Бухоро давлат университети
ўқув-методик кенгаш 6-сонли
йиғилишининг к ў ч и р м а с и

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К У Н Т А Р Т И Б И:

4. Турли масалалар.

4.1. Математика анализ кафедраси таянч докторант Ғ.Қурбонов ва Дифференциал тенгламалар кафедраси ўқитувчиси З.Ҳамдамовларнинг 5130100- Математика таълим йўналиши талабалари учун “Аналитик геометриядан мисол ва масалалар”(2 қисм) деб номланган методик қўлланмани нашрга тавсия этиш.

Э Ш И Т И Л Д И:

Г.Тоирова (кенгаш котибаси) Математика анализ кафедраси таянч докторант Ғ.Қурбонов ва Дифференциал тенгламалар кафедраси ўқитувчиси З.Ҳамдамовларнинг 5130100- Математика таълим йўналиши талабалари учун “Аналитик геометриядан мисол ва масалалар”(2 қисм) деб номланган методик қўлланмани нашрга тавсия этиш учун тайёрланганлигини маълум қилди. Ушбу методик қўлланма тақризчи: БМТИ доцент Ғ.Юнусов ф.-м.ф.н. доценти Б.Мамуровлар томонидан ижобий баҳо берилганлигини таъкидлади. Методик қўлланма муҳокамаси ҳақидаги Физика-математика факультети (2021 йил 4 январь) ва Математика анализ кафедраси (2021 йил 4 январь) йиғилиш қарори билан таништирди.

Юқоридагиларни инобатга олиб ўқув-методик кенгаш.

Қ А Р О Р Қ И Л А Д И:

1. Математика анализ кафедраси таянч докторант Ғ.Қурбонов ва Дифференциал тенгламалар кафедраси ўқитувчиси З.Ҳамдамовларнинг 5130100- Математика таълим йўналиши талабалари учун “Аналитик геометриядан мисол ва масалалар”(2 қисм) деб номланган методик қўлланма нашрга тавсия этилсин.

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Кўчирма аслига тўғри

Ўқув-услий кенгаш котибаси **Тоирова Г.И.**

G‘.G‘. QURBONOV
Z.N. HAMDAMOV

ANALITIK GEOMETRIYADAN MISOL VA MASALALAR

II qism

“Durdona” nashriyoti
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Ushbu o‘quv-metodik qo‘llanma Oliy ta’lim muassasalarining ,,Matematika” ta’lim yo‘nalishida tahsil olayotgan talabalar uchun mo‘ljallab yozilgan. Qo‘llanmada asosan tekislikda ikkinchi tartibli chiziqlar va ularning qutb koordinatalar sistemasidagi tenglamalari, tekislikda ikkinchi tartibli chiziqlarning umumiy tenglamalari, ikkinchi tartibli chiziq va to‘g‘ri chiziqning o‘zaro vaziyati va ularning tenglamalarini soddalashtirish, ikkinchi tartibli sirtlar va ularning to‘g‘ri chiziqli yasovchilari, ikkinchi tartibli sirtlarlarning tenglamalarini kanonik ko‘rinishga keltirish, chiziqli va affin fazolar kabi tushunchalar batafsil yoritilgan. Barcha mavzularda nazariy ma’lumotlar, namunaviy masalalar yechimlari va talabalar mustaqil bajarishlari uchun mo‘ljallangan topshiriqlar keltirilgan. Bundan tashqari, qo‘llanmada keltirilgan mavzular bo‘yicha egallangan bilimlarni mustahkamlash uchun test topshiriqlari hamda ularning javoblari ham o‘z aksini topgan.

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MUNDARIJA

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,,Analitik geometriya” fani oliy matematikaning asosiy bo‘limlaridan biri hisoblanadi. Mazkur fanning 2-modulida Ikkinchi tartibli chiziqlar va sirtlar: tekislikda ikkinchi tartibli chiziqlar, tekislikda ikkinchi tartibli chiziqlarning qutb koordinatalar sistemasidagi tenglamalari va umumiy tenglamalari, ikkinchi tartibli chiziq va to‘g‘ri chiziqning o‘zaro vaziyati va ularning tenglamalarini soddalashtirish, ikkinchi tartibli sirtlarning to‘g‘ri chiziqli yasovchilari, ikkinchi tartibli sirtlarning tenglamalarini kanonik ko‘rinishga keltirish, chiziqli va affin fazolarni o‘rganish ko‘zda tutilgan.

Ushbu o‘quv - metodik qo‘llanma Analitik geometriya fani dasturida keltirilgan ”Ikkinchi tartibli chiziqlar va sirtlar” bo‘limi bo‘yicha barcha mavzularga doir nazariy ma’lumotlar hamda misol va masalalarini qamrab olgan bir qo‘llanmadir. U Oliy ta’lim muassasalarining “Matematika” ta’lim yo‘nalishida tahsil olayotgan talabalar uchun mo‘ljallab yozilgan.

Mazkur o‘quv - metodik qo‘llanmada yuqorida sanab o‘tilgan mavzularga oid qisqacha nazariy ma’lumotlar bayon qilingan. Ularga doir misol va masalalar dastlab sodda va muayyan tasavvur hosil qilinadigan, so‘ngra murakkabroq masalalarni yechishga alohida e’tibor qaratilgan. Misol va masalalarni sharhlab, ularni yechib ko‘rsatishdan ko‘zlangan maqsad Analitik geometriya kursidan olingan nazariy bilimlardan misol va masalalarni yechishda foydalana olish ko‘nikmasini shakllantirishdir. Talabalar namuna sifatida yechib ko‘rsatilgan masalalarda qo‘llanilgan usullardan foydalanib mustaqil bajarishlari uchun ko‘plab misol va masalalar keltirilgan.

Ma’lumki, ikkinchi tartibli chiziqlar va sirtlarning turli tenglamalari orasida o‘xshash va farqli jihatlari mavjud. Qo‘llanmada keltirilgan ma’lumotlarda Analitik geometriyaga xos bo‘lgan usullar alohida ta’kidlab o‘tilgan.

Qo‘llanmani o‘qish jarayonida talabalar o‘zlarining Analitik geometriya, Chiziqli algebra va analitik geometriya fanlaridan olgan bilimlarini to‘ldiradilar. Undan matematikaning ko‘plab sohalari bo‘yicha ilmiy-tadqiqot ishlari olib borayotgan magistrantlar, tayanch doktorantlar va mustaqil izlanuvchilar ham foydalanishlari mumkin.

1-MAVZU: TEKISLIKDA IKKINCHI TARTIBLI CHIZIQLAR.

Reja:

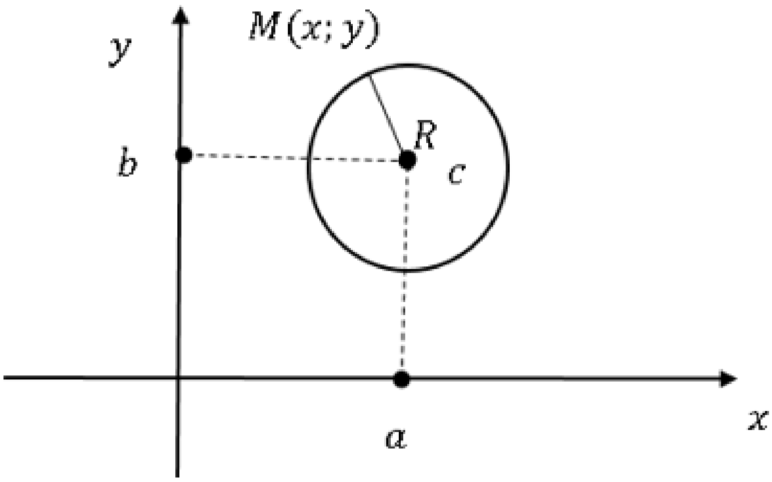
1. Ellips.
2. Giperbola.
3. Parabola.

***Tayanch iboralar**:* ellips, giperbola, parabola, diametr, vatar, fokus, parametr, asimptota, direkrtisa, ekssentrisitet.

1.1.Ellips.

Ta'rif. Tekislikda berilgan nuqtadan bir xil uzoqlikdagi nuqtalarning geometrik o'rniga aylana deyiladi.

Aylananing ta'rifidan foydalanib uning tenglamasini keltirib chiqaramiz. Bizga Dekart koordinatalar sistemasi berilgan bo'lsin. Koordinatalar sistemasida $C(a;b)$ nuqta berilgan bo'lsin. $C(a;b)$ nuqtadan bir xil (R) uzoqlikdagi $M(x;y)$ nuqtalar to'plamiga aylana deyilar ekan.



1.1.1-chizma

$|CM| = R$ aylana tenglamasi bo'ladi.

$|CM| = \sqrt{(x-a)^2 + (y-b)^2}$ ekanligi kelib chiqadi.

$$\sqrt{(x-a)^2 + (y-b)^2} = R \Longrightarrow$$

$$(x-a)^2 + (y-b)^2 = r^2 \qquad (1.1)$$

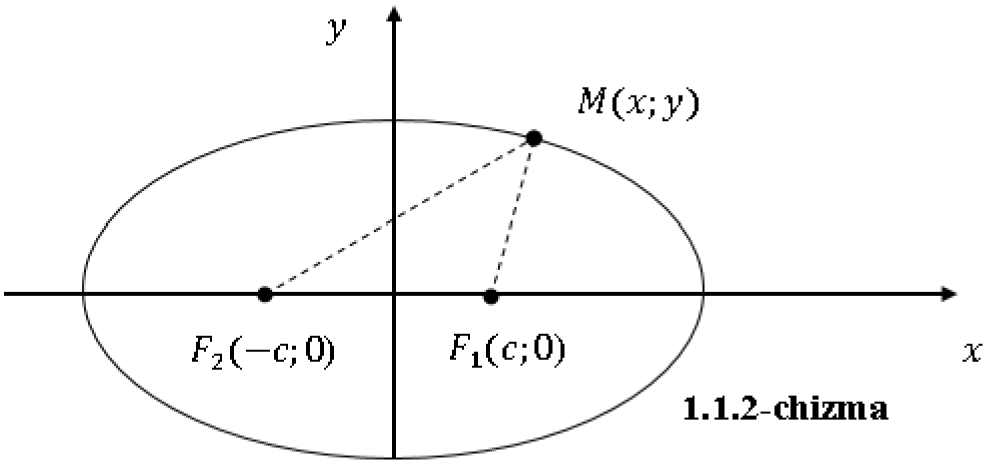
(1.1) tenglama markazi $C(a;b)$ nuqtada radiusi R ga teng bo'lgan aylana tenglamasini keltirib chiqardik.

Xususiy holda aylana markazi koordinatalar boshida bo‘lsa, ya’ni $O(0;0)$ da bo‘lsa, $x^2+y^2=R^2$ bo‘ladi. Bu tenglama markazi koordinatalar boshida radiusi R ga teng bo‘lgan aylana tenglamasi.

Ta’rif. Tekislikda qo‘zg‘almaydigan ikki nuqtagacha masofalarning yig‘indisi o‘zgarmas bo‘lgan nuqtalarning geometrik o‘rni ***ellips*** deyiladi.

Bizga qo‘zg‘almas ikkita nuqta berilgan bo‘lsin. Shu qo‘zg‘almas ikki nuqtaga ***fokus*** deyiladi.

Tekislikda ikkita F_1 va F_2 nuqtalar berilgan bo‘lsin. F_1 va F_2 nuqtalardan to‘g‘ri chiziq o‘tkazamiz va to‘g‘ri chiziqqa yo‘nalish berib uni absissa o‘qi deymiz. F_1 va F_2 nuqtalarning o‘rtasidan ordinata o‘qini o‘tkazamiz.



1.1.2-chizma

$|F_1F_2|=2c$ ga teng bo‘lsin, bundan kelib chiqadiki $F_1(c;0)$, $F_2(-c;0)$ bo‘ladi. Ellepsning ta’rifini qanoatlantiruvchi $M(x;y)$ nuqta bo‘lsin. U holda

$$|F_1M|+|F_2M|=2a \text{ bo‘ladi.}$$

$$|F_1M|=\sqrt{(x-c)^2+(y-0)^2},\ |F_2M|=\sqrt{(x+c)^2+(y-0)^2} \Longrightarrow$$

$$\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$$

$$\sqrt{(x-c)^2+y^2}=2a-\sqrt{(x+c)^2+y^2}$$

$$x^2-2xc+c^2+y^2=$$

$$=4a^2-4a\sqrt{(x+c)^2+y^2}+x^2+2xc+c^2+y^2$$

$$-2xc=4a^2-4a\sqrt{(x+c)^2+y^2}+2xc$$

$$4a^2-4a\sqrt{(x+c)^2+y^2}+4xc=0$$

$$a^2-a\sqrt{(x+c)^2+y^2}+xc=0$$

$$a^2+xc=a\sqrt{(x+c)^2+y^2}$$

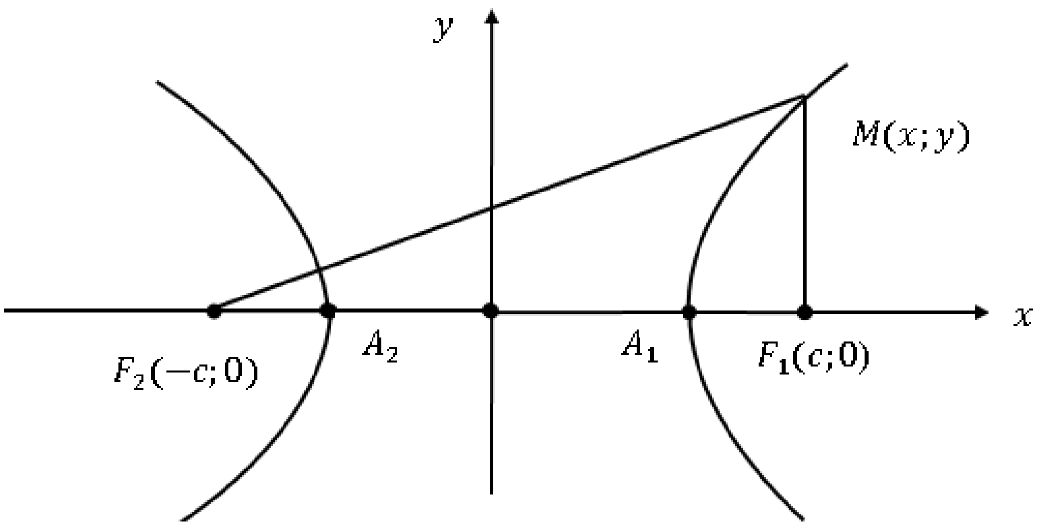
$$a^4 + 2a^2xc + x^2c^2 = a^2(x^2 + 2xc + c^2 + y^2)$$
$$a^4 + 2a^2xc + x^2c^2 = a^2x^2 + 2a^2xc + a^2c^2 + a^2y^2$$
$$a^4 + x^2c^2 = a^2x^2 + a^2c^2 + a^2y^2$$
$$a^4 + x^2c^2 - a^2x^2 - a^2c^2 - a^2y^2 = 0$$
$$a^4 + x^2c^2 - a^2x^2 - a^2c^2 - a^2y^2 = 0$$
$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$
$$a^2 + b^2 = c^2 \Longrightarrow\ b^2 = a^2 - c^2 \Longrightarrow b^2x^2 + a^2y^2 = a^2b^2 \Longrightarrow$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (1.2)$$
ekanligi kelib chiqadi.

Fokuslar orasidagi masofani katta o‘qqa nisbatiga ekssentrisitet deyiladi.
$$\varepsilon = \frac{2c}{2a} \Longrightarrow\ \varepsilon = \frac{c}{a} \qquad (1.3)$$
Ellipsning kichik o‘qiga parallel va uning markazidan a/ε masofadan o‘tuvchi parallel to‘g‘ri chiziqlar ellipsning ***direktrisalari*** deyiladi.
$$x = \pm\frac{c}{\varepsilon} = \pm\frac{a}{c/a} = \pm\frac{a^2}{c}$$
1-Misol. $x^2 + 4y^2 = 4$ tenglama ellipsni ifodalashini ko‘rsating va uning barcha xarakteristikalarini toping.

Yechish: Dastlab berilgan tenglamani ikkala tomonini 4 soniga bo‘lamiz:
$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$
bu yerdan berilgan tenglama yarim o‘qlari $a = 2$ va $b = 1$ bo‘lgan ellipsni ifodalashini ko‘ramiz. Unda $c^2 = a^2 - b^2 = 3$ bo‘lgani uchun qaralayotgan ellipsning fokuslari $F_1(-\sqrt{3}; 0)$ va $F_2(\sqrt{3}; 0)$ nuqtalarda joylashganligini ko‘ramiz. Bu natijalardan foydalanib, ellipsning ekssentrisiteti va direktrisalarini topamiz:
$$\varepsilon = \frac{c}{a} = \frac{\sqrt{3}}{2}, \quad x = \pm\frac{a}{\varepsilon} = \pm\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \pm\frac{\sqrt{3}}{4}.$$
Ellipsga tegishli $M(x; y)$ nuqtaning fokal radiuslari
$$r_1 = a + \varepsilon x = 2 + \frac{\sqrt{3}}{2}x, \quad r_2 = a + \varepsilon x = 2 + \frac{\sqrt{3}}{2}x$$
formulalar bilan topiladi.

1.2. Giperbola.

Fokus deb ataladigan ikki nuqtagacha bo‘lgan masofalarining ayirmasi o‘zgarmas songa teng bo‘lgan nuqtalarning geometrik o‘rniga ***giperbola*** deyiladi. Fokuslar - F_1, F_2, ular orasidagi masofa $|F_1F_2| = 2c$. Fokuslar yotgan to‘g‘ri chiziqqa yo‘nalish berib absissa o‘qi deylik. Absissa o‘qini 2 ta fokusdan o‘tkazaylik. Fokuslarning o‘rtasidan absissa o‘qiga perpendikulyar qilib ***ordinata*** o‘qini o‘tkazaylik.



