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# Non-Stationary Vibration of a Viscoelastic Cylindrical Shell with a Viscous Fluid 

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#### Abstract

As part of this study, we consider the problem of no stationary interaction of a viscoelastic cylindrical shell of limited length with a viscous fluid. To illustrate the relationship between the forces and displacements of the shell, the Boltzmann-Volterra heredity integral was used. In this case, general solutions of the linearized Navier-Stokes equations for a viscous fluid are applied. We apply the Laplace transform to the equations in time, and the Fourier transform in coordinates on the constrained interval. It has been established that with increasing time, the influence of the compressibility of the liquid manifests itself as an increase in displacement.


Keywords: shell, viscous fluid, heredity integral, axisymmetric problem, Laplace transforms.

## 1. Introduction.

The study of viscoelastic body's dynamic reactivity to non-stationary influences is very important right now, and there's a lot of interest in it. It should be noted that numerical methods are often used to calculate bodies interacting with a medium. Along with, analytical methods make it possible to reveal many features of dynamic deformation that cannot be obtained numerically. Works [1,2] are devoted to the study of the problem of interaction of a viscoelastic shell with a viscous compressible fluid. In [3-5], an axisymmetric problem is considered, and in [5, 6] the general (non-axisymmetric) problem of wave propagation in an isotropic homogeneous shell filled with a viscous fluid is studied. In [7], a solution was obtained for the plane case of no stationary interaction of a cylindrical shell with an ideal fluid. We look at the non-stationary interaction of a viscoelastic cylindrical shell of limited length with a viscous fluid in this work. The Boltzmann-Voltaire heredity integral was employed to describe the relationship between the shell's forces and displacements [8,9]. General solutions of the linearized Stokes-Navier equations for a viscous fluid are used in this situation [10, 11]. The shell motion equation is described using the Kirchhoff-Love assumptions.

## 2. Methods.

### 2.1. Problem Statements and Solution Method

Consider the radius, length, and thickness of a viscoelastic cylindrical shell. The motion equations for a shell that satisfy the Kirchhoff-Love hypothesis are given in the form

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$$
\begin{align*}
& \frac{1}{R^{2}} \frac{\partial u_{r}}{\partial \theta}+\left(\frac{1-v_{1}}{2} \frac{\partial^{2}}{\partial z^{2}}+\frac{1}{R^{2}} \frac{\partial^{2}}{\partial \theta^{2}}\right) u_{0}+\frac{1-v_{1}}{2 R} \frac{\partial^{2} u_{z}}{\partial z \partial \theta}- \\
& -\int_{0}^{t} R_{p}(t-\tau)\left(\frac{1}{R^{2}} \frac{\partial u_{r}(r, \tau)}{\partial \theta}+\left(\frac{1-v_{1}}{2} \frac{\partial^{2}}{\partial z^{2}}+\frac{1}{R^{2}} \frac{\partial^{2}}{\partial \theta^{2}}\right) u_{\theta}(r, \tau)+\frac{1-v_{1}}{2 R} \frac{\partial^{2} u_{z}(r, \tau)}{\partial z \partial \theta}\right) d \tau=  \tag{1}\\
& \quad=\frac{1-v_{1}^{2}}{E_{0} h}\left(\rho_{1} h \frac{\partial^{2} u_{r}}{\partial t^{2}}+\left.q_{\theta}\right|_{r=R}+\left.p_{r \theta}\right|_{r=R}\right) ; \\
& \begin{array}{l}
\frac{v_{1}}{R^{2}} \frac{\partial u_{r}}{\partial z}+\frac{1-v_{1}}{2 R} \frac{\partial^{2} u_{\theta}}{\partial z \partial \theta}+\frac{1}{R^{2}} \frac{\partial^{2} u_{\theta}}{\partial \theta^{2}}+\left(\frac{\partial^{2}}{\partial z^{2}}+\frac{1-v_{1}}{2 R^{2}} \frac{\partial^{2}}{\partial \theta^{2}}\right) u_{z}- \\
-\int_{0}^{t} R_{p}(t-\tau)\left(\frac{v_{1}}{R^{2}} \frac{\partial u_{r}(r, \tau)}{\partial z}+\frac{1-v_{1}}{2 R} \frac{\partial^{2} u_{\theta}(r, \tau)}{\partial z \partial \theta}+\frac{1}{R^{2}} \frac{\partial^{2} u_{\theta}(r, \tau)}{\partial \theta^{2}}+\left(\frac{\partial^{2}}{\partial z^{2}}+\frac{1-v_{1}}{2 R^{2}} \frac{\partial^{2}}{\partial \theta^{2}}\right) u_{z}(r, \tau)\right) d \tau= \\
\quad=\frac{1-v_{1}^{2}}{E_{0} h}\left(\rho_{1} h \frac{\partial^{2} u_{z}}{\partial t^{2}}+\left.q_{z}\right|_{r=R}+\left.p_{r z}\right|_{r=R}\right) ;
\end{array}
\end{align*}
$$

