue: 1 in January-2022

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$$v_z = \frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial z} - \frac{\partial^2 \chi_2}{\partial r^2} - \frac{1}{r} \frac{\partial \chi_2}{\partial r} - \frac{1}{r^2} \frac{\partial \chi_2}{\partial \theta} \right) \quad (6)$$

For non-stationary problems, the solution of equations (5) will be sought in the form

$$\psi_g = f_{1g}(r, z, t) \cos m\theta; \quad \chi_{1g} = f_{2g}(r, z, t) \sin m\theta; \quad \chi_{2g} = f_{3g}(r, z, t) \cos m\theta \quad (7)$$

We apply the integral Laplace transform with respect to time t and the Fourier transform with respect to coordinate z to equations (5) on a finite interval [8]. Then equations (5), taking into account relations (3), are reduced to the following equation:

$$\frac{d^2 f_{gi}^{FL}}{dr^2} + \frac{1}{r} \frac{df_{gi}^{FL}}{dr} + \left(\eta_i^2 - \frac{m^2}{r^2} \right) f_{gi}^{FL} = 0 \quad (i = 1, 2, 3) \quad (8)$$

Here

$$\eta_1 = \sqrt{\beta^2 + \frac{3\lambda^2}{3a_0 + \lambda v}}; \quad \eta_2 = \eta_3 = \sqrt{\beta^2 + \frac{\lambda}{v}}$$

$$f_1^{FL} = \int_0^\infty e^{-\lambda t} \left[\int_0^t f_{1g}(r, z, t) \sin \beta z dz \right] dt;$$

$$f_2^{FL} = \int_0^\infty e^{-\lambda t} \left[\int_0^t f_{2g}(r, z, t) \sin \beta z dz \right] dt;$$

$$f_3^{FL} = \int_0^\infty e^{-\lambda t} \left[\int_0^t f_{3g}(r, z, t) \sin \beta z dz \right] dt;$$

β and λ – Fourier and Laplace transform parameters; index FL denotes an image of the corresponding size.

Solution of ordinary differential equations (8) with variable coefficients, represent in the form

$$f_i^{FL} = A_i Z_m(\eta_i r), \quad (9)$$

where $Z_m(x)$ – cylindrical functions.

For potentials (5) in the image area, solutions (9) lead to the following expressions:

$$\begin{aligned} \psi_g^{FL} &= [A_1 I_m(\eta_1 r) + B_1 K_m(\eta_1 r)] \cos m\theta; \quad \chi_{1g}^{FL} = [A_3 I_m(\eta_2 r) + B_3 K_m(\eta_2 r)] \sin m\theta; \\ \chi_{2g}^{FL} &= [A_2 I_m(\eta_3 r) + B_2 K_m(\eta_3 r)] \cos m\theta, \end{aligned} \quad (10)$$

where $I_m(x)$ - modified Bessel functions; $K_m(x)$ - Macdonald functions.

Let us find a solution for the axisymmetric case ($m = 0$) Translating (9) into the image area and substituting (10), we obtain

$$v_r^{FL} = \lambda [A_1 \eta_1 I_1(\eta_1 r) - B_1 \eta_1 K_1(\eta_1 r) - \beta \eta_2 A_2 I_1(\eta_2 r) + \beta \eta_2 B_2 K_1(\eta_2 r)];$$

$$v_z^{FL} = \lambda \left[\beta A_1 I_0(\eta_1 r) + \beta B_1 K_0(\eta_1 r) - \beta \eta_2^2 A_2 I_0(\eta_2 r) - \eta_2^2 B_2 K_0(\eta_2 r) \right]; \quad (11)$$

$$p^{FL} = \frac{4}{3} p_0 \nu \lambda \left(\eta_1^2 - \beta^2 - \frac{3\lambda}{4\nu} \right) \left[A_1 I_0(\eta_1 r) + B_1 K_0(\eta_1 r) \right]$$

Values (11) allow us to write the load (10) due to the velocity field in the form

$$p_{rz}^{FL} \Big|_{r=R} = a_1 A_1 + a_2 B_1 + a_3 A_2 + a_4 B_2; \quad p_{rr}^{FL} \Big|_{r=R} = b_1 A_1 + b_2 B_1 + b_3 A_2 + b_4 B_2;$$