1.2.1-chizma

$$|F_1M| - |F_2M| = 2a$$

$$|F_1M| = \sqrt{(x-c)^2 + y^2},\ |F_2M| = \sqrt{(x+c)^2 + y^2}$$

$$\left|\sqrt{(x-c)^2 + y^2} - \sqrt{(x+c)^2 + y^2}\right| = 2a$$

$$\sqrt{(x-c)^2 + y^2} = \pm 2a + \sqrt{(x+c)^2 + y^2}$$

$$x^2 - 2xc + c^2 + y^2 =$$

$$= x^2 + 2xc + c^2 + y^2 \pm 4a\sqrt{(x+c)^2 + y^2} + 4a^2$$

$$\pm 4a\sqrt{x^2 + 2xc + c^2 + y^2} = 4a^2 + 4xc$$

$$a^2x^2 + 2a^2xc + a^2c^2 + a^2y^2 = a^4 + 2a^2xc + x^2c^2$$

$$x^2(a^2 - c^2) - a^2(a^2 - c^2) + a^2y^2 = 0$$

$$x^2(a^2 - c^2) - a^2y^2 = a^2(a^2 - c^2) \Rightarrow c > 0 \Rightarrow\ c^2 - a^2 = b^2$$

$$\Delta MF_1F_2 \Rightarrow\ F_1M - F_2M = 2a\ \Rightarrow\ |F_1M - F_2M| < |F_1F_2| \Rightarrow$$

$$2a < 2c \Rightarrow a < c$$

$$x^2b^2 - a^2y^2 = a^2b^2 \Rightarrow$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (1.4)$$

ekanligi kelib chiqadi.

Bu yerda $2a$ −haqiqiy o‘q, $2b-$ mavhum o‘q, $2c-$ fokuslar orasidagi masofa.

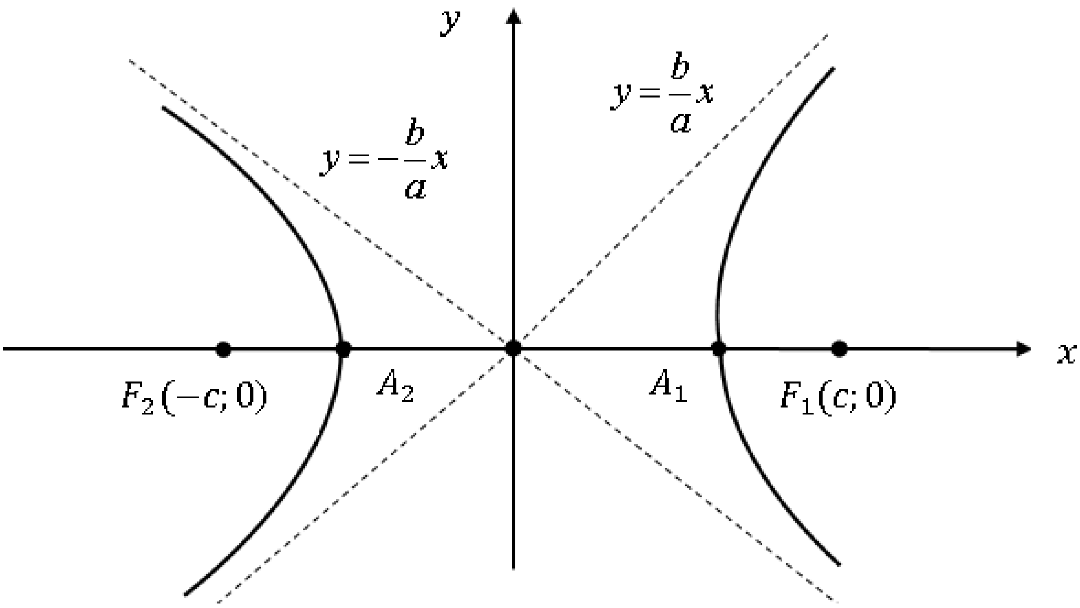
Giperbolaning ekssentrisiteti deb, giperbola fokuslari orasidagi masofaning haqiqiy o‘qqa nisbatiga aytiladi.

$$\varepsilon = \frac{2c}{2a} = \frac{c}{a} \Rightarrow c^2 - a^2 = b^2 \Rightarrow c > a \Rightarrow \varepsilon > 1$$

Giperbolaning mavhum o‘qiga parallel va uning markazidan

$$x = \frac{c}{\varepsilon} = \frac{a}{c/a} = \frac{a^2}{c}$$

masofada yotuvchi ikki to‘g‘ri chiziqqa ***direktrisalari*** deyiladi.



1.2.2-chizma

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow d\colon x = \pm\frac{c}{\varepsilon} = \pm\frac{a^2}{c}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \Rightarrow y = \pm\frac{b^2}{c} \Rightarrow \frac{c^2}{c} < a \Rightarrow a < c.$$

Ushbu $$y = \pm\frac{b}{a}x \qquad (1.5)$$

tenglamalar bilan aniqlanuvchi to‘g‘ri chiziqlarga ***asimptota*** deyiladi. Agar $a = b$ bo‘lsa, ***teng yonli geperbola*** deyiladi.

Fokal radiuslar. Ellips uchun

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \Rightarrow$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right) \Rightarrow a^2 - c^2 = b^2$$

$$|MF_1| = r_1 = \sqrt{(x-c)^2 + y^2} = \sqrt{(x-c)^2 + b^2 - \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{x^2 - 2xc + c^2 + b^2 - \frac{b^2x^2}{a^2}} = \sqrt{x^2 - 2xc + c^2 + b^2 - \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{\frac{a^2 - b^2}{a^2}x^2 - 2xc + c^2 + b^2} = \sqrt{\frac{c^2}{a^2}x^2 - 2xc + a^2} =$$

$$= \left|\frac{c}{a}x - a\right| = |\varepsilon x - a|.$$

$$|MF_2| = r_2 = \sqrt{(x+c)^2 + y^2} = \left|\frac{c}{a}x + a\right| = |\varepsilon x + a| \implies$$

$$0 < c < a$$

$$r_1 = a - \frac{c}{a}x, \quad r_2 = a + \frac{c}{a}x \implies r_1 = r_2 = 2a$$

Ellips yoki giperbola uchun fokal radiusi deganda, uning biror nuqtasidan fokuslarigacha bo‘lgan masofalar, tushiniladi.

Giperbola uchun

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \implies \frac{x^2}{a^2} + 1 = \frac{y^2}{b^2} \implies y^2 = b^2\left(1 + \frac{x^2}{a^2}\right)$$

$$|MF_1| = r_1 = \sqrt{(x-c)^2 + y^2} = \sqrt{x^2 - 2xc + c^2 + \left(\frac{x^2}{a^2} + 1\right)b^2} =$$

$$= \sqrt{x^2 - 2xc + c^2 + b^2 + \frac{b^2x^2}{a^2}} =$$

$$= \sqrt{\frac{(a^2 - b^2)}{a^2}x^2 - \frac{a^2}{a^2}2xc + \frac{c^2}{a^2} + \frac{b^2}{a^2}b^2 + a^2b^2} = \left|\frac{c}{a}x - a\right|.$$

$$|MF_2| = r_2 = \sqrt{(x+c)^2 + y^2} = \left|\frac{c}{a}x + a\right| \implies c^2 - a^2 = b^2 \implies$$

$$c < a < 0$$

$$r_1 = \left|\frac{c}{a}x - a\right| = \frac{c}{a}x - a, \quad r_2 = \frac{c}{a}x + a \qquad (1.6)$$

bo‘ladi.

2-Misol. Kanonik tenglamasi bilan berilgan quyidagi giperbolaning barcha xarakteristikalarini toping:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Bu giperbolaning absissasi $x = 8$, ordinatasi $y > 0$ bo‘lgan M nuqtasining fokal radiuslarini aniqlang.

Yechish: Berilgan tenglamani (1.4) kanonik tenglama bilan taqqoslab, giperbolaning haqiqiy va mavhum yarim o‘qlari $a = 4$, $b = 3$ ekanligini ko‘ramiz. Bu holda $c^2 = a^2 + b^2 = 16 + 9 = 25 \Longrightarrow c = 5$ bo‘lgani uchun giperbolaning fokuslari $F_1(-5; 0)$ va $F_2(5; 0)$ nuqtalarda joylashganligini aniqlaymiz. Berilgan giperbolaning asimptotalari

$$y = \pm\frac{b}{a}x = \pm\frac{3}{4}x = \pm 0{,}75x,$$

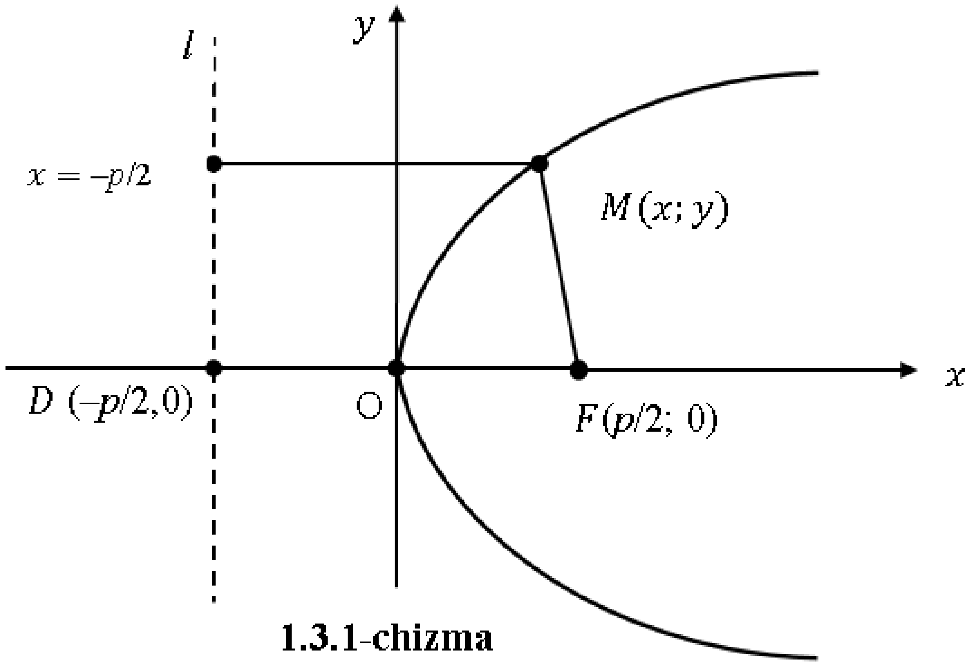
ekssentrisiteti $\varepsilon = c/a = 5/4 = 1{,}25$, direktrisalarining tenglamasi esa $x = \pm a/\varepsilon = \pm 4/1{,}25 = \pm 3{,}2$ bo‘ladi. Endi giperbolaning berilgan $M(8; 0)$ nuqtasining fokal radiuslarini topamiz. Bu nuqta giperbolaning o‘ng shoxida joylashgan va shu sababli (1.6) formulani " + " ishora bilan qaraymiz:

$$r_1 = a + \varepsilon x = 4 + 1{,}25 \cdot 8 = 14,$$
$$r_2 = -a + \varepsilon x = -4 + 1{,}25 \cdot 8 = 6.$$

1.3. Parabola.

Berilgan nuqtadan va berilgan to‘g‘ri chiziqdan bir xil uzoqlikda joylashgan nuqtalarning geometrik o‘rniga ***parabola*** deyiladi.

Ta’rifdan foydalanib, parabolaning kanonik tenglamasini keltirib chiqaraylik. Bizga F nuqta va l to‘g‘ri chiziq berilgan bo‘lsin. F nuqtadan o‘tib l to‘g‘ri chiziqqa perpendikulyar qilib Ox o‘qini olaylik. l to‘g‘ri chiziqqa parallel va F nuqta bilan l to‘g‘ri chiziqni o‘rtasidan Oy o‘qini o‘tkazaylik. Kanonik tenglamasini topmoqchi bo‘lgan parabola ustidan ixtiyoriy $M(x; y)$ nuqta olaylik. F nuqtadan l to‘g‘ri chiziqqacha bo‘lgan masofa P bo‘lsin. U holda F nuqtaning koordinatalari $F(\frac{p}{2}; 0)$ va l to‘g‘ri chiziqning tenglamasi $x = -\frac{p}{2}$ bo‘ladi. F nuqtadan M nuqtagacha bo‘lgan masofa $|FM| = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}$ bo‘ladi. M nuqtadan l to‘g‘ri chiziqqacha bo‘lgan masofa esa $d = \left|x + \frac{p}{2}\right|$ bo‘ladi.



1.3.1-chizma

$$|FM| = |Fd|$$

$$\sqrt{\left(x - \frac{p}{2}\right)^2 + y^2} = \left|x + \frac{p}{2}\right| \Rightarrow x^2 - 2x\frac{p}{2} + \frac{p^2}{4} + y^2 =$$

$$= x^2 + 2x\frac{p}{2} + \frac{p^2}{4} \Rightarrow y^2 - xp = xp \Rightarrow$$

$$y^2 = 2px \qquad (1.7)$$

tenglama ***parabola tenglamasi*** hisoblanadi.

3-Misol. Ox o‘qi parabolaning simmetriya o‘qi bo‘lib, uning uchi koordinatalar boshida yotadi. Parabola uchidan fokusigacha bo‘lgan masofa 4 birlikka teng. Parabola va uning direktrisasi tenglamasini toping.

Yechish: Dastlab, masala shartiga asosan, parabolaning p parametrini topamiz:

$$|OF| = 4 \ \Rightarrow p/2 = 4 \ \Rightarrow p = 8.$$

Unda, (1.7) formulaga asosan, parabola tenglamasini topamiz:

$$y^2 = 2px \ \ \Rightarrow y^2 = 2 \cdot 8x = 16x.$$

Bu yerdan direktrisa tenglamasi $x = -p/2 \Rightarrow \ x = -4$ ekanligini ko‘ramiz.

Shuni ta’kidlab o‘tish kerakki, $y = ax^2 + bx + c\ (a \neq 0)$ kvadrat uchhadning grafigi uchi koordinatalari

$$x_0 = -\frac{b}{2a}, \quad y_0 = \frac{4ac - b^2}{4a}$$

bo‘lgan $M_0(x_0; y_0)$ nuqtada, simmetriya o‘qi esa Oy o‘qiga parallel va $x = -b/2a$ tenglamaga ega bo‘lgan vertikal to‘g‘ri chiziqdan tashkil topgan paraboladan iboratdir. Agar $a > 0$ bo‘lsa, parabola shoxlari yuqoriga, $a < 0$ bo‘lsa, pastga yo‘nalgan bo‘ladi.

Mustaqil yechish uchun topshiriqlar.

1.1. Ellipsga doir misollar.

1.1.1. Markazi $C(x; y)$ va radiusi R ga teng bo‘lgan aylana tenglamasini tuzing.

1) $C(2; -3)$, $R = 5$; 2) $C(-5; 4)$, $R = 2$;
3) $C(7; 1)$, $R = 3$; 4) $C(-2; 9)$, $R = 4$;
5) $C(-4; 6)$, $R = 7$; 6) $C(6; -3)$, $R = 6$.

1.1.2. Quyidagi har bir holda aylananing kanonik tenglamasini tuzing. Markazi va radiusini aniqlang.

1) $x^2 + y^2 - 6x = 0$;
2) $x^2 + y^2 + 6x - 8y = 0$;
3) $x^2 + y^2 - 10x + 24y - 56 = 0$;
4) $3x^2 + 3y^2 + 6x - 4y - 1 = 0$;
5) $x^2 + y^2 - 2x + 4y = 0$;
6) $3x^2 + 3y^2 - 2x + 7y + 1 = 0$.

1.1.3. 1) $x^2 + y^2 - 1 = 0$; 2) $x^2 + y^2 - 10x + 24y - 56 = 0$;
3) $9x^2 + 9y^2 - 3 = 0$; 4) $3x^2 + 3y^2 + 6x - 4y - 1 = 0$;
aylanalarga nisbatan $A(3; 1)$, $B(1; 0)$, $C(-2; 0)$ va $D(-2; 1)$ nuqtalarning vaziyatini aniqlang.

1.1.4. Koordinatalari quyidagi:

1) $(x-1)^2 + (y-3)^2 \geq 25$; 2) $16 \leq (x-1)^2 + (y+3)^2 \leq 25$;
3) $(x-1)^2 + (y-2)^2 \leq 25$, 4)$(x-4)^2 + (y-6)^2 \leq 9$;
5) $x^2 + y^2 - 6x \leq 0, y \geq 0$; 6) $x^2 + y^2 - 4x \leq 0, |x| \geq 1$.

tengsizliklarni qanoatlantiruvchi nuqtalar tekislikda qanday joylashadi?

1.1.5. Ox o‘qiga $M(6; 0)$ nuqtada urinuvchi va $N(9; 9)$ nuqta orqali o‘tadigan aylananing tenglamasi tuzilsin.

1.1.6. Markazi $C(2; 3)$ nuqtada yotadigan va $x - 2y + 1 = 0$ to‘g‘ri chiziqqa urinadigan aylananing tenglamasi tuzilsin.

1.1.7. $A(-4; 1)$, $B(3; 2)$, $C(-2; -5)$, $D(5; 0)$ va $E(3; -6)$ nuqtalar berilgan.

1) A, B, C nuqtalardan; 2) A, B, D nuqtalardan;
3) A, B, E nuqtalardan; 4) B, C, D nuqtalardan;
5) B, C, E nuqtalardan; 6) C, D, E nuqtalardan.

o‘tuvchi aylana tenglamasini tuzing.

1.1.8. Berilgan $A(2;7)$, $B(-2;1)$ nuqtalar orqali o‘tadigan va radiusi $r=\sqrt{26}$ bo‘lgan aylananing tenglamasini tuzing.

1.1.9. Quyidagilardan foydalanib aylana tenglamasini tuzing:

1) markazi koordinata boshida va radiusi $R=3$ bo‘lgan;
2) markazi $C(2;-3)$ nuqtada va radiusi $R=7$ bo‘lgan;
3) markazi koordinata boshida va $C(6;-8)$ nuqtadan o‘tuvchi;
4) markazi $A(2;6)$ nuqtada va $C(-1;2)$ nuqtadan o‘tuvchi;
5) $A(3;2)$ va $B(-1;6)$ nuqtalar diametrning uchlari bo‘lgan;
6) markazi koordinata boshida va $3x-4y+20=0$ to‘g‘ri chiziqqa urinuvchi;
7) markazi $C(1;-1)$ nuqtada va $5x-12y+9=0$ to‘g‘ri chiziqga urinuvchi;
8) $A(3;1)$ va $B(-1;3)$ nuqtalardan o‘tadigan va markazi $3x-y-2=0$ to‘g‘ri chiziqda yotuvchi;
9) $A(1;1)$, $B(1;-1)$ va $C(2;0)$ uchta nuqtadan o‘tuvchi;
10) $M_1(-1;5)$, $M_2(-2;-2)$ va $M_3(5;5)$ uchta nuqtadan o‘tuvchi.