Here $u_{z}, u_{r}, u_{\theta}$ - components of the vector of viscoelastic displacements of points of the middle surface of the shell; $E_{0}$ and $v_{1}$ - instants modulus of elasticity and Poisson's ratio; $p_{z}, p_{r}, p_{\theta}-$ given non-stationary effects on an absolutely rigid surface; $p_{r z}, p_{r r}, p_{r \theta}-$ components of the stress tensor of a viscous fluid, taking into account compressibility;

$$
\begin{aligned}
& p_{r r}=-p+\lambda_{1}\left(\frac{\partial v_{r}}{\partial r}+\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{\partial v_{z}}{\partial z}+\frac{v_{r}}{r}\right)+2 \mu \frac{\partial v_{r}}{\partial r} \\
& p_{r \theta}=\mu\left(\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}+\frac{\partial v_{\theta}}{\partial r}-\frac{v_{\theta}}{r}\right) ; \quad p_{r z}=\mu\left(\frac{\partial v_{z}}{\partial r}+\frac{\partial v_{r}}{\partial z}\right)
\end{aligned}
$$

$v_{z}, v_{r}, v_{\theta}, p-$ parameters of the velocity field resulting from elastic shell deformations; $\mu$ viscosity coefficient; $\lambda_{1}=-\frac{2}{3} \mu$ - second viscosity coefficient.
Taking into account the assumption that for a thin shell, the radial stresses are equal to zero, the generalized Hooke's law can be written as:

$$
\sigma_{z}=\frac{\tilde{E}_{n}\left(1-R^{*}\right)}{1-\tilde{v}_{n}^{2}}\left(\varepsilon_{z}+\tilde{v}_{n} \varepsilon_{\theta}\right) ; \sigma_{\theta}=\frac{\tilde{E}_{n}\left(1-R^{*}\right)}{1-\tilde{v}_{n}^{2}}\left(\tilde{v}_{n} \varepsilon_{z}+\varepsilon_{\theta}\right) ; \sigma_{z \theta}=\sigma_{\theta z}=\frac{\tilde{E}_{n}\left(1-R^{*}\right)}{1+\tilde{v}_{n}} \varepsilon_{z \theta},
$$

Where $R^{*}$ - integral operator with relaxation kernel $\Gamma^{*}(\mathrm{t})$ acting on a function $\varphi$ :

$$
\begin{aligned}
& \tilde{E}_{n}=E_{0 n}\left(1-R_{n}^{\bullet}\right) ; \quad \tilde{v}_{n}=v_{0 n}+\frac{1-2 v_{0 n}}{2} R_{n}^{\bullet} ; \\
& R_{n}^{\bullet} f(t)=m_{n} \int_{-\infty}^{t} Э_{-1 / 2}^{(n)}\left(-\beta_{n}, t-\tau\right) f(\tau) d \tau
\end{aligned}
$$

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or
$\tilde{E}_{n} f(t)=E_{0 n}\left[f(t)-\int_{0}^{t} R_{E n}(t-\tau) f(\tau) d \tau\right]$
$\tau$ - the time preceding the moment of observation; $\varphi(t)$ - arbitrary function of time; $R_{E n}(t-\tau)$ relaxation core; $E_{n}$ - instant modulus of elasticity; $v_{n}$ - Poisson's ratio; $m, \beta_{n}$ - material parameters.