Here

$$a_1 = 2\eta_1 \beta \mu \lambda I_1(\eta_1 R); \quad a_2 = 2\eta_1 \beta \mu K_1(\eta_1 R);$$

$$a_3 = \eta_2 (\eta_2^2 + \beta^2) \mu \lambda I_1(\eta_2 R); \quad a_4 = -\eta_2 (\eta_2^2 + \beta^2) \mu \lambda K_1(\eta_2 R);$$

$$b_1 = -\lambda \left(\frac{2}{3} \mu \beta^2 + p_0 \lambda \right) I_0(\eta_1 R) + \frac{2\mu \lambda \eta_1}{R} I_1(\eta_1 R);$$

$$b_2 = -\lambda \left(\frac{2}{3} \mu \beta^2 + p_0 \lambda \right) K_0(\eta_1 R) + \frac{2\mu \lambda \eta_1}{R} K_1(\eta_1 R);$$

$$b_3 = 2\mu \lambda \beta \eta_2^2 \left[I_0(\eta_2 R) - \frac{I_1(\eta_2 R)}{\eta_2 R} \right]; \quad b_4 = 2\mu \lambda \beta \eta_2^2 \left[K_0(\eta_2 R) - \frac{K_1(\eta_2 R)}{\eta_2 R} \right].$$

The solution of integro-differential equations (1) taking into account (7) takes the form

$$u_r^{FL} = M_1 A_1 + M_2 B_1 + M_3 A_2 + M_4 B_2 + H_1;$$

$$u_\theta^{FL} = M_5 A_1 + M_6 B_1 + M_7 A_2 + M_8 B_2 + H_2,$$

$$u_z^{FL} = M_9 A_1 + M_{10} B_1 + M_{11} A_2 + M_{12} B_2 + H_3,$$

where

$$m_1 = \frac{a_1 c_3 - b_1 c_2}{\Delta_1} \Gamma_k^R; \quad m_2 = \frac{a_2 c_3 - b_2 c_2}{\Delta_1} \Gamma_k^R; \quad m_3 = \frac{a_3 c_3 - b_3 c_2}{\Delta_1} \Gamma_k^R; \quad m_4 = \frac{a_4 c_3 - b_4 c_2}{\Delta_1} \Gamma_k^R;$$

$$m_5 = \frac{b_1 c_1 - a_1 c_2}{\Delta_1} \Gamma_k^R; \quad m_6 = \frac{b_2 c_1 - a_2 c_2}{\Delta_1} \Gamma_k^R; \quad m_7 = \frac{b_3 c_1 - a_3 c_2}{\Delta_1} \Gamma_k^R; \quad m_8 = \frac{b_4 c_1 - a_4 c_2}{\Delta_1} \Gamma_k^R;$$

$$H_1 = \frac{c_3 h_1 - c_2 h_2}{\Delta_1}; \quad H_2 = \frac{c_1 h_2 - c_2 h_1}{\Delta_1}; \quad H_3 = \frac{c_2 h_1 - c_3 a_1}{\Delta_1}; \quad h_1 = q_r^{FL} \Big|_{r=R}; \quad h_2 = q_z^{FL} \Big|_{r=R};$$

$$c_1 = -\left(\frac{E_0 h \beta^2}{1 - \nu_1^2} + \lambda^2 p_1 h \right); \quad c_2 = \frac{\nu_1 E_0 h \beta^2}{R(1 - \nu_1^2)};$$

$$c_3 = - \left[\frac{E_0 h}{R^2 (1 - \nu_1^2)} + \frac{E_0 h^2 \beta^4}{12 (1 - \nu_1^2)} + \lambda^2 p_1 h \right]; \quad \Delta_1 = c_1 c_3 - c_2^2$$

To determine the coefficients A_1, A_2, B_1, B_2 we use kinematic conditions (3), which in the image area have the form

$$v_r^{FL} = \lambda u_r^{FL}; \quad v_z^{FL} = \lambda u_z^{FL}; \quad r = R$$

as well as conditions at infinity or on the axis ($r=0$).

3. Results and analysis

Since at $r \rightarrow \infty$ $I_0(x)$ and $I_1(x) \rightarrow \infty$, then in solutions (11), (10) it is necessary to put $A_1 = A_2 = 0$. Coefficient values B_1 and B_2 we determine from conditions (11):

$$B_1 = \frac{1}{\Delta_2} (\Pi_2 k_8 - \Pi_1 k_6); \quad B_2 = \frac{1}{\Delta_2} (\Pi_1 k_5 - \Pi_2 k_7), \tag{12}$$

where

$$k_5 = -\eta_1 K_1(\eta_1 R) - m_6; \quad k_6 = \beta \eta_2 K_1(\eta_2 R) - m_8;$$

$$k_7 = \beta K_0(\eta_1 R) - m_2; \quad k_8 = -\eta_2^2 K_0(\eta_2 R) - m_4; \quad \Delta_2 = k_5 k_8 - k_6 k_7$$

With internal interaction on the shell axis ($r = R$) functions $K_0(x), K_1(x) \rightarrow \infty$, therefore, in solutions (10), (11) one should put:

$$B_1 = B_2 = 0$$

Values A_1 and A_2 we also determine from the boundary conditions (12)

$$A_1 = \frac{1}{\Delta_3} (\Pi_2 k_4 - \Pi_1 k_2); \quad A_2 = \frac{1}{\Delta_3} (\Pi_1 k_1 - \Pi_2 k_3);$$

Here

$$k_1 = \eta_1 I_1(\eta_1 R) - m_5; \quad k_2 = -\beta \eta_2 I_1(\eta_2 R) - m_7;$$

$$k_4 = -\eta_2^2 I_0(\eta_2 R) - m_3; \quad \Delta_3 = k_1 k_4 - k_2 k_3.$$

If we consider the problem of the interaction of a fluid located between elastic coaxial cylinders with radius R_1 and R_2 ($R_1 > R_2$), then, due to the limited distance between the surfaces, in solutions (11) and (12) all coefficients should be preserved.