1.1.10. Ikki parallel to‘g‘ri chiziq tenglamasi $2x+y-5=0$, $2x+y+15=0$ va bu to‘g‘ri chiziqlarning biri bilan $A(2;1)$ nuqtada urinuvchi aylana berilgan. Aylananing tenglamasini tuzing.

1.1.11. Ikki aylana markazidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing:

1) $(x-3)^2+y^2=9$ va $(x+2)^2+(y-1)^2=1$;
2) $(x+2)^2+(y-1)^2=16$ va $(x+2)^2+(y+5)^2=25$;
3) $x^2+y^2-4x+6y=0$ va $x^2+y^2-6x=0$;
4) $x^2+y^2-x+2y=0$ va $x^2+y^2+5x+2y-1=0$.

1.1.12. $A(1;-1)$ nuqta va ikki $x^2+y^2+2x-2y-23=0$, $x^2+y^2-6x+12y-35=0$ aylananing kesishgan nuqtasi orqali o‘tuvchi aylana tenglamasini tuzing.

1.1.13. $x^2+y^2+3x-y=0$, $3x^2+3y^2+2x+y=0$ aylananing kesishgan nuqtalari orqali o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

1.1.14. Quyidagi malumotlarga ko‘ra ellipsning kanonik tenglamasi tuzilsin:

1) yarim o‘qlari mos ravishda 5 va 4ga teng;
2) katta o‘qi 10, fokuslari orasidagi masofa 8 ga teng;
3) katta o‘qi 26 va ekssentrisiteti $\varepsilon=\frac{12}{13}$.

1.1.15. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellips fokuslarining koordinatalari topilsin.

1.1.16. $\frac{x^2}{25}+\frac{y^2}{169}=1$ ellips fokuslarining koordinatalari topilsin.

1.1.17. $\frac{x^2}{9}+\frac{y^2}{25}=1$ ellipsga nisbatan quyidagi nuqtalarning: 1) $A_1(1;2)$; 2) $A_2(-1;3)$; 3) $A_3(6;1)$; 4) $A_4(-1;7)$ vaziyati aniqlansin.

1.1.18. O‘qlari koordinata o‘qlari bilan ustma-ust tushuvchi va $P(2;2)$, $Q(3;1)$ nuqtalar orqali o‘tuvchi ellips tenglamasi tuzilsin.

1.1.19. Katta o‘qi 2 birlikka teng, fokuslari $F_1(0;1)$, $F_2(1;0)$ nuqtalarda bo‘lgan ellips tenglamasi tuzilsin.

1.1.20. Ellips fokuslarining biridan katta o‘qi uchlarigacha masofalar mos ravishda 7 va 1 ga teng. Bu ellips tenglamasini tuzing.

1.1.21. $\frac{x^2}{36}+\frac{y^2}{20}=1$ ellips direktrisalarining tenglamalarini yozing.

1.1.22. Ellipsning direktrisalari $x=\pm 8$ to‘g‘ri chiziqlar, uning kichik o‘qi 8 ga teng ekanligi ma’lum bo‘lsa, ellips tenglamasini tuzing.

1.1.23. $\frac{x^2}{100}+\frac{y^2}{36}=1$ ellipsda o‘ng fokusigacha masofa chap fokusigacha bo‘lgan masofasiga nisbatan 4 marta katta bo‘lgan nuqta topilsin.

1.1.24. Quyidagilarni bilgan holda fokusi absissa o‘qida yotib, koordinata boshiga nisbatan simmetrik bo‘lgan ellips tenglamasini tuzing:

1) uning yarim o‘qlari 5 va 2 ga teng;
2) uning katta o‘qi 10 ga teng, fokuslar orasidagi masofa esa $2c=8$;
3) uning kichik o‘qi 24 ga teng, fokuslar orasidagi masofa esa $2c=10$;
4) fokuslar orasidagi masofa $2c=6$ ekssentrisiteti $\varepsilon=\frac{3}{5}$;
5) uning katta o‘qi 20 ga teng va ekssentrisiteti $\varepsilon=\frac{3}{5}$;
6) uning kichik o‘qi 10 ga teng , ekssentrisiteti $\varepsilon=\frac{12}{13}$;
7) direktrisalar orasidagi masofa 5 ga va fokuslar orasidagi masofa $2c=4$ ga teng;
8) uning katta o‘qi 8 ga teng, direktrisalar orasidagi masofa 16 ga teng;
9) uning kichik o‘qi 6 ga teng va direktrisalar orasidagi masofa 13 ga teng;
10) direktrisalar orasidagi masofa 32 ga va ekssentrisiteti $\varepsilon=\frac{1}{2}$.

1.1.25. Quyidagilarni bilgan holda fokusi ordinata o‘qida yotib, koordinata boshiga nisbatan simmetrik bo‘lgan ellips tenglamasini tuzing:
1) uning yarim o‘qlari mos ravishda 7 va 2 ga teng;
2) uning katta o‘qi 10 ga teng, fokuslar orasidagi masofa esa $2c = 8$;
3) fokuslar orasidagi masofa $2c = 24$ va ekssentrisiteti $\varepsilon = \frac{12}{13}$;
4) uning kichik o‘qi 16 va ekssentrisiteti $\varepsilon = \frac{3}{5}$;
5) fokuslar orasidagi masofa $2c = 6$ va direktrisalar orasidagi masofa $16\frac{2}{3}$ ga teng;
6) direktrisalar orasidagi masofa $10\frac{2}{3}$ ga va ekssentrisiteti $\varepsilon = \frac{3}{4}$ ga teng.

1.1.26. Ellipsning yarim o‘qlarini toping:
1) $\frac{x^2}{16}+\frac{y^2}{9}=1$; 2) $\frac{x^2}{4}+y^2=1$; 3) $x^2+25y^2=25$;
4) $x^2+5y^2=15$; 5) $4x^2+9y^2=25$; 6) $9x^2+25y^2=1$;
7) $x^2+4y^2=1$; 8) $16x^2+y^2=16$; 9) $25x^2+9y^2=1$;
10) $x^2+y^2=1$.

1.1.27. Ellips tenglamasi $9x^2+25y^2=225$ berilgan bo‘lsa, quyidagilarni toping: 1) yarim o‘qlarini; 2) fokuslarni;
3) ekssentrisiteti; 4) direktrisa tenglamalarini tuzing.

1.1.28. Ellips tenglamasi $9x^2+5y^2=45$ berilgan bo‘lsa, quyidagilarni toping:
1) yarim o‘qlarini; 2) fokuslarni; 3) ekssentrisiteti;
4) direktrisa tenglamalarini tuzing.

1.1.29. Quyidagi nuqtalardan qaysi birlari ushbu $8x^2+5y^2=77$ ellipsda yotadi:
1) $A_1(-2;3)$; 2) $A_2(2;-2)$; 3) $A_3(2;-4)$; 4) $A_4(-1;3)$;
5) $A_5(-4;-3)$; 6) $A_6(3;-1)$; 7) $A_7(3;-2)$; 8) $A_8(2;1)$;
9) $A_9(0;15)$; 10) $A_{10}(0;-16)$.
Shulardan qaysilari ichki, qaysilari tashqi nuqtalar?

1.1.30. Fokuslari absissa o‘qida joylashgan bo‘lib, koordinata boshiga nisbatan simmetrrik bo‘lgan ellipsning tenglamasini tuzing agar quyidagilar berilgan bo‘lsa:
1) ellipsdan $M_1(-2\sqrt{5};2)$ nuqta va uning kichik yarim o‘qi $b=3$;
2) ellipsdan $M_1(2;-2)$ nuqta va uning katta yarim o‘qi $a=4$;
3) ellipsdan $M_1(4;-\sqrt{3})$ nuqta va $M_2(2\sqrt{2};3)$ nuqta;

4) ellipsdan $M_1(\sqrt{15}; -1)$ nuqta va fokuslar orasidagi masofa $2c = 8$;
5) ellipsdan $M_1(2; -\frac{5}{3})$ nuqta va uning ekssentrisiteti $\varepsilon = \frac{2}{3}$;
6) ellipsdan $M_1(8; 12)$ nuqta va chap fokusgacha bo'lgan masofa $r_1 = 20$ ga teng;
7) ellipsdan $M_1(-\sqrt{5}; 2)$ nuqta va uning direktrisalari orasidagi masofa 10 ga teng.

1.2. Giperbolaga doir misollar.

1.2.1. $x^2 - y^2 = 1$ giperbolaga nisbatan $A(4; 1)$, $B(1; -2)$, $C(\sqrt{2}; 1)$ nuqtalarning vaziyati aniqlansin.

1.2.2. Quyidagi malumotlarga ko'ra:
1) haqiqiy o'qi $a = 5$ mavhum o'qi $b = 3$;
2) fokuslari orasidagi masofa 10 ga, haqiqiy o'qi esa 8 ga teng giperbolaning kanonik tenglamasi tuzilsin.

1.2.3. Quyidagi ma'lumotlarga ko'ra:
1) ekssentrisiteti $\varepsilon = \frac{12}{13}$ haqiqiy o'qi 48 ga teng;
2) haqiqiy o'qi 16 ga, asimptotasi bilan absissa o'qi orasidagi φ burchak tangensi $\frac{3}{4}$ ga teng;
giperbolaning kanonik tenglamasi tuzilsin.

1.2.4. Teng tomonli giperbolaning ekssentrisiteti hisoblansin.

1.2.5. Giperbola asimptotalarining tenglamalari $y = \pm\frac{5}{12}x$ va giperbolada yotuvchi $M(24; 5)$ nuqta berilgan. Giperbola tenglamasi tuzilsin.

1.2.6. $\frac{x^2}{25} - \frac{y^2}{144} = 1$ giperbolaning fokuslarini aniqlang.

1.2.7. $\frac{x^2}{225} - \frac{y^2}{64} = -1$ giperbolaning fokuslarini aniqlang.

1.2.8. Quyidagi ma'lumotlarga ko'ra:
1) direktrisalari orasidagi masofa $\frac{32}{5}$ ga teng va ekssentrisiteti $\varepsilon = \frac{5}{4}$;
2) asimptotalari orasidagi burchak 60^0 ga teng va $c = 2\sqrt{3}$ giperbolaning kanonik tenglamasi tuzilsin.

1.2.9. $\frac{x^2}{49} + \frac{y^2}{24} = 1$ ellips bilan fokusdosh va ekssentrisiteti $\varepsilon = \frac{5}{4}$ bo'lgan giperbolaning tenglamasi yozilsin.

1.2.10. $\frac{x^2}{9}-\frac{y^2}{16}=1$ giperbola berilgan:
1) fokuslarining koordinatalari;
2) ekssentrisiteti;
3) asimptotalarining va direktrisalarining tenglamalari;
4) qo‘shma giperbolaning tenglamasi va uning ekssentrisiteti hisoblansin.
1.2.11. Giperbola haqida quyidagilar ma’lum bo‘lsa, uning yarim o‘qlari hisoblansin:
1) fokuslari orasidagi masofa 8 ga va direktrisalari orasidagi masofa 6 ga teng;
2) direktrisalari $x=\pm 3\sqrt{2}$ tenglamalar bilan berilgan va asimptotalari orasidagi burchak to‘g‘ri burchak;
3) asimptotalari $y=\pm 2$ tenglamalar bilan berilgan va fokuslari markazdan 5 birlik masofada;
4) asimptotalari $y=\pm\frac{5}{3}x$ tenglamalar bilan berilgan va giperbola $N(6;9)$ nuqtadan o‘tadi.
1.2.12. Teng tomonli giperbola $x^2-y^2=8$ berilgan. Unga fokusdosh bo‘lib, $M(-5;3)$ nuqtadan o‘tuvchi giperbolaning tenglamasi topilsin.
1.2.13. Fokusi absissa o‘qida joylashgan va koordinata boshiga nisbatan simmetrik bo‘lgan giperbola tenglamasini tuzing, agar quyidagilar ma’lum bo‘lsa:
1) uning o‘qlari $2a=10$ va $2b=8$;
2) fokuslar orasidagi masofa $2a=10$ va mavhum o‘qi $2b=8$;
3) fokuslar orasidagi masofa $2c=6$ va ekssentrisiteti $\varepsilon=\frac{3}{2}$;
4) haqiqiy o‘qi $2a=16$ va ekssentrisiteti $\varepsilon=\frac{5}{4}$;
5) asimptota tenglamasi $y=\pm\frac{4}{3}x$ va fokuslar orasidagi masofa $2c=20$;
6) direktrisalar orasidagi masofa 22 ga teng va fokuslar orasidagi masofa $2c=26$;
7) direktrisalar orasidagi masofa $\frac{32}{5}$ ga teng va mavhum o‘qi $2b=6$;
8) direktrisalar orasidagi masofa $\frac{8}{3}$ ga teng va ekssentrisiteti $\varepsilon=\frac{3}{2}$;

9) asimptota tenglamasi $y = \pm\frac{3}{4}x$ va direktrisalar orasidagi masofa $12\frac{4}{5}$ ga teng.

1.2.14. Fokusi ordinata o‘qida joylashgan va koordinata boshiga nisbatan simmetrik bo‘lgan giperbola tenglamasini tuzing, agar quyidagilar ma’lum bo‘lsa:

1) uning yarim o‘qlari $a = 6$, $b = 18$;

2) fokuslar orasidagi masofa $2c = 10$ va ekssentrisiteti $\varepsilon = \frac{5}{3}$;

3) asimptota tenglamasi $y = \pm\frac{12}{5}x$ va uchlari orasidagi masofa 48;

4) direktrisalar orasidagi masofa $7\frac{1}{7}$ va ekssentrisiteti $\varepsilon = \frac{7}{5}$;

5) asimptota tenglamasi $y = \pm\frac{4}{3}x$ va direktrisalar orasidagi masofa $6\frac{2}{5}$ ga teng.

1.2.15. Quyida berilgan giperbolalarni a va b yarim o‘qlarini toping:

1) $\frac{x^2}{9} - \frac{y^2}{4} = 1$; 2) $\frac{x^2}{16} - y^2 = 1$; 3) $x^2 - 4y^2 = 16$;

4) $x^2 - y^2 = 1$; 5) $4x^2 - 9y^2 = 25$; 6) $25x^2 - 16y^2 = 1$;

7) $9x^2 - 16y^2 = 1$.

1.2.16. Giperbola tenglamasi berilgan $16x^2 - 9y^2 = 144$ bo‘lsa, quyidagilarni toping:

1) yarim o‘qlari a va b larni; 2) fokuslarini; 3) ekssentrisitetini;

4) asimptota tenglamasini; 5) direktrisa tenglamasini.

1.2.17. Giperbola tenglamasi berilgan $16x^2 - 9y^2 = -144$ bo‘lsa, quyidagilarni toping:

1) yarim o‘qlari a va b larni; 2) fokuslarini; 3) ekssentrisitetini;

4) asimptota tenglamasini; 5) direktrisa tenglamasini.

1.2.18. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ giperbola asimtotasi va $9x + 2y - 24 = 0$ to‘g‘ri chiziq bilan chegaralangan uchburchakning yuzini toping.

1.2.19. Quyidagi tenglamalar qanday chiziqlarni ifodalashini aniqlang:

1) $y = +\frac{2}{3}\sqrt{x^2 - 9}$; 2) $y = -3\sqrt{x^2 + 1}$

3) $y = -\frac{4}{3}\sqrt{x^2 + 9}$; 4) $y = +\frac{2}{5}\sqrt{x^2 + 25}$.

va bu chiziqlarni chizmasini chizing.

1.2.20. $\frac{x^2}{80}-\frac{y^2}{20}=1$ giperbolaning $M_1(10;-\sqrt{5})$ nuqtasi berilgan. Fakal radiusi M_1 nuqta bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.
1.2.21. $\frac{x^2}{16}-\frac{y^2}{9}=1$ giperbolani $M_1(-5;\frac{9}{4})$ nuqta qanoatlantirishi ko‘rinib turibdi, M_1 nuqtaning fokal radiusini toping.
1.2.22. $\frac{x^2}{64}-\frac{y^2}{36}=1$ giperboladagi nuqtadan o‘ng fokusgacha bo‘lgan masofa 4,5 ga teng bo‘lsa, shu nuqtani koordinatasini toping.
1.2.23. $\frac{x^2}{9}-\frac{y^2}{16}=1$ giperboladagi nuqtadan chap fokusgacha bo‘lgan masofa 7 ga teng bo‘lsa, shu nuqtani koordinatasini toping.
1.2.24. Fokuslari absissa o‘qida yotib koordinata boshiga nisbatan simmetrik bo‘lgan giperbola tenglamasini tuzing, quyidagilar berilgan bo‘lsa:
1) giperbolaning $M_1(6;-1)$ va $M_2(-8;2\sqrt{2})$ nuqtalari;
2) giperbolaning $M_1(-5;3)$ nuqtasi va ekssentrisiteti $\varepsilon=\sqrt{2}$;
3) giperbolaning $M_1(\frac{9}{2};-1)$ nuqtasi va $y=+\frac{2}{3}x$ asimtota tenglamasi;
4) giperbolaning $M_1(-3;\frac{5}{2})$ nuqtasi va $y=+\frac{4}{3}$ direktrisa tenglamasi;
5) $y=\pm\frac{3}{4}x$ asimtota tenglamasi va $x=\pm\frac{16}{5}$ direktrisa tenglamasi.
1.2.25. Teng tomonli giperbolaning ekssentrisitetini hisoblang.
1.2.26. $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellipsning fokusi giperbolaning fokusi bilan ustma - ust tushadi. Agar giperbolaning ekssentrisiteti $\varepsilon=2$ ga teng bo‘lsa, giperbolaning tenglamasini tuzing.
1.2.27. Fokusi $\frac{x^2}{100}+\frac{y^2}{64}=1$ ellipsning uchida yotuvchi, direktrisasi esa ellipsning fokusidan o‘tuvchi giperbola tenglamasini tuzing.
1.2.28. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning fokusidan uning asimptotasigacha bo‘lgan masofa b ga teng bo‘lishini isbotlang.
1.2.29. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning ixtiyoriy nuqtasidan uning ikki asimptotasigacha bo‘lgan masofalar ko‘paytmasi har doim $\frac{a^2b^2}{a^2+b^2}$ ga teng bo‘lishini isbotlang.