As the kernel of the integral operator, we will use the fractional-exponential Rabotnov function [5]
$m_{n} Э_{-1 / 2}^{(n)}(-\beta, t)=\frac{m_{n}}{t^{1 / 2}} \sum_{j=0}^{\infty} \Gamma[(j+1) / 2]\left(-\beta_{n}\right)^{j} t^{j / 2}$
где $\Gamma(\mathrm{j})=\int_{0}^{\infty} \exp (-y) y^{j-1} d y$ - gamma function.
The kinematic conditions are satisfied on the surface of the deformable shell
$v_{z}=\frac{\partial u_{r}}{\partial t}, v_{r}=\frac{\partial u_{r}}{\partial t}, v_{\theta}=\frac{\partial u_{\theta}}{\partial t} \quad(r=R)$
We also assume that the ends of the shell have a pivotally movable support and there are no deformations at the initial moment of time [6].

In [1,2,3], a general solution of the Navier - Stokes equations for a viscous fluid was obtained. According to [3], we obtain

$$
\begin{align*}
& \vec{v}=\frac{\partial}{\partial t}\left[\vec{\nabla} \psi+\vec{\nabla} \times \overrightarrow{e_{3}} \chi_{1}+\vec{\nabla} \times\left(\vec{\nabla} \times \overrightarrow{e_{3}} \chi_{2}\right)\right] \\
& p=p_{0}\left(\frac{\lambda_{1}+2 \mu}{p_{0}} \Delta-\frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} \psi \tag{4}
\end{align*}
$$

$$
\frac{\partial p}{\partial t}=\frac{p_{0}}{a_{0}^{2}}\left(\frac{\lambda_{1}+2 \mu}{p_{0}} \Delta-\frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} \psi
$$

Here the potentials $\psi, \chi_{1}, \chi_{2}$ satisfy the equations

$$
\begin{align*}
& {\left[\Pi \Delta-\frac{1}{a_{0}^{2}} \frac{\partial^{2}}{\partial t^{2}}\right] \psi=0, \Pi=\left(1+\frac{\lambda_{1}+2 \mu}{p_{0}}\right)}  \tag{5}\\
& \frac{\partial \chi_{1}}{\partial t}-v \Delta \chi_{1}=0, \frac{\partial \chi_{2}}{\partial t}-v \Delta \chi_{2}=0
\end{align*}
$$

From (4), following the works [3,4,12], we obtain the representation of the components of the velocity vector through the potentials

$$
v_{r}=\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial r}+\frac{1}{r} \frac{\partial \chi_{1}}{\partial \theta}+\frac{\partial^{2} \chi_{2}}{\partial r \partial z}\right) ; v_{\theta}=\frac{\partial}{\partial t}\left(\frac{1}{r} \frac{\partial \psi}{\partial \theta}-\frac{\partial \chi_{1}}{\partial r}+\frac{1}{r} \frac{\partial^{2} \chi_{2}}{\partial \theta \partial z}\right) ;
$$

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$v_{z}=\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial z}-\frac{\partial^{2} \chi_{2}}{\partial r^{2}}-\frac{1}{r} \frac{\partial \chi_{2}}{\partial r}-\frac{1}{r^{2}} \frac{\partial \chi_{2}}{\partial \theta}\right)$

For non-stationary problems, the solution of equations (5) will be sought in the form
$\psi_{g}=f_{1 g}(r, z, t) \cos m \theta ; \quad \chi_{1 g}=f_{2 g}(r, z, t) \sin m \theta ; \chi_{2 g}=f_{3 g}(r, z, t) \cos m \theta$.
We apply the integral Laplace transform with respect to time $t$ and the Fourier transform with respect to coordinate z to equations (5) on a finite interval [8]. Then equations (5), taking into account relations (3), are reduced to the following equation:
$\frac{d^{2} f_{g i}^{F L}}{d r^{2}}+\frac{1}{r} \frac{d f_{g i}^{F L}}{d r}+\left(\eta_{l}^{2}-\frac{m^{2}}{r^{2}}\right) f_{g i}^{F L}=0 \quad(i=1,2,3)$
Here
$\eta_{1}=\sqrt{\beta^{2}+\frac{3 \lambda^{2}}{3 a_{0}+\lambda v}} ; \quad \eta_{2}=\eta_{3}=\sqrt{\beta^{2}+\frac{\lambda}{v}} ;$
$f_{1}^{F L}=\int_{0}^{\infty} e^{-\lambda t}\left[\int_{0}^{t} f_{1 g}(r, z, t) \sin \beta z d z\right] d t ;$
$f_{2}^{F L}=\int_{0}^{\infty} e^{-\lambda t}\left[\int_{0}^{t} f_{2 g}(r, z, t) \sin \beta z d z\right] d t ;$
$f_{3}^{F L}=\int_{0}^{\infty} e^{-\lambda t}\left[\int_{0}^{t} f_{3 g}(r, z, t) \sin \beta z d z\right] d t ;$
$\beta$ and $\lambda$ - Fourier and Laplace transform parameters; index $F L$ denotes an image of the corresponding size.