In this case, these solutions must additionally satisfy the boundary condition on the second shell, which formally coincides with (12). Then A_1, A_2, B_1, B_2 will be determined by the system

$$A_1 k_{11} + B_1 k_{12} + A_2 k_{13} + B_2 k_{14} = \Pi_{11}; A_1 k_{15} + B_1 k_{16} + A_2 k_{17} + B_2 k_{18} = \Pi_{12};$$

$$A_1 k_{21} + B_1 k_{22} + A_2 k_{23} + B_2 k_{24} = \Pi_{21}; A_1 k_{25} + B_1 k_{26} + A_2 k_{27} + B_2 k_{28} = \Pi_{22};$$

Here index 1 corresponds to a shell with radius R_1 , and index 2 is for a shell with a radius R_2 . As a numerical example, solutions of the internal interaction were studied, when surface deformations were caused by a change in pressure according to the law $\Delta p_0 = -ap_0 \cos \omega t$

The transition from the image to the original was carried out numerically using piecewise polynomial functions with the following parameter values: $E = 2.1 \cdot 10^{11}$ Pa; $\nu = 0.58 \frac{cm^2}{c}$; $p_1 = 2.5 \cdot 10^4 \Pi a$; $p_2 = 1.86 \cdot 10^4 \Pi a$

$$R = 5.0 cm; l = 20 cm; h = 0.20 cm;$$

On fig. 1 shows the change in the contour stresses of the shell from time to time for various thicknesses.

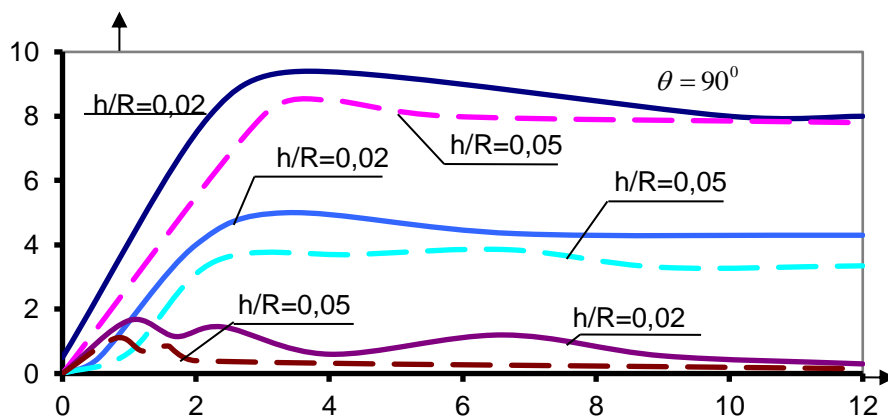


Fig.1. The dependence of the dimensionless annular voltage from at different h/R

On fig.2. shows the change in the radial contour stresses of the shell from length for a viscous compressible fluid (solid line) and for a viscous incompressible fluid (dashed line), respectively, at $t = 0.4$ and $t = 16$

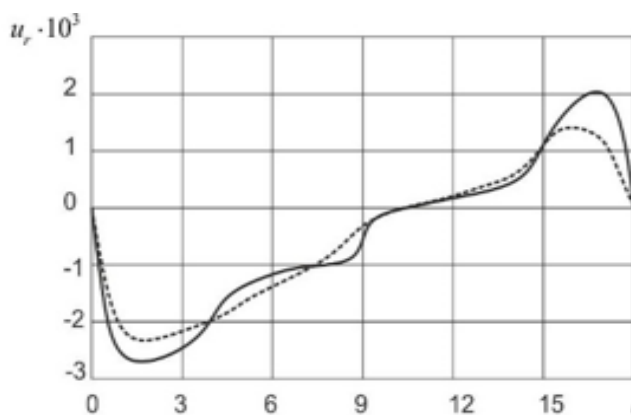


Fig.2. Variation of radial contour stress on shell length

Conclusions

1. Based on the Laplace and Fourier integral transformation approach, a method for estimating non-stationary oscillations of a shell with a viscous fluid has been devised in this study.
2. As time passes, the liquid's compressibility appears to have less of an impact.

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