1.2.30. Agar yarim o‘qlari a va b, markazi $C(x_0; y_0)$ nuqta va fokusi quyidagi chiziqlarda:
1) Ox o‘qiga parallel;
2) Oy o‘qiga parallel,
bo‘lsa, giperbola tenglamasini tuzing.

1.3. Parabolaga doir misollar.

1.3.1. Quyidagi nuqtalardan qaysilari $y^2 = 6x$ parabolaga tegishli:
1) $A(-2; 4)$; 2) $B(1; 5)$; 3) $C(3; 1)$; 4) $A(-2; 4)$.
1.3.2. Quyidagi parabolalardan qaysilarining fokusi a) $F_1(3; 0)$, b) $F_2(-3; 0)$, c) $F_3(0; 3)$ va d) $F_4(0; -3)$ nuqtadan o‘tadi:
1) $y^2 = 3x$; 2) $y^2 = -3x$; 3) $x^2 = 3y$; 4) $x^2 = -3y$;
5) $y^2 = 6x$; 6) $y^2 = -6x$; 7) $x^2 = 6y$; 8) $x^2 = -6y$;
9) $y^2 = 12x$; 10) $y^2 = -12x$; 11) $x^2 = 12y$; 12) $x^2 = -12y$.
1.3.3. Quyidagi parabolalardan qaysilarining direktrisa tenglamasi a) $x = 5$, b) $x = -5$, c) $y = 5$ va d) $y = -5$:
1) $y^2 = 5x$; 2) $y^2 = -5x$; 3) $x^2 = 5y$; 4) $x^2 = -5y$;
5) $y^2 = 10x$; 6) $y^2 = -10x$; 7) $x^2 = 10y$; 8) $x^2 = -10y$;
9) $y^2 = 20x$; 10) $y^2 = -20x$; 11) $x^2 = 20y$; 12) $x^2 = -20y$.
1.3.4. $y^2 = 4x$ parabola fokusining koordinatalarini aniqlang.
1.3.5. $x^2 = 4y$ parabola fokusining koordinatalarini aniqlang.
1.3.6. $y^2 = -8x$ parabola fokusining koordinatalarini aniqlang.
1.3.7. $y^2 = 6x$ parabola direktrisasi tenglamasini tuzing.
1.3.8. Parabolaning fokusidan uchigacha bo‘lgan masofa 3 ga teng, uning kanonik tenglamasini tuzing.
1.3.9. Parabolaning fokusidan direktrisasigacha bo‘lgan masofa 2 ga teng, uning kanonik tenglamasini tuzing.
1.3.10. Parabolaning fokusi $F(3; 0)$ nuqtada va $x = -1$ direktrisasining tenglamasi bo‘lsa, parabola tenglamasini tuzing.
1.3.11. Parabolaning uchidan fokusigacha bo‘lgan masofa 3 ga teng va parabola Ox o‘qiga nisbatan simmetrik bo‘lib, Oy o‘qiga urinsa parabola tenglamasini tuzing.
1.3.12. Fokusi $M(5; 0)$ nuqtada bo‘lib, ordinatalar o‘qi direktrisa bo‘lsa, parabola tenglamasini tuzing.
1.3.13. Parabola Ox o‘qiga nisbatan simmetrik bo‘lib, $M(1; -4)$ nuqtadan va koordinatalar boshidan o‘tadigan parabola tenglamasini tuzing.

1.3.14. Parabolaning fokusi $M(0;2)$ nuqtada va uchi koordiniatalar boshida yotsa, parabola tenglamasini tuzing.

1.3.15. Parabola Oy o‘qiga nisbatan simmetrik bo‘lib, $M(6;-2)$ nuqtadan va koordinatalar boshidan o‘tadi, parabola tenglamasini tuzing.

1.3.16. $y^2 = 8x$ paraboladagi fokal radius vektori 20 ga teng bo‘lgan nuqta topilsin.

1.3.17. Ox o‘qiga nisbatan simmetrik, $A(9;6)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing.

1.3.18. Ox o‘qiga nisbatan simmetrik, $B(-1;3)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing.

1.3.19. Oy o‘qiga nisbatan simmetrik, $C(1;1)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing.

1.3.20. Oy o‘qiga nisbatan simmetrik, $D(4;-8)$ nuqtadan va uchi koordinatalar boshidan o‘tuvchi parabola tenglamasini tuzing.

1.3.21. Koordinata boshidan o‘tib, Oy o‘qiga simmetrik va fokusi $F(0;-3)$ nuqtada bo‘lgan parabola tenglamasini tuzing.

1.3.22. $y^2 = 24x$ parabola tenglamasidan F fokusini va direktrisa tenglamasini toping.

1.3.23. $y^2 = -24x$ parabola tenglamasidan F fokusini va direktrisa tenglamasini toping.

1.3.24. $x^2 = -24y$ parabola tenglamasidan F fokusini va direktrisa tenglamasini toping.

1.3.25. $y^2 = 20x$ parabola tenglamasi berilgan, agar M nuqtaning absissasi 7 ga teng bo‘lsa, M fokal radiusni toping.

1.3.26. $y^2 = 12x$ parabola tenglamasi berilgan, agar M nuqtaning ordinatasi 6 ga teng bo‘lsa, M fokal radiusni toping.

1.3.27. $y^2 = 16x$ parabola tenglamasi berilgan. Fokal radius 13 ga teng bo‘ladigan M nuqtani toping.

1.3.28. $x^2 = 16y$ parabola tenglamasi berilgan. Fokal radius 13 ga teng bo‘ladigan M nuqtani toping.

1.3.29. $x^2 = -16y$ parabola tenglamasi berilgan. Fokal radius 13 ga teng bo‘ladigan M nuqtani toping.

1.3.30. Agar $F(-7;0)$ fokus va direktirisa tenglamasi $x - 7 = 0$ berilgan bo‘lsa, parabola tenglamasini tuzing.

2-MAVZU: TEKILIKDA IKKINCHI TARTIBLI CHIZIQLARNING QUTB KOORDINATALAR SISTEMASIDAGI TENGLAMALARI.

Reja:

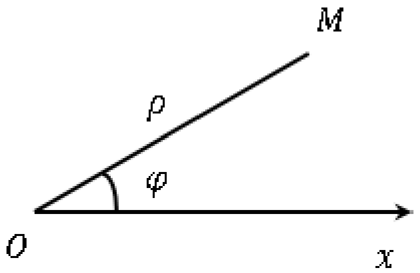
1. Qutb koordinatalar. Konus kesimlari.
2. Qutb koordinatalardagi tenglamalar.
3. Konus kesimlarining Dekart koordinatalardagi kanonik ko'rinishli tenglamalari.

Tayanch iboralar: qutb, ellips, giperbola, parabola, diametr, vatar, fokus, parametr, direktrisa, asimptota, ekssentrisitet.

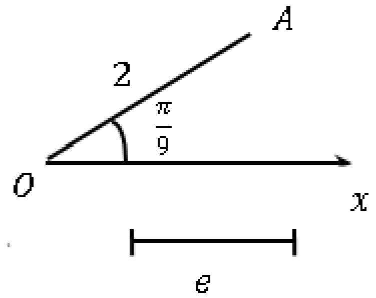
2.1. Qutb koordinatalar.

Qutb koordinatalar sistemasining asosiy elementlari va undan chiquvchi nur, ya'ni qutb O va ***qutb o'qi*** Ox dir (2.1.1- chizma).

M nuqtaning tekislikdagi o'rni bu nuqtaning qutbdan bo'lgan masofasi – radius – vektori ρ va radius – vektorning qutb o'qi bilan tashkil etgan qutb burchgi φ bilan aniqlanadi.



2.1.1-chizma



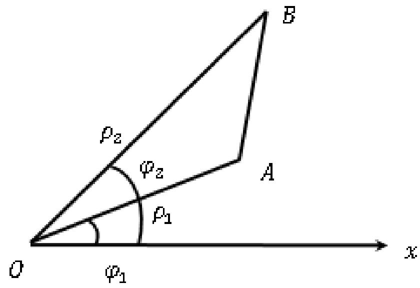
2.1.2-chizma

Ikki koordinata (ρ, φ) birgina nuqtani aniqlaydi. 2.1.2 - chizmada A nuqta $\rho = 2,\ \varphi = \frac{\pi}{9}$ koordinatalari bo'yicha yasalgan.

Agar biz tekislikdagi nuqtalar va qo'sh koordinatalar (ρ, φ) orasidagi o'zaro qiymatli moslikni o'rnatmoqchi bo'lsak, u holda ρ ga faqat musbat qiymatlar, φ ga esa 0 bilan 2π orasidagi qiymatlar berish kifoya (nurni soat strelkasiga qarshi aylantirganda musbat burchaklar hosil bo'ladi). Agar bu cheklashlarga rioya qilinmasa, u holda birgina nuqtaning o'zi $(\rho; \varphi + 2\pi n)$ yoki $(-\rho; \varphi(2n+1)\pi)$ koordinatalar bilan aniqlanadi, n – ixtiyoriy butun son. Qutb koordinatalar sistemasida berilgan ikkita nuqta $A(\rho_1, \varphi_1)$ va $B(\rho_2, \varphi_2)$ orasidagi masofa

$$AB = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cdot \cos(\varphi_2 - \varphi_1)} \qquad (2.1)$$

formula bilan hisoblanadi.



2.1.3-chizma

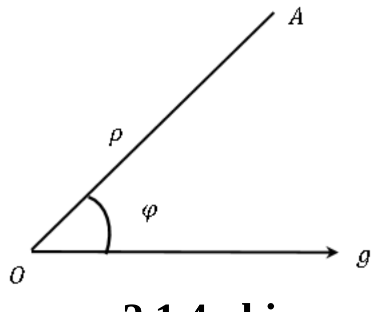
Dekart koordinatalari bilan ish ko‘rganimiz singari, egri chiziqning qutb koordinatalaridagi tenglamasi haqida gapirish mumkin, chunonchi, agar egri chiziqdagi har bir nuqtaning qutb koordinatalari ushbu

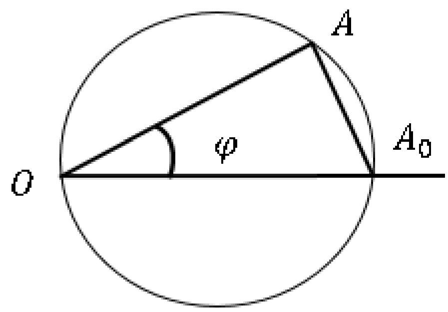
$$F(\rho; \varphi) = 0$$

tenglamani qanoatlantirsa, bu tenglama egri chiziqning ***qutb koordinatalardagi tenglamasi*** deb ataladi. Aksincha, ρ, φ sonlarning bu tenglamani qanoatlantiradigan istalgan jufti egri chiziq nuqtalaridan birining qutb koordinatalari bo‘ladi.

Misol tariqasida qutbdan o‘tgan va markazi qutb o‘qidagi R radiusli aylananing qutb koordinatalaridagi tenglamasini tuzaylik. To‘g‘ri burchakli OAA_0 uchburchakdan $OA = OA_0 cos\varphi$ ni hosil qilamiz (2.1.5- chizma). Bu yerdan aylana tenglamasini hosil qilamiz:

$$\rho = 2Rcos\varphi.$$



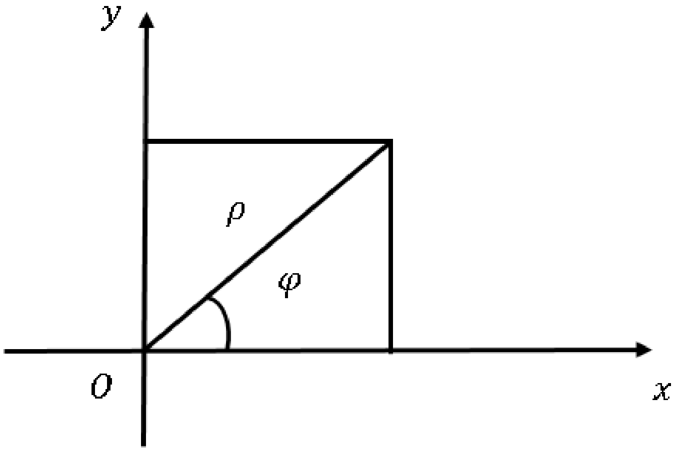


2.1.4-chizma **2.1.5-chizma**

$\rho\varphi$ tekislikda Dekart koordinatalari sistemasi xy ni kiritamiz, buning uchun qutb O ni dekart koordinatalari sistemasining boshi, qutb o‘qini musbat yarim o‘q x sifatida va musbat yarim y ning musbat yo‘nalishini shunday tanlab olamizki, burchaklarni hisoblash uchun tanlab olingan yo‘nalish bilan muvofiq holda qutb o‘qi bilan $+\frac{\pi}{2}$ burchakni hosil qilsin.

Nuqtaning qutb va Dekart koordinatalari orasida quyidagicha bog‘lanishning mavjudligi ravshan:

$$x = \rho cos\varphi,\ y = \rho sin\varphi \qquad (2.2)$$



2.1.6-chizma

Bu egri chiziqning qutb koordinatalar sistemasidagi tenglamasini bilgan holda, uning Dekart koordinatalaridagi tenglamasini va aksincha, hosil qilish imkonini beradi.

Misol tariqasida ixtiyoriy to‘g‘ri chiziqning qutb sistemasidagi tenglamasini tuzaylik. To‘g‘ri chiziqning Dekart koordinatalaridagi tenglamasi

$$ax + by + c = 0, \qquad c < 0.$$

Bu tenglama ρ bilan φ ni x va y o‘rniga (2.2) formula bo‘yicha kiritsak, natijada:

$$\rho(a\,cos\varphi + b\,sin\varphi) + c = 0.$$

So‘ngra ushbularni faraz qilsak:

$$\frac{a}{\sqrt{a^2+b^2}} = cos\alpha, \qquad \frac{b}{\sqrt{a^2+b^2}} = sin\alpha, \qquad \frac{c}{\sqrt{a^2+b^2}} = -\rho_0$$

to‘g‘ri chiziqning ushbu ko‘rinishidagi tenglamani hosil qilamiz:

$$\rho\cos(\alpha - \varphi) = \rho_0.$$

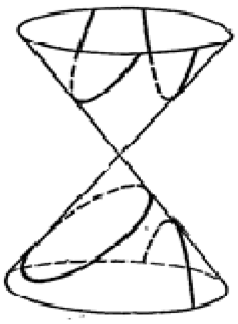
2.2. Konus kesimlari. Qutb koordinatalardagi tenglamalar.

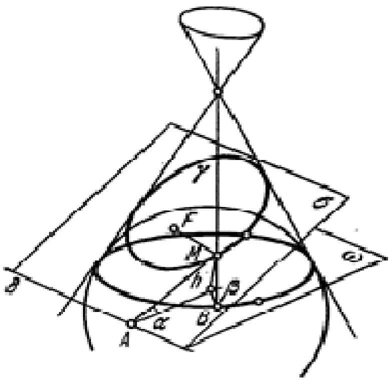
Doiraviy konusni uning uchidan o‘tmaydigan tekislik bilan kesish natijasida hosil qilingan egri chiziq ***konus kesimi*** deyiladi (2.1.7-chizma). Konus kesimlari qator ajoyib xossalarga ega. Ularning biri quyidagidan iborat.

Aylanadan boshqa har qanday konus kesimi shunday nuqtalarning geometrik o‘rnidan iboratki, ularning berilgan F nuqta va berilgan δ to‘g‘ri chiziqqacha masofalarning nisbati o‘zgarmasdir.

Bu xossani isbot qilaylik.F nuqta konus kesimining fokusi, δ to‘g‘ri chiziq esa ***direktrisasi*** deyiladi. Aytaylik, σ tekislikning konus bilan kesishgan egri chizig‘i γ bo‘lsin(2.1.8 – chizma).

Konus ichiga σ tekislikka urinadigan sfera chizaylik; sferaning tekislikka urinish nuqtasini F orqali belgilaylik. ω bilan sferaning konusga urinish aylanasini belgilaylik. γ egri chiziqda ixtiyoriy P nuqta olamiz. Bu P nuqta orqali konusning yasovchisini o‘tkazib, uning ω tekislik bilan kesishgan nuqtasini B orqali belgilaymiz. Nihoyat, P nuqtadan σ, ω tekisliklarning kesishgan to‘g‘ri chizig‘i δ ga perpendikulyar tushiramiz.





2.1.7-chizma **2.1.8-chizma**

Shu γ chiziqning F nuqta bilan δ to‘g‘ri chiziqqa nisbatan yuqorida aytilgan xossaga egaligini isbot qilish talab qilinadi. Haqiqatdan ham, $FP = BP$, chunki bu kesmalar sferaga bitta nuqtadan o‘tkazilgan urinmalardir. So‘ngra $h(P)$ bilan P nuqtadan ω tekislikkacha masofani belgilasak, u holda:

$$AP = \frac{h(P)}{sin\alpha}, \qquad BP = \frac{h(P)}{sin\beta}.$$

bunda α bilan ω, δ tekisliklar orasidagi burchak belgilangan.

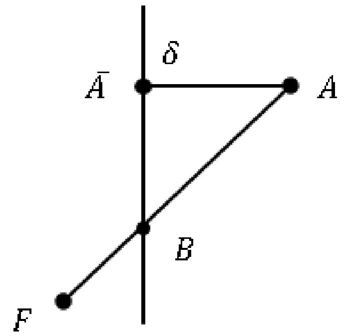
Bulardan ushbu tengliklarni hosil qilamiz:

$$\frac{AP}{FP} = \frac{AP}{BP} = \frac{sin\beta}{sin\alpha}$$

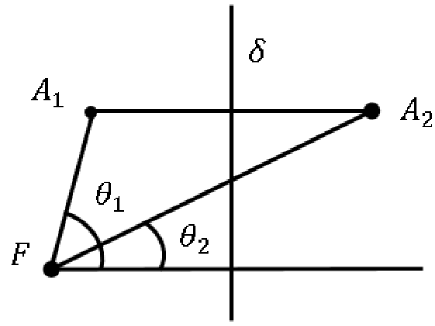
ya’ni $\frac{AP}{FP}$ nisbat P nuqtaga bog‘liq emas degan xulosaga kelamiz. Xossa isbot qilindi.