Solution of ordinary differential equations (8) with variable coefficients, represent in the form
$f_{i}^{F L}=A_{i} Z_{m}\left(\eta_{i} r\right)$,
where $Z_{m}(x)$ - cylindrical functions.
For potentials (5) in the image area, solutions (9) lead to the following expressions:
$\psi_{g}^{F L}=\left[A_{1} I_{m}\left(\eta_{1} r\right)+B_{1} K_{m}\left(\eta_{1} r\right)\right] \cos m \theta ; \chi_{1 g}{ }^{F L}=\left[A_{3} I_{m}\left(\eta_{2} r\right)+B_{3} K_{m}\left(\eta_{2} r\right)\right] \sin m \theta ;$
$\chi_{2 g}{ }^{F L}=\left[A_{2} I_{m}\left(\eta_{3} r\right)+B_{2} K_{m}\left(\eta_{3} r\right)\right] \cos m \theta$,
where $I_{m}(x)$-modified Bessel functions; $K_{m}(x)$ - Macdonald functions.
Let us find a solution for the axisymmetric case $(m=0)$ Translating (9) into the image area and substituting (10), we obtain
$v_{r}^{F L}=\lambda\left[A_{1} \eta_{1} I_{1}\left(\eta_{1} r\right)-B_{1} \eta_{1} K_{1}\left(\eta_{1} r\right)-\beta \eta_{2} A_{2} I_{1}\left(\eta_{2} r\right)+\beta \eta_{2} B_{2} K_{1}\left(\eta_{2} r\right)\right] ;$

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$$
\begin{align*}
& v_{z}^{F L}=\lambda\left[\beta A_{1} I_{0}\left(\eta_{1} r\right)+\beta B_{1} K_{0}\left(\eta_{1} r\right)-\beta \eta_{2}^{2} A_{2} I_{0}\left(\eta_{2} r\right)-\eta_{2}^{2} B_{2} K_{0}\left(\eta_{2} r\right)\right]  \tag{11}\\
& p^{F L}=\frac{4}{3} p_{0} v \lambda\left(\eta_{1}^{2}-\beta^{2}-\frac{3 \lambda}{4 v}\right)\left[A_{1} I_{0}\left(\eta_{1} r\right)+B_{1} K_{0}\left(\eta_{1} r\right)\right]
\end{align*}
$$

Values (11) allow us to write the load (10) due to the velocity field in the form
$\left.p_{r z}^{F L}\right|_{r=R}=a_{1} A_{1}+a_{2} B_{1}+a_{3} A_{2}+a_{4} B_{2} ;\left.\quad p_{r r}^{F L}\right|_{r=R}=b_{1} A_{1}+b_{2} B_{1}+b_{3} A_{2}+b_{4} B_{2} ;$
Here
$a_{1}=2 \eta_{1} \beta \mu \lambda I_{1}\left(\eta_{1} R\right) ; a_{2}=2 \eta_{1} \beta \mu K_{1}\left(\eta_{1} R\right) ;$
$a_{3}=\eta_{2}\left(\eta_{2}^{2}+\beta^{2}\right) \mu \lambda I_{1}\left(\eta_{2} R\right) ; a_{4}=-\eta_{2}\left(\eta_{2}^{2}+\beta^{2}\right) \mu \lambda K_{1}\left(\eta_{2} R\right) ;$
$b_{1}=-\lambda\left(\frac{2}{3} \mu \beta^{2}+p_{0} \lambda\right) I_{0}\left(\eta_{1} R\right)+\frac{2 \mu \lambda \eta_{1}}{R} I_{1}\left(\eta_{1} R\right)$;
$b_{2}=-\lambda\left(\frac{2}{3} \mu \beta^{2}+p_{0} \lambda\right) K_{0}\left(\eta_{1} R\right)+\frac{2 \mu \lambda \eta_{1}}{R} K_{1}\left(\eta_{1} R\right) ;$
$b_{3}=2 \mu \lambda \beta \eta_{2}^{2}\left[I_{0}\left(\eta_{2} R\right)-\frac{I_{1}\left(\eta_{2} R\right)}{\eta_{2} R}\right] ; b_{4}=2 \mu \lambda \beta \eta_{2}^{2}\left[K_{0}\left(\eta_{2} R\right)-\frac{K_{1}\left(\eta_{2} R\right)}{\eta_{2} R}\right]$.