Konus kesimidagi nuqtaning fokus va direktrisagacha masofalarining nisbati λ ning qiymatiga qarab egri chiziq ***ellips*** ($\lambda < 1$), ***parabola*** (λ =1), ***giperbola*** ($\lambda > 1$) deb ataladi. λ son konus kesimining ***ekssentrisiteti*** deyiladi.

F – konus kesimining fokusi, δ – uning direktrisasi bo‘lsin (2.1.9-chizma). Ellips bilan parabola ($\lambda \leq 1$) holi uchun egri chiziqning hamma nuqtalari direktrisadan bir tarafda joylashadi, chunonchi: F fokus qayerdan joy olsa, bu nuqtalar ham o‘sha yerdan joy oladi.



2.1.9-chizma



2.1.10-chizma

Haqiqatdan ham, direktrisaning ikkinchi tarifidagi nuqtalar uchun:

$$\frac{AF}{A\bar{A}} > \frac{AB}{A\bar{A}} \geq 1.$$

Girepbola bilan ish ko‘rganda esa ($\lambda > 1$) direktrisaning ikkala tarafidan joylashga nuqtalar mavjud. Giperbola ikki tarmoqdan iborat bo‘lib, direktrisa ularni bir – biridan ajratib turadi.

$\rho\varphi$ koordinatalar sistemasining qutbi sifatida konus kesimining fokusini qabul qilib, qutb o‘qini esa shunday o‘tkazamizki, u direktrisaga perpendikulyar bo‘lsin va uning bilan kesishadigan bo‘lsin. Koordinatalarning ana shunday qutb sistemasidan konus kesimining tenglamasini tuzamiz.

Fokusdan direktrisagacha masofa P bo‘lsin. Konus kesimidagi ixtiyoriy A nuqtadan fokusgacha masofa ρ ga va direktrisagacha masofa esa A va F nuqtalarning direktrisadan bir tarafda yoki turli tarafda bo‘lishiga qarab $P - \rho cos\varphi$ yoki $\rho cos\varphi - P$ ga teng. Bulardan konus kesimining tenglamasini hosil qilamiz: ellips bilan parabola uchun:

$$\frac{\rho}{P - \rho cos\varphi} = \lambda \qquad (2.3)$$

va giperbola uchun:

$$\frac{\rho}{P - \rho cos\varphi} = \pm\lambda \qquad (2.4)$$

$("+"$ ishora giperbolaning bir tarmog‘i, $"-"$ ishora esa ikkinchi tarmog‘iga mos keladi).

$(2.3), (2.4)$ tenglamalarni ρ ga nisbatan yechib, ushbuni hosil qilamiz:

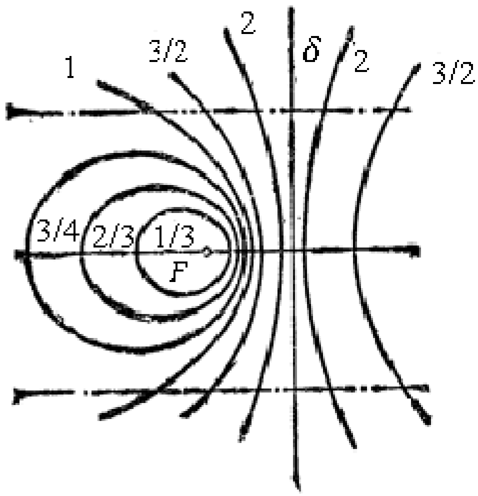
$$\rho = \frac{\lambda P}{1 + \lambda cos\varphi},$$

bu ***ellips bilan parabola tenglamasi*** va

$$\rho = \frac{\pm\lambda P}{1 \pm \lambda cos\varphi}$$

giperbola tenglamasidir.

Ekssentrisitetning qabul qilgan qiymatlariga qarab konus kesimining ko‘rinishi 2.1.11 - chizmada ko‘rsatilgan.



2.1.11-chizma

2.3. Konus kesimlarining Dekart koordinatalardagi kanonik ko‘rinishli tenglamalari.

Yuqorida biz konus kesimlarning $\rho\varphi$ qutb koordinatalaridagi tenglamalarini hosil qilgan edik. Endi Dekart koordinatalar sistemasiga o‘tamiz, buning uchun qutb O ni koordinatalar boshi va qutb o‘qini musbat yarim o‘q x sifatida qabul qilamiz.

(2.3) va (2.4) tenglamalardan istalgan konus kesimi uchun ushbuni hosil qilamiz:

$$\rho^2 = \lambda^2(P - \rho\, cos\varphi)^2.$$

Bundan esa qutb va Dekart koordinatalar orasidagi bog‘lanishni aniqlovchi formulalarini nazarga olsak:

$$x^2 + y^2 = \lambda^2(P - x)^2$$

yoki

$$(1 - \lambda^2)x^2 + 2P\lambda^2 x + y^2 - \lambda^2 P^2 = 0 \qquad (2.5)$$

ni hosil qilamiz.

Koordinatalar boshini x o‘qi bo‘ylab kerakligicha siljitish natijasida bu tenglama ancha soddalashadi.

Avval ellips bilan giperbolaga to‘g‘ri kelgan holni tahlil qilamiz. Bu holda (2.5) tenglamani quyidagicha yozish mumkin:

$$(1 - \lambda^2)\left(x + \frac{P\lambda^2}{1 - \lambda^2}\right) + y^2 - \frac{P^2\lambda^2}{1 - \lambda^2} = 0.$$

Endi ushbu formulalar bo‘yicha yangi x', y' koordinatalarni kiritaylik:

$$x + \frac{P\lambda^2}{1 - \lambda^2} = x', \qquad y = y',$$

bu esa koordinatalar boshini $(-\frac{P\lambda^2}{1-\lambda^2}, 0)$ nuqtaga ko‘chirishga mos keladi. Egri chiziq tenglamasi bu holda ushbu ko‘rinishni oladi:

$$(1 - \lambda^2)x'^2 + y'^2 - \frac{P^2\lambda^2}{1 - \lambda^2} = 0$$

yoki qisqalik uchun

$$\frac{P^2\lambda^2}{(1 - \lambda^2)^2} = a^2, \quad \frac{P^2\lambda^2}{|1 - \lambda^2|} = b^2$$

deb faraz qilsak, ushbu tenglamalarni hosil qilamiz;

ellips uchun:

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} - 1 = 0,$$

giperbola uchun:

$$\frac{x'^2}{a^2} - \frac{y'^2}{b^2} - 1 = 0.$$

a, b parametrlar ellips (giperbola)ning yarim o‘qlari deyiladi.

Parabola ($\lambda = 1$) olgan holda (2.5) tenglama ushbu ko‘rinishda qabul qilinadi:

$$2Px + y^2 - P^2 = 0$$

yoki

$$y^2 - 2P\left(-x + \frac{P}{2}\right) = 0;$$

yangi

$$x' = -x + \frac{P}{2}, \qquad y' = y$$

koordinatalarni kiritish natijasida tenglama

$$y'^2 - 2Px' = 0$$

ko‘rinishga keltiriladi.

Konus kesimlarining x', y' koordinatalarga nisbatan hosil qilinadigan tenglamalari ***kanonik tenglamalar*** deyiladi.

1-misol. Berilgan $A\left(2; \frac{\pi}{12}\right)$ va $B\left(1; \frac{5\pi}{12}\right)$ nuqtalar orasidagi masofani toping.

Yechish: Berilgan A va B nuqtalar orasidagi masofani (2.1) formuladan foydalanib topamiz:

$$|AB| = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cdot \cos(\varphi_2 - \varphi_1)} =$$

$$= \sqrt{2^2 + 1^2 - 2 \cdot 2 \cdot 1 \cdot \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)} =$$

$$= \sqrt{5 - 4 \cdot \cos\frac{\pi}{3}} = \sqrt{5 - 4 \cdot \frac{1}{2}} = \sqrt{3}.$$

Demak, qutb koordinatalar sistemasida A va B nuqtalar orasidagi masofa $\sqrt{3}$ ga teng ekan.

Mustaqil yechish uchun topshiriqlar.

2.1.1. Qutb koordinatalari quyidagi qiymatlarga ega bo‘lgan nuqtalar yasalsin:

1) $\left(3; \frac{\pi}{6}\right)$; 2) $\left(1; \frac{5\pi}{3}\right)$; 3) $\left(5; \frac{7\pi}{6}\right)$; 4) $\left(0{,}5; \frac{\pi}{2}\right)$; 5) $\left(2{,}5; \frac{2\pi}{3}\right)$; 6) $(6; \pi)$; 7) $\left(3; \frac{\pi}{3}\right)$; 8) $\left(\sqrt{3}; -\frac{\pi}{6}\right)$; 9) $\left(-2; \frac{\pi}{4}\right)$.

2.1.2. Qutb koordinatalari quyidagi tenglamalardan birini qanoatlantirgan nuqtalar qanday joylashgan:

1) $\rho = 1$; 2) $\rho = 5$; 3) $\rho = a$; 4) $\varphi = \frac{\pi}{6}$; 5) $\varphi = \frac{\pi}{3}$; 6) $\varphi = \frac{\pi}{2}$; 7) $\varphi = const$.

2.1.3. a) Qutbga nisbatan, b) qutb o‘qiga nisbatan, ushbu 1) $\left(1; \frac{\pi}{4}\right)$; 2) $\left(3; \frac{2\pi}{3}\right)$; 3) $\left(\frac{2}{3}; -\frac{\pi}{6}\right)$; 4) $M(\rho; \varphi)$ nuqtalarga simmetrik bo‘lgan nuqtalarning qutb koordinatalari topilsin.

2.1.4. Tomoni a ga teng bo‘lgan muntazam oltiburchak uchlarining qutb koordinatalari aniqlansin; oltiburchakning uchlaridan biri qutb, shu uchidan o‘tuvchi tomoni qutb o‘qi deb olinsin.
2.1.5. Qutb burchaklari 0^0, 15^0, 30^0, 45^0, 60^0, 75^0, 90^0 ga teng bo‘lgan, mos radius – vektorlari $\rho = a \cdot sin2\varphi$ tenglamadan hisoblanuvchi nuqtalar yasalsin. Olingan nuqtalar uzluksiz egri chiziq bilan tutashtirilsin.
2.1.6. Berilgan ikki nuqta orasidagi masofa hisoblansin:
1) $A\left(2;\frac{\pi}{12}\right)$ va $B\left(1;\frac{5\pi}{12}\right)$; 2) $C\left(4;\frac{\pi}{5}\right)$ va $D\left(6;\frac{6\pi}{5}\right)$; 3) $E\left(3;\frac{11\pi}{18}\right)$ va $F\left(4;\frac{\pi}{9}\right)$.
2.1.7. Qutb koordinatalar sistemasida uchburchakning uchlari berilgan: $A\left(5;\frac{\pi}{2}\right)$, $B\left(8;\frac{5\pi}{6}\right)$, $C\left(3;\frac{7\pi}{6}\right)$. Bu uchburchakning muntazam ekanligi tekshirilsin.
2.1.8. Qutb o‘qiga joylashgan va $A\left(4\sqrt{2};\frac{\pi}{4}\right)$ nuqtadan 5 birlik masofada yotgan nuqta topilsin.
2.1.9. Uchlaridan biri qutb bilan ustma –ust tushgan uchburchakning yuzini hisoblash formulasini keltirib chiqaring.
2.1.10. Uchlaridan biri qutbda, qolgan ikki uchining qutb koordinatalari $\left(4;\frac{\pi}{9}\right)$ va $\left(1;\frac{5\pi}{18}\right)$ bo‘lgan uchburchakning yuzi hisoblansin.
2.1.11. Qutb koordinatalar sistemasida o‘zining $A\left(9;\frac{\pi}{10}\right)$, $B\left(12;\frac{4\pi}{15}\right)$ va $C\left(10;\frac{3\pi}{5}\right)$ uchlari bilan berilgan uchburchakning yuzi hisoblansin.
2.1.12. Qutb koordinatalar sistemasiga nisbatan, radiusi a ga teng va markazi:
1) qutbda; 2) $(a;0)$; 3) $(\rho_1;\varphi_1)$ nuqtada bo‘lgan aylananing tenglamasi tuzilsin.
2.1.13. Qutb koordinatalar sistemasiga nisbatan markazi qutb bilan va fokal o‘qi qutb o‘qi bilan ustma – ust tushgan ellipsning tenglamasi tuzilsin.
2.1.14. $\rho = \frac{288}{16-7cos^2\varphi}$ ellipsning uzunligi 10 birlikka teng bo‘lgan diametri fokal o‘qqa qanday burchak ostida og‘ishgan?
2.1.15. Ellipsning fokal o‘qini qutb o‘qi deb va qutbni 1) ellipsning chap fokusiga joylashtirib; 2) ellipsning o‘ng fokusiga joylashtirib, ellipsning tenglamasini tuzing.

2.1.16. $\rho = \frac{3\sqrt{2}}{2-cos\varphi}$ ellips yarim o‘qlarining uzunligi va ikkala fokus orasidagi masofa hisoblansin.
2.1.17. Markazi qutb bilan va haqiqiy o‘qi qutb o‘qi bilan ustma – ust tushgan giperbolaning tenglamasi tuzilsin.
2.1.18. $\rho = \frac{48}{4cos^2\varphi-1}$ giperbolaning asimptotalari orasidagi burchak hisoblansin.
2.1.19. Giperbolaning fokal o‘qini qutb o‘qi qabul qilinib va qutbni giperbolaning o‘ng fokusida olib, uning tenglamasi tuzilsin.
2.1.20. $\rho = \frac{2}{1-\sqrt{2}cos\varphi}$ giperbola asimptotalarining va direktrisalarining tenglamalari tuzilsin.
2.1.21. Parabolaning o‘qini qutb o‘qi va uchini qutb deb olib, uning tenglamasi tuzilsin.
2.1.22. $\rho = \frac{8cos\varphi}{sin^2\varphi}$ parabolada shunday nuqta topilsinki, uning parabola dipektrisasidan bo‘lgan masofa shu nuqtaning radius – vektoriga teng bo‘lsin.
2.1.23. Fokusi qutb bilan ustma – ust tushgan va o‘qi qutb o‘qidan iborat bo‘lgan parabolaning tenglamasi tuzilsin.
2.1.24. $\rho = \frac{p}{1-cos\varphi}$ parabolada shunday nuqta topilsinki, u
1) eng kichik radius – vektorga;
2) parabolaning parametriga teng bo‘lgan radius – vektorga ega bo‘lsin.
2.1.25. Parabolaning ixtiyoriy fokal vatarining uchlaridan uning o‘qiga tushirilgan perpendikulyarning ko‘paytmasi o‘zgarmas miqdor ekanligini isbotlang.
2.1.26. To‘g‘ri burchakli koordinatalar sistemasiga nisbatan quyidagi egri chiziqlarning eng sodda tenglamalari tuzilsin:
1) $\rho = \frac{25}{13-12cos\varphi}$; 2) $\rho = \frac{1}{3-3cos\varphi}$;
3) $\rho = \frac{9}{4-5cos\varphi}$; 4) $\rho = \frac{4}{\sqrt{5}-cos\varphi}$.
2.1.27. $P(2;-1)$ nuqta polyarasining $x^2 + 6xy + y^2 + 6x + 2y - -1 = 0$ egri chiziqqa nisbatan tenglamasi tuzilsin.
2.1.28. Quyidagi nuqtalarning polyarasi topilsin:
1) $(-3;5)$ nuqtaning $4x^2 + 2xy - y^2 + 6x + 2y + 3 = 0$ egri chiziqqa nisbatan;
2) $(0;1)$ nuqtaning $6x^2 - xy - 2y^2 + 4y = 0$ egri chiziqqa nisbatan.

2.1.29. Quyidagi nuqtalarning
1) $(1;-2)$ nuqtaning $2x^2-4xy+5y^2-8x+6=0$ egri chiziqqa nisbatan;
2) $(0;0)$ nuqtaning $x^2-2xy+2y^2-4x-6y+3=0$ egri chiziqqa nisbatan polyarasini toping.
2.1.30. $18x-17y-41=0$ to‘g‘ri chiziqning $2x^2-xy-3y^2-$ $-x-6y-15=0$ egri chiziqqa nisbatan qutbi topilsin.

3-MAVZU: TEKISLIKDA IKKINCHI TARTIBLI CHIZIQLARNING UMUMIY TENGLAMALARI.

Reja:
1. Koordinata boshini ko‘chirish yordamida ikkinchi tartibli chiziqlarni sinflarga ajratish.
2. Markaziy va nomarkaziy chiziqlar.

Tayanch iboralar*:* ellips, giperbola, parabola, diametr, vatar, fokus, parametr, asimptota, direktrisa, notrivial.

3.1. Koordinata boshini ko‘chirish yordamida ikkinchi tartibli chiziqlarni sinflarga ajratish.

Tekislikda Dekart koordinatalar sistemasida
$$a_{11}x^2+2a_{12}xy+a_{22}y^2+2a_{13}x+2a_{23}y+a_{33}=0 \quad (3.1)$$
tenglama bilan berilgan ikkinchi tartibli chiziqni tekshirish bilan shug‘ullanamiz. Bu ishni koordinatalar sistemasini o‘zgartirish va (3.1) tenglamani soddalashtirish yordamida amalga oshiramiz. Birinchi navbatda parallel ko‘chirishda (3.1) tenglama koeffitsiyentlarini qanday o‘zgarishini tekshiramiz. Buning uchun
$$x'=x-x_0, \quad y'=y-y_0 \quad (3.2)$$
formulalar yordamida almashtirishlarni bajaramiz. Bu holda koordinata o‘qlarining yo‘nalishlari o‘zgarmaydi, faqat koordinata boshi $O'(x_0;y_0)$ nuqtaga ko‘chadi. Bu formulalardan x,y larni topib va (3.1) ga qo‘yib,
$$a_{11}(x'+x_0)^2+2a_{12}(x'+x_0)(y'+y_0)+a_{22}(y'+y_0)^2+$$
$$+2a_{13}(x'+x_0)+2a_{23}(y'+y_0)+a_{33}=0 \quad (3.3)$$
tenglamani hosil qilamiz. Bu tenglamadan
$$a_{11}(x'^2+2x_0x'+x_0^2)+2a_{12}(x'y'+x'y_0+y'x_0+x_0y_0)+$$
$$+a_{22}(y'^2+y_0y'+y_0^2)+2a_{13}x'+2a_{13}x_0+2a_{23}y'+$$

$$+2a_{23}y_0 + a_{33} = 0$$

$$a_{11}{x'}^2 + 2a_{12}x'y' + a_{22}{y'}^2 + (2a_{11}x_0 + 2a_{12}y_0 + 2a_{13})x' +$$
$$+(2a_{12}x_0 + 2a_{22}y_0 + 2a_{23})y' + a_{11}x_0^2 + 2a_{12}x_0y_0 + a_{22}y_0^2 +$$
$$+2a_{13}x_0 + 2a_{23}y_0 + a_{33} = 0 \qquad (3.4)$$

kelib chiqadi.

Bu formulalardan ko‘rinib turibdiki, parallel ko‘chirishda ikkinchi darajali hadlar oldidagi koeffitsiyentlar o‘zgarmaydi.

Agar $O'(x_0; y_0)$ nuqtaning koordinatalari

$$\begin{cases} a_{11}x + a_{12}y + a_{13} = 0 \\ a_{21}x + a_{22}y + a_{23} = 0 \end{cases} \qquad (3.5)$$

sistemani qanoatlantirsa, (3.3) tenglamada birinchi darajali hadlar qatnashmaydi.

Bundan tashqari, agar $O'(x_0; y_0)$ nuqtaning koordinatalari (3.5) sistemani qanoatlantirsa, $O'(x_0; y_0)$ nuqta ikkinchi tartibli chiziq uchun simmetriya markazi bo‘ladi. Haqiqatan ham, bu holda koordinatalar markazini $O'(x_0; y_0)$ nuqtaga ko‘chirsak, tenglamada birinchi darajali hadlar qatnashmaydi. Shuning uchun yangi koordinatalar sistemasida

$$F(x'; y') = F(-x'; -y')$$

tenglik o‘rinli bo‘ladi. Demak, $O'(x_0; y_0)$ nuqta chiziq uchun simmetriya markazidir. Va aksincha, agar birorta A nuqta chiziq uchun simmetriya markazi bo‘lsa, uning koordinatalari (3.5) sistemani qanoatlantirishini ko‘rsatamiz. Koordinata boshini A nuqtaga joylashtirib, yangi x, y koordinatalar sistemasini kiritamiz. Agar $F(x; y)$ nuqta chiziqqa tegishli bo‘lsa,

$$F(x; y) = 0$$

tenglik o‘rinli bo‘ladi. Koordinata boshi simmetriya markazi bo‘lgani uchun $F(-x; -y) = 0$ tenglik ham o‘rinli bo‘ladi. Yuqoridagilarni hisobga olsak quyidagi ta’rifning geometrik ma’nosi yaxshi tushunarli bo‘ladi.

***Ta’rif*.** Tekislikdagi $M_0(x_0; y_0)$ nuqtaning koordinatalari (3.5) sistemani qanoatlantirsa, bu nuqtada (3.1) tenglama bilan berilgan ikkkinchi tartibli chiziqning markazi deyiladi.

Tabiiyki, (3.5) sistema yagona yechimga ega bo‘lishi, cheksiz ko‘p yechimga ega bo‘lishi yoki umuman yechimga ega bo‘lmasligi mumkin.

Agar, $a_{11}a_{22} - a_{21}^2 \neq 0$

munosabat o‘rinli bo‘lsa, (3.5) sistema yagona yechimga ega bo‘ladi. Agar,

$$\frac{a_{11}}{a_{21}}=\frac{a_{12}}{a_{22}}=\frac{a_{13}}{a_{23}}$$

munosabat o‘rinli bo‘lsa sistema cheksiz ko‘p yechimga,

$$\frac{a_{11}}{a_{21}}=\frac{a_{12}}{a_{22}}\neq\frac{a_{13}}{a_{23}}$$

munosabat bajarilsa sistema yechimga ega emas. Bularni e’tiborga olib, biz ikkinchi tartibli chiziqlarni uchta sinfga ajratamiz:

a) yagona markazga ega bo‘lgan chiziqlar;
b) cheksiz ko‘p markazga ega bo‘lgan chiziqlar;
d) markazga ega bo‘lmagan chiziqlar.

3.2. Markaziy va nomarkaziy chiziqlar.

Quyidagi determinantlami kiritamiz

$$\delta=\begin{vmatrix}a_{11} & a_{12}\\ a_{12} & a_{22}\end{vmatrix},\ \Delta=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{01} & a_{02} & a_{33}\end{vmatrix},$$

bu yerda $a_{21}=a_{12}$, $a_{31}=a_{13}$, $a_{32}=a_{23}$ belgilashlar kiritilgan. Yagona markazga ega chiziqlar uchun $\delta\neq 0$, yagona markazga ega bo‘lmagan chiziqlar uchun $\delta=0$. Chiziqlar cheksiz ko‘p markazga ega bo‘lishi uchun $\Delta=0$ tenglik bajarilshi kerak.

Uchinchi tartibli determinantni

$$\Delta=a_{13}\begin{vmatrix}a_{21} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}-a_{23}\begin{vmatrix}a_{11} & a_{12}\\ a_{31} & a_{32}\end{vmatrix}+a_{33}\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}$$

ko‘rinishda yozib olsak, oxirgi determinant δ ga tengdir. Agar $\delta=0$ bo‘lsa, birorta k soni uchun

$$\frac{a_{11}}{a_{21}}=\frac{a_{12}}{a_{22}}=k,\quad \begin{vmatrix}a_{11} & a_{12}\\ a_{31} & a_{32}\end{vmatrix}=k\begin{vmatrix}a_{12} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}$$

munosabat bajariladi. Bu tenglikni hisobga olib

$$\Delta=(a_{13}-ka_{23})\begin{vmatrix}a_{12} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}$$

tenglikni hosil qilamiz. Agar $\Delta=0$ tenglik ham bajarilsa

$$a_{13}-ka_{23}=0 \text{ yoki } \begin{vmatrix}a_{12} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}=0$$

tengliklardan kamida bittasi o‘rinli bo‘ladi. Bu tengliklarning birinchisi o‘rinli bo‘lsa, $\frac{a_{11}}{a_{12}}=\frac{a_{12}}{a_{22}}=k$ munosabatdan $\frac{a_{11}}{a_{12}}=\frac{a_{12}}{a_{22}}=\frac{a_{13}}{a_{23}}$ munosabat kelib chiqadi. Agar,

$$\begin{vmatrix} a_{12} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = 0$$

bo‘lsa, $\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = k$ va $\frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}}$ tengliklardan

$$\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}} = k$$

munosabat kelib chiqadi. Demak, $\delta = 0$ va $\Delta = 0$ tengliklarning bir vaqtda bajarilishi

$$\frac{a_{11}}{a_{12}} = \frac{a_{12}}{a_{22}} = \frac{a_{13}}{a_{23}} = k$$

shartga teng kuchlidir. Natijada biz quyidagi tasdiqni hosil qilamiz:

1-tasdiq. Ikkinchi tartibli chiziq

a) $\delta \neq 0$ bo‘lsa yagona markazga ega,

b) $\delta = 0$ va $\Delta = 0$ bo‘lsa, cheksiz ko‘p markazga ega va markazlar to‘plami bitta to‘g‘ri chiziqni tashkil etadi;

c) $\delta = 0$ va $\Delta = 0$ bo‘lsa markazga ega emas.

2-tasdiq. Yagona markazga ega bo‘lgan ikkinchi tartibli chiziq markazi unga tegishli bo‘lishi uchun $\Delta = 0$ tenglikning bajarilishi zarur va yetarlidir.

Isbot. Ikkinchi tartibli chiziq markazi $M_0(x_0; y_0)$ nuqtada bo‘lib, u chiziqqa tegishli bo‘lsa

$$\begin{cases} a_{11}x_0 + a_{12}y_0 + a_{13} = 0 \\ a_{21}x_0 + a_{22}y_0 + a_{23} = 0 \end{cases} \qquad (3.6)$$

va

$$a_{11}x_0^2 + 2a_{12}x_0y_0 + a_{22}y_0^2 + 2a_{13}x_0 + 2a_{23}y_0 + a_{33} = 0 \qquad (3.7)$$

tengliklar bajariladi. Yuqoridagi (3.6) tenglikning birinchisini x_0 ga, ikkinchisini y_0 ga ko‘paytirib, (3.7) tenglikdan ayirsak,

$$a_{31}x_0 + a_{32}y_0 + a_{33} = 0$$

tenglikni hosil qilamiz. Demak, $(x_0; y_0; 1)$ uchlik

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = 0 \\ a_{21}x + a_{22}y + a_{23}z = 0 \\ a_{31}x + a_{32}y + a_{33}z = 0 \end{cases} \qquad (3.8)$$

bir jinsli sistemaning notrivial yechimidir. Bu esa $\Delta = 0$ shartga teng kuchlidir. Aksincha $\Delta = 0$ bo‘lsa, (3.8) sistema notrivial $(x_0; y_0; z_0)$ yechimga egadir. Bu uchlikda $z_0 \neq 0$, chunki $\delta \neq 0$. Biz $z_0 = 1$ deb hisoblay olamiz, chunki $\delta \neq 0$ bo‘lganligi uchun har bir z_0 uchun $(x_0; y_0)$ juftlik mavjud. Yuqoridagi (3.8) sistemada $z_0 = 1$ bo‘lganda

$(x_0; y_0)$ juftlik markaz koordinatalari ekanligi kelib chiqadi. Bundan tashqari (3.8) sistemadan foydalanib,

$$a_{11}x_0^2 + 2a_{12}x_0y_0 + a_{22}y_0^2 + 2a_{13}x_0 + 2a_{23}y_0 + a_{33} = 0$$

tenglikni olish mumkin.

1-misol. Quyidagi tenglama bilan berilgan chiziqning turi va joylashishi aniqlansin:

$$6xy + 8y^2 - 12x - 26y + 11 = 0.$$

Yechish:

$$I_2 = \begin{vmatrix} 0 & 3 \\ 3 & 8 \end{vmatrix} = -9 < 0,$$

demak bu chiziq - birinchi guruhga tegishli:

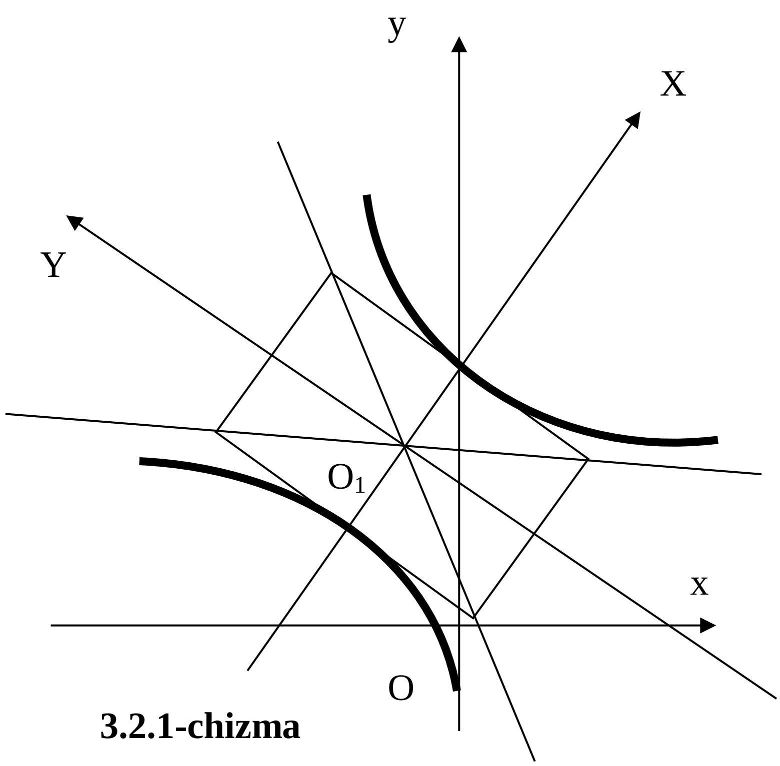
$$K_3 = \begin{vmatrix} 0 & 3 & -6 \\ 3 & 8 & -13 \\ -6 & -13 & 11 \end{vmatrix} = 81 \neq 0,$$

chiziq giperboladan iborat:

$$I_1 = 0 + 8 = 8.$$

Chiziqning xarakteristik tenglamasi:

$$\lambda^2 - 8\lambda - 9 = 0.$$



3.2.1-chizma

Xarakteristik tenglamaning yechimlari:

$$\lambda_1 = 9, \lambda_2 = -1.$$

Almashtirishdan so‘ng tenglama $9X^2 - Y^2 + \frac{81}{-9} = 0$ ko‘rinishga keladi. Kanonik tenglamasi esa:

$$\frac{X^2}{1} - \frac{Y^2}{9} = 1.$$

Markazi quyidagi

$$\begin{cases} 3y - 6 = 0, \\ 3x + 8y - 13 = 0 \end{cases}$$

tenglamalardan topiladi. $O'(-1; 2)$ - nuqta chiziq markazi. $O'X$ o‘qning burchak koeffitsiyenti $k = \frac{9}{3} = 3$.

2-misol. Quyidagi

$$x^2 - 4xy + 4y^2 + 4x - 3y - 7 = 0$$

chiziqning shakli va joylashishi, fokusi va direktrisalari aniqlansin.

Yechish:

$$I_2 = \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} = 0, \qquad K_3 = \begin{vmatrix} 1 & -2 & 2 \\ -2 & 4 & -\frac{3}{2} \\ 2 & -\frac{3}{2} & -7 \end{vmatrix} = -\frac{25}{4}.$$

Demak, berilgan chiziq - parabola:

$$I_1 = 1 + 4 = 5.$$

Parametri:

$$p = \sqrt{\frac{25}{4 \cdot 5^3}} = \frac{1}{2\sqrt{5}}.$$

Kanonik tenglamasi:

$$Y^2 = \frac{1}{\sqrt{5}} X.$$

O‘qining tenglamasi:

$$x - 2y - \frac{1 \cdot 2 - 2 \cdot (-\frac{3}{2})}{1+4} = 0$$

yoki

$$x - 2y + 1 = 0.$$

Parabola uchining koordinatalarini topish uchun tenglamalar:

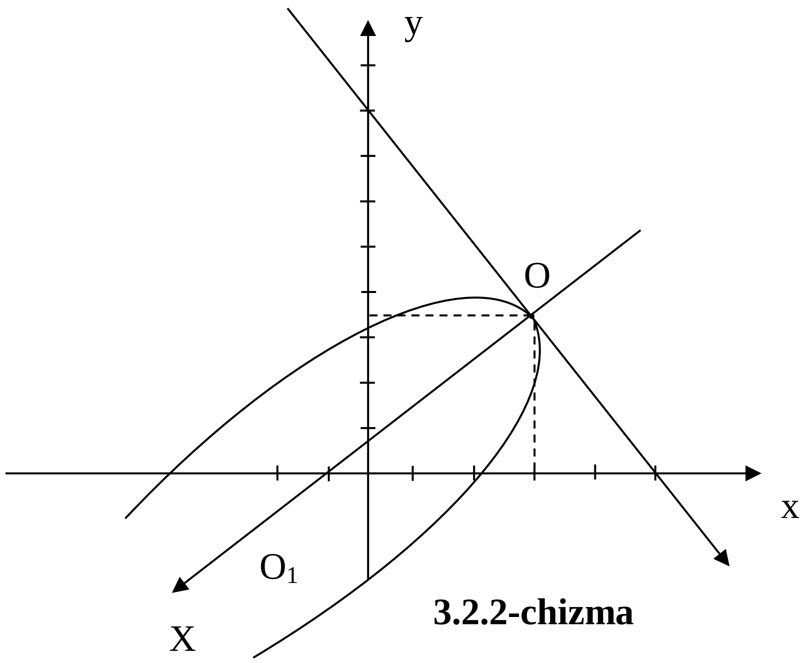
$$\begin{cases} x - 2y + 1 = 0 \\ x^2 - 4xy + 4y^2 + 4x - 3y - 7 = 0, \end{cases}$$

$$\begin{cases} x - 2y = -1 \\ (x - 2y)^2 + 4x - 3y - 7 = 0, \end{cases}$$

$$\begin{cases} x - 2y + 1 = 0 \\ 4x - 3y - 6 = 0. \end{cases}$$

Natijada, parabola uchi $O'(3; 2)$ nuqtada, botiqlik tomoniga yo‘nalgan o‘q vektori esa

$$\left\{\begin{vmatrix}-2 & 4\\ 2 & -\frac{3}{2}\end{vmatrix}, -\begin{vmatrix}1 & -2\\ 2 & -\frac{3}{2}\end{vmatrix}\right\} = \{-5, -\frac{5}{2}\} \downarrow\downarrow (-2, -1)$$



3.2.2-chizma

koordinatalarga ega (3.2.2-chizma). ($X = \sqrt{5}$ bo‘lganda $Y = \pm 1$ teng ekanini bilish foydali).