The solution of integro-differential equations (1) taking into account (7) takes the form
$u_{r}^{F L}=M_{1} A_{1}+M_{2} B_{1}+M_{3} A_{2}+M_{4} B_{2}+\mathrm{H}_{1} ;$
$u_{\theta}^{F L}=M_{5} A_{1}+M_{6} B_{1}+M_{7} A_{2}+M_{8} B_{2}+\mathrm{H}_{2}$,
$u_{z}^{F L}=M_{9} A_{1}+M_{10} B_{1}+M_{11} A_{2}+M_{12} B_{2}+\mathrm{H}_{3}$,
where

$$
\begin{aligned}
& m_{1}=\frac{a_{1} c_{3}-b_{1} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; m_{2}=\frac{a_{2} c_{3}-b_{2} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; m_{3}=\frac{a_{3} c_{3}-b_{3} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; m_{4}=\frac{a_{4} c_{3}-b_{4} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; \\
& m_{5}=\frac{b_{1} c_{1}-a_{1} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; m_{6}=\frac{b_{2} c_{1}-a_{2} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; m_{7}=\frac{b_{3} c_{1}-a_{3} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; m_{8}=\frac{b_{4} c_{1}-a_{1} c_{2}}{\Delta_{1}} \Gamma_{k}^{R} ; \\
& \mathrm{H}_{1}=\frac{c_{3} h_{1}-c_{2} h_{2}}{\Delta_{1}} ; \mathrm{H}_{2}=\frac{c_{1} h_{2}-c_{2} h_{1}}{\Delta_{1}} ; \mathrm{H}_{3}=\frac{c_{2} h_{1}-c_{3} a_{1}}{\Delta_{1}} ; h_{1}=\left.q_{r}^{F L}\right|_{r=R} ; h_{2}=\left.q_{z}^{F L}\right|_{r=R} ; \\
& c_{1}=-\left(\frac{E_{0} h \beta^{2}}{1-v_{1}^{2}}+\lambda^{2} p_{1} h\right) ; c_{2}=\frac{v_{1} E_{0} h \beta^{2}}{R\left(1-v_{1}^{2}\right)} ;
\end{aligned}
$$

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$c_{3}=-\left[\frac{E_{0} h}{R^{2}\left(1-v_{1}^{2}\right)}+\frac{E_{0} h^{2} \beta^{4}}{12\left(1-v_{1}^{2}\right)}+\lambda^{2} p_{1} h\right] ; \quad \Delta_{1}=c_{1} c_{3}-c_{2}^{2}$
To determine the coefficients $A_{1}, A_{2}, B_{1}, B_{2}$ we use kinematic conditions (3), which in the image area have the form

$$
v_{r}^{F L}=\lambda u_{r}^{F L} ; v_{z}^{F L}=\lambda u_{z}^{F L} ; \quad r=R
$$

as well as conditions at infinity or on the axis $(\Gamma=0)$.