Berilgan chiziq parametri $p = \frac{1}{2\sqrt{5}}$ ga teng parabolani ifodalaydi. Parabola uchi $O'(3;2)$ nuqtada. Parabola o‘qining musbat yo‘nalishi $(-2;-1)$ vektor bilan aniqlanadi. Ox va $O'x$ o‘qlar orasidagi burchakni φ desak:

$$\cos\varphi = -\frac{2}{\sqrt{5}},\ \sin\varphi = -\frac{1}{\sqrt{5}}.$$

Demak almashtirish formulalari:

$$x = \frac{-2X + Y}{\sqrt{5}} + 3,\quad y = \frac{-X - 2Y}{\sqrt{5}} + 2,$$

bundan

$$X = \frac{-2x - y + 8}{\sqrt{5}},\qquad Y = \frac{x - 2y + 1}{\sqrt{5}}.$$

Kanonik sistemada fokus koordinatalari: $X = \frac{1}{4\sqrt{5}},\quad Y = 0,$

boshlang‘ich sistemada esa: $x = 2{,}9;\quad y = 1{,}95,$

kanonik sistemada direktrisa tenglamasi: $X = -\frac{1}{4\sqrt{5}},$

boshlang‘ich sistemada esa:

$$\frac{-2x-y+8}{\sqrt{5}}=-\frac{1}{4\sqrt{5}} \qquad \text{yoki} \qquad 8x+4y-33=0.$$

Mustaqil yechish uchun topshiriqlar.

3.1.1. Beshta nuqtadan o‘tuvchi ikkinchi tartibli chiziqning tenglamasi tuzilsin: $(0;0)$, $(0;1)$, $(1;0)$, $(2;-5)$, $(-5;2)$.

3.1.2. Quyidagi egri chiziqlarning markazlari topilsin:

1) $5x^2+8xy+5y^2-18x-18y+11=0$;
2) $2xy-4x+2y+11=0$;
3) $4x^2+4xy+y^2-10x-5y+6=0$;
4) $x^2-2xy+y^2-3x+2y-11=0$.

3.1.3. 1) Parallelogrammga tashqi chizilgan ikkinchi tartibli chiziqning markaziy chiziq ekanligi va markazi parallellogram diagonallarining kesishish nuqtasidan iboratligi isbotlansin.

2) Parallelogrammga ichki chizilgan ikkinchi tartibli chiziqning hamisha markaziy ekanligi va markaz parallellogram diagonallarining kesishish nuqtasida ekanligi ko‘rsatilsin.

3.1.4. Uchburchakka ichki chizilgan ikkinchi tartibli chiziq markazi uchburchakning og‘irlik markazi bo‘lsa, bu chiziqning ellips ekanligi isbotlansin.

3.1.5. To‘g‘ri burchakli koordinatalar sistemasida quyidagi:

1) $5x^2+8xy+5y^2-18x-18y+9=0$;
2) $2xy-4x+2y-3=0$;
3) $x^2-4xy+4y^2-5x+10y+6=0$;
4) $2x^2+3xy-2y^2+5x-2=0$;
5) $x^2-4xy+4y^2-5x+6=0$,

ikkinchi tartibli chiziqlarning markaziy yoki nomarkaziy chiziq ekanligini aniqlang.

3.1.6. Quyidagi beshta nuqtadan o‘tuvchi ikkinchi tartibli egri chiziqning tenglamasi tuzilsin:

$$(0;0),\ (0;2),\ (-1;0),\ (-2;1),\ (-1;3).$$

3.1.7. Quyidagi nuqtalardan qanday ikkinchi tartibli egri chiziq o‘tkazish mumkin: $(0;0)$, $(0;3)$, $(6;0)$, $(2;2)$ va $(-2;1)$.

3.1.8. To‘rtta nuqta berilgan: $(0;15)$, $(3;0)$, $(5;0)$ va $(2;3)$. Bular orqali parabola tipidagi egri chiziq o‘tkazilsin.

Ko‘rsatma. Parabola tipidagi egri chiziq to‘rtta shart bilan aniqlanadi, chunki uning koeffitsiyentlari orasida $a_{11}a_{22}-a_{12}^2=0$ munosabat mavjud bo‘lishi kerak; demak, parabolik egri chiziqning tenglamasi faqat to‘rtta erkli parametrga ega.

3.1.9. Agar koordinatalar boshi $O'(1;0)$ nuqtaga ko‘chirilsa,

$$x^2 - 4xy + 3y^2 - 2x + 1 = 0$$

egri chiziqning tenglamasi qanday shaklni oladi?

3.1.10. $xy - 6x + 2y + 3 = 0$ egri chiziq berilgan. Koordinatalar boshi $(-2;6)$ nuqtaga ko‘chirilgandan so‘ng bu egri chiziqning almashingan tenglamasi topilsin.

3.1.11. $x^2 + 6x - 8y + 1 = 0$ egri chiziqning koordinatalar boshi $(-3;-1)$ nuqtaga ko‘chirilgandan so‘ng almashingan tenglamasi topilsin.

3.1.12. Quyidagi egri chiziqlarning markazlari topilsin:

1) $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$;
2) $3x^2 + 2xy + 3y^2 + 4x + 4y - 4 = 0$;
3) $2x^2 - 3xy - y^2 + 3x + 2y = 0$;
4) $x^2 - 2xy + y^2 - 4x - 6y + 3 = 0$;
5) $x^2 + 2xy + y^2 + 2x + 2y - 4 = 0$;
6) $2x^2 - 4xy + 5y^2 - 8x + 6 = 0$.

3.1.13. a va b parametrlarning qanday qiymatlarida

$$x^2 + 6xy + ay^2 + 3x + by - 4 = 0$$

tenglama:

a) markaziy egri chiziqni;
b) parabola tipidagi egri chiziqni;
c) markazlar chizig‘iga ega bo‘lgan egri chiziqni ifodalaydi?

3.1.14. Quyidagi egri chiziqlarning markazlari topilsin:

1) $5x^2 - 3xy + y^2 + 4 = 0$;
2) $3x^2 - 2xy + 4 = 0$;
3) $7xy - 3 = 0$;
4) $9x^2 - 12xy + 4y^2 - 1 = 0$;
5) $a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + a_{33} = 0$.

3.1.15. Koordinatalar boshi $2x^2 - 6xy + 5y^2 - 2x + 2y - 10 = 0$ egri chiziqning markaziga ko‘chirilsa, uning tenglamasi qanday ko‘rinishni oladi?

3.1.16. Koordinatalar boshini ko‘chirishdan foydalanib, quyidagi egri chiziqlarning tenglamalari soddalashtirilsin:

1) $7x^2 + 4xy + 4y^2 - 40x - 32y + 5 = 0$;
2) $x^2 - 2xy + 2x + 2y + 1 = 0$;

3) $6x^2 - 4xy + 9y^2 - 4x - 32y - 6 = 0$.

3.1.17. Umumiy $(x_0; y_0)$ markazga ega bo‘lgan barcha ikkinchi tartibli egri chiziqlarning umumiy tenglamasi tuzilsin.

3.1.18. Ikkinchi tartibli egri chiziqning koordinatalar boshidan va $A(0;1)$, $B(1;0)$ nuqtalardan o‘tadi. Bundan tashqari uning $C(2;3)$ markazi ma’lum. Shu egri chiziqning tenglamasi tuzilsin.

3.1.19. $x^2 + 2xy - y^2 - 2ax + 4ay + 1 = 0$ egri chiziqlar markazlarining geometrik o‘rni topilsin, bunda $a-$ o‘zgaruvchi parametr.

3.1.20. To‘rtta $(0;0)$, $(2;0)$, $(0;1)$ va $(1;2)$ nuqtalardan o‘tuvchi barcha ikkinchi tartibli markaziy egri chiziqlar markazlarining geometrik o‘rni topilsin.

4-MAVZU: IKKINCHI TARTIBLI CHIZIQ VA TO‘G‘RI CHIZIQNING O‘ZARO VAZIYATI.

Reja:

1. Ikkinchi tartibli egri chiziqning to‘g‘ri chiziq bilan kesilishi.

2. Egri chiziqning diametrlari. Bosh o‘qlar. Asimptotalar. Egri chiziqning qo‘shma yo‘nalishlarga nisbatan tuzilgan tenglamasi; egri chiziqning asimptotalarga nisbatan tenglamasi.

3. Ikkinchi tartibli chiziqlarning urinma tenglamalari.

Tayanch iboralar: ellips, giperbola, parabola, diametr, vatar, fokus, urinma, asimptota, direktrisa, qo‘shma diametr.

4.1. Ikkinchi tartibli egri chiziqning to‘g‘ri chiziq bilan kesilishi.

Ikkinchi tartibli

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0 \qquad (4.1)$$

egri chiziqning

$$Ax + By + C = 0 \qquad (4.2)$$

to‘g‘ri chiziq bilan kesilish nuqtalarining koordinatalari (4.1) va (4.2) tenglamalarni birgalikda yechib aniqlanadi.

Bu tenglamalar sistemasi, umuman aytganda, ikki qo‘sh ildizga ega bo‘lishi kerak, shuning uchun ikkinchi tartibli egri chiziq to‘g‘ri chiziq bilan ikkita (haqiqiy, mavhum yoki ustma – ust tushgan) nuqtada kesishadi. Agar bu ikki nuqta ustma – ust tushsa, to‘g‘ri chiziq egri chiziqqa shu nuqtadagi ***urinma*** deyiladi.

(4.1) egri chiziqqa $(x';y')$ nuqtadagi urinmaning tenglamasi:

$$(a_{11}x' + a_{12}y' + a_{13})x + (a_{21}x' + a_{22}y' + a_{23})y + \\ +(a_{31}x' + a_{32}y' + a_{33}) = 0. \qquad (4.3)$$

Agar berilgan

$$Ax + By + C = 0$$

to‘g‘ri chiziq (4.1) egri chiziqqa urinsa, u holda urinish nuqtasining koordinatalari bu to‘g‘ri chiziq va (4.3) urinma tenglamalari koeffitsiyentlarining proporsionallik, ya’ni:

$$\frac{a_{11}x' + a_{12}y' + a_{13}}{A} = \frac{a_{21}x' + a_{22}y' + a_{23}}{B} = \\ = \frac{a_{31}x' + a_{32}y' + a_{33}}{C} \qquad (4.4)$$

shartidan aniqlanadi.

(4.1) egri chiziq va (4.2) to‘g‘ri chiziq tenglamalarining koordinatalaridan birini yo‘qotganda ikkinchi koordinatani aniqlash uchun ikkinchi darajali emas, balki birinchi darajali tenglama hosil bo‘lishi (aniqlanayotgan koordinataning kvadrati oldidagi koeffitsiyent nolga aylanadi), (4.1) egri chiziqning (4.2) to‘g‘ri chiziq bilan kesilishining xususiy holiga ega bo‘lamiz. Bu holda tekislikning chekli qismida (4.1) egri chiziq bilan (4.2) to‘g‘ri chiziqning birgina umumiy nuqtasi bo‘ladi. Ular faqat bir nuqtada kesishadi deymiz. Bu to‘g‘ri chiziqlarning burchak koeffitsiyentlari

$$a_{11} + 2a_{12}k + a_{22}k^2 = 0$$

tenglamadan aniqlanadi. Agar (4.1) va (4.2) tenglamalar birgalikda bo‘la olmasa, ya’ni umumiy chekli ildizlarga ega bo‘lsa, (4.1) egri chiziq (4.2) to‘g‘ri chiziq bilan hech bir umumiy nuqtaga ega emas deymiz. Bu holda (4.1) va (4.2) tenglamalarning koordinatalaridan birini yo‘qotishda faqat aniqlanayotgan koordinataning kvadrati oldidagi koeffitsiyenti ham nolga aylanadi.

4.2. Egri chiziqning diametrlari. Bosh o‘qlar. Asimptotalar. Egri chiziqning qo‘shma yo‘nalishlarga nisbatan tuzilgan tenglamasi; egri chiziqning asimptotalarga nisbatan tenglamasi.

Agar ikkinchi tartibli egri chiziqning bir xil yo‘nalishdagi hamma vatarlari o‘tkazilsa, bu vatarlar o‘rtalarining geometrik o‘rni biror to‘g‘ri chiziqdan iborat bo‘ladi. Bu to‘g‘ri chiziq berilgan vatarga ***qo‘shma diametr*** deyiladi. Diametrning tenglamasi:

$$(a_{11}x + a_{12}y + a_{13}) + k(a_{21}x + a_{22}y + a_{33}) = 0, \quad (4.5)$$

yoki

$$F_x + kF_y = 0, \quad (4.6)$$

bu yerda k – berilgan yo‘nalishdagi vatarlarning burchak koeffitsiyenti. k ni o‘zgartirish bilan, ya’ni vatarlar yo‘nalishini o‘zgartirish bilan cheksiz ko‘p diametrlar hosil qilamiz, ularning hammasi egri chiziqning markazidan o‘tadi. Parabolaning hamma diametrlari o‘zaro paralleldir.

Vatarning yo‘nalishi va ularga qo‘shma diametrning yo‘nalishi berilgan egri chiziqqa nisbatan qo‘shma yo‘nalishlar deyiladi. Ikki qo‘shma yo‘nalish orasidagi bog‘lanish quyidagicha bo‘ladi:

$$a_{11} + a_{12}(k + k') + a_{22}kk' = 0. \quad (4.7)$$

Qo‘shma diametrlar deb, shunday ikkita diametrga aytiladiki, ularning har biri ikkinchisiga parallel vatarlarni teng ikkiga bo‘ladi. Parabolaning qo‘shma diametrlari yo‘q, chunki hamma diametrlar bir xil yo‘nalishga ega.

Qo‘shma vatarlarga perpendikulyar bo‘lgan diametrlar egri chiziqning bosh o‘qlari deyiladi; ularning yo‘nalishlari ***bosh yo‘nalishlar*** deyiladi.

To‘g‘ri burchakli koordinatalar sistemasida bosh yo‘nalishlar

$$a_{12}k^2 + (a_{11} - a_{22})k - a_{12} = 0, \quad (4.8)$$

yoki

$$tg2\varphi = \frac{2a_{12}}{a_{11} - a_{22}} \quad (4.9)$$

tenglamadan aniqlanadi, bunda φ – bosh yo‘nalishlardan birining x o‘qi yo‘nalishi bilan tashkil etgan burchagi.

Qiyshiq burchakli koordinatalar sistemasida bosh yo‘nalishlar tenglamasi:

$$(a_{12} - a_{22}cos\omega)k^2 + (a_{11} - a_{22})k - (a_{12} - a_{11}cos\omega) = 0. \quad (4.10)$$

Aylanadan tashqari har qanday ikkinchi tartibli egri chiziq ikkita bosh yo‘nalishga ega; aylananing bosh yo‘nalishlari aniq emas (cheksiz ko‘p).

Parabolaning hamma diametrlari uchun burchak koeffitsiyenti

$$k = -\frac{a_{11}}{a_{22}} = \frac{a_{12}}{a_{22}} \quad (4.11)$$

formula bilan aniqlanadi yoki parabolaning ikkinchi darajali hadlarining koeffitsiyentlari:

$$a_{11} = \alpha^2, \;\; a_{12} = \alpha\beta, \;\; a_{22} = \beta^2$$

bilan belgilansa, u holda burchak koeffitsiyenti

$$k = -\frac{\alpha}{\beta} \tag{4.12}$$

bo‘ladi.

Parabolaning bosh o‘qi uning diametrlaridan biri bo‘lgani uchun u ham shu yo‘nalishga egadir va to‘g‘ri burchakli koordinatalarda

$$F_x + \frac{\beta}{\alpha} F_y = 0 \tag{4.13}$$

tenglama bilan bilan ifodalanadi.

Parabolaning ikkinchi bosh yo‘nalishi uning diametriga perpendikulyardir, lekin parabolaning ikkinchi bosh o‘qi yo‘q.

Agar ikki qo‘shma yo‘nalishga nisbatan egri chiziqning tenglamasi tuzilsa, ya’ni koordinata o‘qlari deb, bu egri chiziqqa nisbatan qo‘shma yo‘nalishlarga ega bo‘lgan ikki to‘g‘ri chiziq olinsa, egri chiziq tenglamasiga koordinatalarning ko‘paytmasidan tuzilgan had kirmaydi ($a_{12} = 0$). Parabola tenglamasida, bundan tashqari ikkinchi darajali hadlardan biri yo‘qoladi ($a_{11} = 0$ yoki $a_{22} = 0$).

Agar markaziy egri chiziqning tenglamasi ikki qo‘shma diametrga (yoki bosh o‘qlarga) nisbatan yozilsa, uning tenglamasi

$$a'_{11}x^2 + a'_{22}y^2 + \frac{\Delta}{\delta} = 0 \tag{4.14}$$

ko‘rinishni oladi.

Koordinatalar boshini parabolaning uchiga, ya’ni parabolaning bosh o‘qi bilan kesishish nuqtasiga olib ($a'_{33} = 0$), bosh o‘qni absissalar o‘qi ($a'_{23} = 0$, $a'_{12} = 0$ va $a'_{11} = 0$) va parabola uchidan o‘tgan urinmani (u parabola o‘qiga perpendikulyar) ordinatalar o‘qi deb olinsa, parabolaning eng sodda tenglamasi hosil bo‘ladi

$$a'_{22}y^2 + 2a'_{13}x = 0. \tag{4.15}$$

Koordinata o‘qlari yuqoridagidek olinsa, markaziy egri chiziq

$$a'_{11}x^2 + a'_{22}y^2 + 2a'_{13}x = 0 \tag{4.16}$$

tenglama bilan ifodalanadi.

Egri chiziqning diametrlaridan o‘z – o‘ziga qo‘shma bo‘lganlariga uning asimptotalari deb qarash mumkin. Asimptotalarning burchak koeffitsiyentlari

$$a_{11} + 2a_{12}k + a_{22}k^2 = 0 \tag{4.17}$$

tenglamadan aniqlanadi.

Asimptotalar faqat markaziy egri chiziqlarda bo‘lishi mumkin: giperbola ikkita haqiqiy asimptotaga ega, ellips ikkita mavhum asimptotaga ega. Egri chiziq ikkita kesishuvchi to‘g‘ri chiziqlarga ajralsa, u holda asimptotalar bu to‘g‘ri chiziqlar bilan ustma – ust tushadi. Giperbolaning asimptotalari koordinata o‘qlari deb olinsa, u holda bu giperbolaning tenglamasi,

$$2a'_{12}xy + a'_{33} = 0 \qquad (4.18)$$

shaklni oladi.