## 3. Results and analysis

Since at $r \rightarrow \infty I_{0}(x)$ and $I_{1}(x) \rightarrow \infty$, then in solutions (11), (10) it is necessary to put $A_{1}=A_{2}=0$ Coefficient values $B_{1}$ and $B_{2}$ we determine from conditions (11):

$$
\begin{equation*}
B_{1}=\frac{1}{\Delta_{2}}\left(\Pi_{2} k_{8}-\Pi_{1} k_{6}\right) ; \quad B_{2}=\frac{1}{\Delta_{2}}\left(\Pi_{1} k_{5}-\Pi_{2} k_{7}\right), \tag{12}
\end{equation*}
$$

where

$$
\begin{gathered}
k_{5}=-\eta_{1} K_{1}\left(\eta_{1} R\right)-m_{6} ; \quad k_{6}=\beta \eta_{2} K_{1}\left(\eta_{2} R\right)-m_{8} ; \\
k_{7}=\beta K_{0}\left(\eta_{1} R\right)-m_{2} ; k_{8}=-\eta_{2}^{2} K_{0}\left(\eta_{2} R\right)-m_{4} ; \Delta_{2}=k_{5} k_{8}-k_{6} k_{7}
\end{gathered}
$$

With internal interaction on the shell axis $(r=R)$ functions $K_{0}(x), K_{1}(x) \rightarrow \infty$, therefore, in solutions (10), (11) one should put:
$B_{1}=B_{2}=0$
Values $A_{1}$ and $A_{2}$ we also determine from the boundary conditions (12)

$$
A_{1}=\frac{1}{\Delta_{3}}\left(\Pi_{2} k_{4}-\Pi_{1} k_{2}\right) ; A_{2}=\frac{1}{\Delta_{3}}\left(\Pi_{1} k_{1}-\Pi_{2} k_{3}\right) ;
$$

Here
$k_{1}=\eta_{1} I_{1}\left(\eta_{1} R\right)-m_{5} ; k_{2}=-\beta \eta_{2} I_{1}\left(\eta_{2} R\right)-m_{7} ;$
$k_{4}=-\eta_{2}^{2} I_{0}\left(\eta_{2} R\right)-m_{3} ; \Delta_{3}=k_{1} k_{4}-k_{2} k_{3}$.
If we consider the problem of the interaction of a fluid located between elastic coaxial cylinders with radius $R_{1}$ and $R_{2}\left(R_{1}>R_{2}\right)$, then, due to the limited distance between the surfaces, in solutions (11) and (12) all coefficients should be preserved.
In this case, these solutions must additionally satisfy the boundary condition on the second shell, which formally coincides with (12). Then $A_{1}, A_{2}, B_{1}, B_{2}$ will be determined by the system

$$
\begin{aligned}
& A_{1} k_{11}+B_{1} k_{12}+A_{2} k_{13}+B_{2} k_{14}=\Pi_{11} ; A_{1} k_{15}+B_{1} k_{16}+A_{2} k_{17}+B_{2} k_{18}=\Pi_{12} ; \\
& A_{1} k_{21}+B_{1} k_{22}+A_{2} k_{23}+B_{2} k_{24}=\Pi_{21} ; A_{1} k_{25}+B_{1} k_{26}+A_{2} k_{27}+B_{2} k_{28}=\Pi_{22} ;
\end{aligned}
$$

Here index 1 corresponds to a shell with radius $R_{1}$, and index 2 is for a shell with a radius $R_{2}$. As a numerical example, solutions of the internal interaction were studied, when surface deformations were caused by a change in pressure according to the law $\Delta p_{0}=-a p_{0} \cos \omega t$
The transition from the image to the original was carried out numerically using piecewise polynomial functions with the following parameter values: $E==2.1 \cdot 10^{11} \quad \mathrm{~Pa}$; $v=0.58 \mathrm{~cm}^{2} / c ; p_{1}=2.5 \cdot 10^{4} \Pi a ; p_{2}=1.86 \cdot 10^{4} \Pi a$
$R=5.0 c m ; l=20 c m ; h=0.20 c m ;$
On fig. 1 shows the change in the contour stresses of the shell from time to time for various thicknesses.


Fig.1. The dependence of the dimensionless annular voltage from at different $h / R$

On fig.2.shows the change in the radial contour stresses of the shell from length for a viscous compressible fluid (solid line) and for a viscous incompressible fluid (dashed line), respectively, at $t=0.4$ and $t=16$


Fig.2. Variation of radial contour stress on shell length

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## Conclusions

1. Based on the Laplace and Fourier integral transformation approach, a method for estimating nonstationary oscillations of a shell with a viscous fluid has been devised in this study.
2. As time passes, the liquid's compressibility appears to have less of an impact.

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