4.3. Ikkinchi tartibli chiziqlarning urinma tenglamalari.

Tekislikdagi $F(x;y) = 0$ oshkormas tenglama bilan berilgan egri chiziqning $M_0(x_0;y_0)$ nuqtasidagi urinma tenglamasi

$$F_x{}'(x_0;y_0)(x - x_0) - F_y{}'(x_0;y_0)(y - y_0) = 0 \qquad (4.19)$$

Ellips uchun

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad F_x{}'(x) = \frac{2x}{a^2}, \quad F_y{}'(y) = \frac{2y}{a^2} \Rightarrow$$

$$\frac{2x}{a^2}(x - x_0) + \frac{2y}{a^2}(y - y_0) = 0 \Rightarrow$$

$$-\frac{2xx_0}{a^2} + \frac{2x^2}{a^2} + \frac{2y^2}{a^2} - \frac{2yy_0}{a^2} = 0 \Rightarrow$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{xx_0}{a^2} + \frac{yy_0}{a^2}.$$

Ushbu tenglamaga

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1 \qquad (4.20)$$

ellipsning urinma tenglamasi deyiladi.

Giperbola uchun

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad F_x{}'(x) = \frac{2x}{a^2}, \quad F_y{}'(y) = \frac{2y}{a^2} \Rightarrow$$

$$F_x{}'(x_0;y_0) = \frac{2x_0}{a^2}, \quad F_y{}'(x_0;y_0) = -\frac{2y_0}{a^2} \Rightarrow$$

$$\frac{2x_0}{a^2}(x - x_0) + \frac{2y_0}{a^2}(y - y_0) = 0 \Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{a^2} = \frac{{x_0}^2}{a^2} - \frac{{y_0}^2}{a^2}$$

$$\frac{xx_0}{a^2} - \frac{yy_0}{a^2} = 1 \qquad (4.21)$$

munosabatlar o‘rinlidir. Hosil bo’lgan tenglamaga ***giperbolaning urinma tenglamasi*** deyiladi.

1-misol. $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$ egri chiziqning shunday ikkita qo‘shma diametrlari topilsinki, ularning biri koordinatalar boshidan o‘tsin.

Yechish: $\delta \neq 0$ bo‘lgani uchun, berilgan egri chiziq markaziy egri chiziq hisoblanadi. Uning ixtiyoriy diametrining tenglamasi

$$(x - y - 2) + k(-x + 2y - 3) = 0$$

ko‘rinishda bo‘ladi, bu yerda $k -$ unga qo‘shma diametrning burchak koeffitsiyenti. Izlangan diametr koordinatalar boshidan o‘tganligi uchun uning tenglamasidagi ozod had nolga teng bo‘lishi, ya’ni $-2 - -3k = 0$ va $k = -\frac{2}{3}$ bo‘lishi kerak. Parametrning bu qiymatini diametrning umumiy tenglamasiga qo‘yib va soddalashtirib, $5x - 7y = 0$ tenglamani olamiz. Bu izlangan diametrlardan birining tenglamasidir, uning burchak koeffitsiyenti $k' = \frac{5}{7}$, demak, bunga qo‘shma diametrning tenglamasi

$$(x - y - 2) + \frac{5}{7}(-x + 2y - 3) = 0,$$

yoki

$$2x + 3y - 29 = 0$$

ko‘rinishida bo‘ladi.

2-misol. $x^2 - 2xy + y^2 + x - 2y + 3 = 0$ parabolaning o‘qi topilsin.

Yechish: Berilgan parabolaning hamma diametrlari $k = 1$ burchak koeffitsiyentga ega. Parabolaning o‘qi diametriga perpendikulyar vatarga, ya’ni $k_1 = -1$ burchak koeffitsiyentli vatarlarga qo‘shma bo‘lgan diametrdir, bu holatda koordinatalar sistemasi to‘g‘ri burchakli deb faraz qilinadi. Bu parabola har qanday diametrining tenglamasi

$$2x - 2y + 1 + k(-2x + 2y - 2) = 0$$

bo‘ladi: $k = -1$ bo‘lganda parabola o‘qining $4x - 4y + 3 = 0$ tenglamasi hosil bo‘ladi.

3-misol. Koordinatalar boshidan o‘tuvchi ikkinchi tartibli egri chiziqning ikki juft qo‘shma diametrlari ma’lum:

$$\begin{cases} x - 3y - 2 = 0 \\ 5x - 5y - 4 = 0 \end{cases} \text{va} \quad \begin{cases} 5y + 3 = 0 \\ 2x - y - 1 = 0. \end{cases}$$

Bu egri chiziqning tenglamasi tuzilsin.

Yechish: Qo‘shma diametrlarning burchak koeffitsiyentlari

$$c_{11} + a_{12}(k_1 + k_2) + a_{22}k_1k_2 = 0$$

tenglamani qanoatlantiradi. Berilgan diametrlarning burchak koeffitsiyentlari $k_1 = \frac{1}{3}$ va $k_2 = 1;\ k_1' = 0$ va $k_2' = 2.$ Bu qiymatlarni ko‘rsatilgan tenglamaga qo‘yib, quyidagilarni hosil qilamiz:

$$\begin{cases} 3a_{11}x + 4a_{12} + a_{22} = 0 \\ a_{11} + 2a_{12} = 0 \end{cases}, \qquad a_{11}: a_{12}: a_{22} = 2: (-1): (-2).$$

Izlangan egri chiziq markazining koordinatalarini ikki diametr tenglamalarini birgalikda yechib aniqlashimiz mumkin:

$$x_0 = \frac{1}{5}; \quad y_0 = \frac{3}{5}.$$

Bu koordinatalar $Fx_0 = 0$ va $Fy_0 = 0$ tenglamalarni, ya’ni berilgan holda $2x_0 - y_0 + a_{13} = 0$ va $-x_0 - 2y_0 + a_{23} = 0$ tenglamalarni qanoatlantirishi kerak; x_0 va y_0 ning qiymatlarini bu tenglamalarga qo‘yib, $a_{13} = -1$ va $a_{23} = -1$ ni olamiz. Bundan tashqari egri chiziq koordinatalar boshidan o‘tadi, demak, $a_{33} = 0$ va egri chiziqning tenglamasi:

$$2x^2 - 2xy - 2y^2 - 2x - 2y = 0 \text{ yoki } x^2 - xy - y^2 - x - y = 0.$$

Mustaqil yechish uchun topshiriqlar.

4.1. Ikkinchi tartibli egri chiziqning to‘g‘ri chiziq bilan kesilishi. Ikkinchi tartibli to‘g‘ri chiziqning diametriga doir misollar.

4.1.1. $5x^2 - 3xy + y^2 - 3x + 2y - 5 = 0$ chiziqning $x - 2y - 1 = = 0$ to‘g‘ri chiziq bilan kesishishidan hosil qilingan vatarning o‘rtasidan o‘tadigan diametr tenglamasi yozilsin.

4.1.2. $4xy - 5y^2 + 2x + 6y + 1 = 0$ egri chiziqning $(-4; 2)$ nuqta orqali o‘tadigan diametri tenglamasi yozilsin.

4.1.3. $5x^2 - 6xy + 3y^2 - 2x = 0$ egri chiziqning $2x - 3y = 0$ to‘g‘ri chiziqqa parallel bo‘lgan diametri tenglamasi yozilsin.

4.1.4. Ikki egri chiziqning umumiy diametri topilsin:

$x^2 - 2xy - y^2 - 2x - 2y = 0,\ \ x^2 - 2xy + y^2 - 2x - 2y = 0.$

4.1.5. $5x^2 - 3xy + y^2 - 3x + 2y - 5 = 0$ chiziq va ikkita $A(2; 1)$ va $B(1; 4)$ nuqta berilgan. A nuqtadan o‘tuvchi diametrga qo‘shma bo‘lgan B nuqtadan shunday vatar o‘tkazilsin.

4.1.6. $\frac{x^2}{9}-\frac{y^2}{4}=1$ giperbolaning $N(5;1)$ nuqtada teng ikkiga bo‘linadigan vatarining tenglamasi tuzilsin.

4.1.7. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning o‘qlari ular teng ikkiga bo‘ladigan vatarlarga perpendikulyar bo‘lgan yagona diametrlari ekanligini tekshiring.

4.1.8. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaga ichki chizilgan kvadratning uchlari topilsin va qanday giperbolalarga ichki kvadrat chizish mumkinligi tekshirilsin.

4.1.9. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning $F(c;0)$ fokusi orqali katta o‘qiga perpendikulyar bo‘lgan vatar o‘tkazilgan. Bu vatar uzunligini toping.

4.1.10. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsga ichki chizilgan kvadrat tomonining uzunligi hisoblansin.

4.1.11. $\frac{x^2}{100}+\frac{y^2}{64}=1$ ellipsning $2x-y+7=0$, $2x-y-1=0$ vatarlarining o‘rtalari orqali o‘tadigan to‘g‘ri chiziq tenglamasini tuzing.

4.1.12. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning $M(2;1)$ nuqtada teng ikkiga bo‘linuvchi vatar tenglamasi tuzilsin.

4.1.13. $y^2=2px$ parabolaning fokusi orqali uning o‘qiga perpendikulyar bo‘lgan vatar o‘tkazilgan. Bu vatarning uzunligini aniqlang.

4.1.14. $y^2=4x$ parabolaning $M(3;1)$ nuqtada teng ikkiga bo‘ladigan vatarini toping.

4.1.15. $x+y-3=0$ to‘g‘ri chiziq va $x^2=4y$ parabola kesishgan nuqtani toping.

4.1.16. $3x+4y-12=0$ to‘g‘ri chiziq va $y^2=-9x$ parabola kesishgan nuqtani toping.

4.1.17. $3x-2y+6=0$ to‘g‘ri chiziq va $y^2=6x$ parabola kesishgan nuqtani toping.

4.1.18. $\frac{x^2}{100}+\frac{y^2}{225}=1$ ellips bilan $y^2=24x$ parabolaning kesishish nuqtalarini toping.

4.1.19. $\frac{x^2}{20}-\frac{y^2}{5}=-1$ giperbola va $y^2=3x$ parabola kesishgan nuqtalarini toping.

4.1.20. Ikki parabolaning kesishish nuqtalarini toping:
$y = x^2 - 2x + 1, \quad x = y^2 - 6y + 7.$

4.1.21. $x + 2y - 7 = 0$ to‘g‘ri chiziq bilan $x^2 + 4y^2 = 25$ ellipsning kesishish nuqtalarini toping.

4.1.22. $3x + 10y - 25 = 0$ to‘g‘ri chiziq va $\frac{x^2}{25} + \frac{y^2}{4} = 1$ tenglama bilan berilgan ellipsning kesishish nuqtalarini toping.

4.1.23. $3x - 4y - 40 = 0$ to‘g‘ri chiziq va $\frac{x^2}{16} + \frac{y^2}{9} = 1$ tenglama bilan berilgan ellipsning kesishish nuqtalarini toping.

4.1.24. Agar to‘g‘ri chiziq va ellips quyidagi tenglamalar bilan berilgan bo‘lsa, ularning o‘zaro kesishishi, urinishi yoki umumiy nuqtaga ega emasligini aniqlang:

1) $2x - y - 3 = 0,$ $\frac{x^2}{16} + \frac{y^2}{9} = 1.$

2) $2x + y - 10 = 0,$ $\frac{x^2}{9} + \frac{y^2}{4} = 1.$

3) $3x + 2y - 20 = 0,$ $\frac{x^2}{40} + \frac{y^2}{10} = 1.$

4.1.25. $y = -kx + m$ to‘g‘ri chiziq m ning qanday qiymatlarida $\frac{x^2}{20} + \frac{y^2}{5} = 1$ ellipsni: 1) kesib o‘tadi; 2) unga urinadi; 3) ushbu ellips tashqarisidan o‘tadi.

4.1.26. $\frac{x^2}{20} - \frac{y^2}{5} = 1$ giperbola va $2x - y - 10 = 0$ to‘g‘ri chiziq kesishmasining nuqtalarini toping.

4.1.27. $\frac{x^2}{25} - \frac{y^2}{16} = 1$ giperbola va $4x - 3x - 16 = 0$ to‘g‘ri chiziq kesishmasining nuqtalarini toping.

4.1.28. $\frac{x^2}{9} - \frac{y^2}{4} = 1$ giperbola va $2x - y + 1 = 0$ to‘g‘ri chiziq kesishmasining nuqtalarini toping.

4.1.29. m ning qanday qiymatlarida $y = 5x + m$ to‘g‘ri chiziq:
1) $\frac{x^2}{9} - \frac{y^2}{36} = 1$ giperbola bilan kesishadi; 2) urinma bo‘ladi;
3) giperbolaning tashqarisidan o‘tadi.

4.1.30. k va m ning qanday qiymatida $y = kx + m$ to‘g‘ri chiziq $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaga urinadi.

4.2. Egri chiziqning diametrlari. Bosh o‘qlar. Asimptotalar. Egri chiziqning qo‘shma yo‘nalishlarga nisbatan tuzilgan tenglamasi; egri chiziqning asimptotalarga nisbatan tenglamasiga doir misollar.

4.2.1. Berilgan to‘rtta $A(1;0)$, $B(3;2)$, $C(0;2)$, $D(0;-2)$ nuqta orqali o‘tadigan; AB va CD vatarlari o‘zaro qo‘shma yo‘nalishlari ekanligini bilgan holda, ikkinchi tartibli chiziqning tenglamasini yozing.

4.2.2. $3x^2 - 2xy + y^2 + 6x - 10 = 0$, $3x^2 - 2xy - y^2 + 6x - -10 = 0$ ikkinchi tartibli chiziqlar berilgan. Har bir chiziq uchun shunday qo‘shma diametrlar juftini topingki, birinchi juft diametrlar ikkinchi juft diametrlarga parallel bo‘lsin.

4.2.3. Giperbolaning asimptotalari topilsin:
$10x^2 + 21xy + 9y^2 - 41x - 39y + 4 = 0.$

4.2.4. Quyidagi
1) $x^2 - 3xy - 10y^2 + 6x - 8y = 0;$
2) $3x^2 + 2xy - y^2 + 8x + 10y - 14 = 0;$
3) $3x^2 + 7xy + 4y^2 + 5x + 2y - 6 = 0;$
4) $10xy - 2y^2 + 6x + 4y + 21 = 0,$
giperbolalarning asimptotalari topilsin.

4.2.5. $x^2 - 2y^2 - 5x + 4y + 6 = 0$ chiziqning absissa o‘qi bilan kesishish nuqtalaridagi urinmalari tenglamalari tuzilsin.

4.2.6. $x^2 + xy + y^2 + 2x + 3y - 3 = 0$ chiziqning $3x + 3y - 5 = 0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmalari tenglamalari tuzilsin.

4.2.7. $4x^2 + 4xy + y^2 - 6x + 4y + 2 = 0$ ikkinchi tartibli chiziqning Oy o‘qiga parallel urinmalarining tenglamalari yozilsin.

4.2.8. $M(3;4)$ nuqtadan $2x^2 - 4xy + y^2 - 2x + 6y - 3 = 0$ egri chiziqqa o‘tkazilgan urinma tenglamalari tuzilsin.

4.2.9. $(\alpha x + \beta y + \gamma)^2 + 2(Ax + By + C) = 0$, $\begin{vmatrix} \alpha & \beta \\ A & B \end{vmatrix} \neq 0$ tenglama berilgan.
1) Bu tenglamaning parabolani aniqlashi;
2) $\alpha x + \beta y + \gamma$ to‘g‘ri chiziqning diametr ekanligi;
3) $Ax + By + C = 0$ to‘g‘ri chiziqning diametr bilan parabolaning kesishish nuqtasidagi urinmasi ekanligi ko‘rsatilsin.

4.2.10. Parabola: $(\alpha x+\beta y)^2+2a_{13}x+2a_{23}y+a_{33}=0$, $\begin{vmatrix} \alpha & \beta \\ a_{13} & a_{23} \end{vmatrix} \neq 0$ tenglama bilan berilgan. Parabolaga uning ixtiyoriy $(x_0; y_0)$ nuqtasidan o‘tkazilgan urinma tenglamasini va unga mos kelgan qo‘shma diametr tenglamasini yozing.

4.2.11. Ikkinchi tartibli chiziqqa tashqi chizilgan parallelogrammning diagonallari berilgan chiziqning qo‘shma diametrlari ekanligi isbotlansin.

4.2.12. $\frac{x^2}{5}-\frac{y^2}{4}=1$ giperbolaga $M(5;-4)$ nuqtada urinadigan to‘g‘ri chiziq tenglamasi yozilsin.

4.2.13. $x^2-y^2=8$ giperbolaga $N(3;-1)$ nuqtada urinadigan to‘g‘ri chiziq tenglamasi yozilsin.

4.2.14. $x^2-\frac{y^2}{4}=1$ giperbolaga $M(1;4)$ nuqta orqali o‘tadigan urinmalarning tenglamalari yozilsin.

4.2.15. Berilgan $\frac{x^2}{9}-\frac{y^2}{36}=1$ giperbolaga:

1) $3x-y-17=0$ to‘g‘ri chiziqqa parallel;

2) $2x+5y+11=0$ to‘g‘ri chiziqqa perpendikulyar qilib o‘tkazilgan urinmalarning tenglamalari tuzilsin.

4.2.16. Giperbola asimptotalarining tenglamalari $y=\pm\frac{1}{2}x$ va urinmalardan birining tenglamasi $5x-6y-8=0$ ma’lum bo‘lsa, giperbola tenglamasini tuzing.

4.2.17. $\frac{x^2}{20}-\frac{y^2}{5}=1$ giperbolaga urinma va $4x+3y-7=0$ to‘g‘ri chiziqga perpendikulyar bo‘lgan tenglamani tuzing.

4.2.18. $\frac{x^2}{16}-\frac{y^2}{64}=1$ giperbolaga urinma va $10x-3y+9=0$ to‘g‘ri chiziqqa parallel bo‘lgan tenglamani tuzing.

4.2.19. $\frac{x^2}{16}-\frac{y^2}{8}=1$ giperbolaga urinma va $2x+4y-5=0$ to‘g‘ri chiziqga parallel bo‘lgan tenglamani tuzing va ular orasidagi masofa d ni toping.

4.2.20. $\frac{x^2}{24}-\frac{y^2}{18}=1$ giperbola va $3x+2y+1=0$ to‘g‘ri chiziqqa eng yaqin bo‘lgan M_1 nuqtani toping va M_1 nuqtadan to‘g‘ri chiziqqacha bo‘lgan d masofani hisoblang.

4.2.21. $N(-1;-7)$ nuqtadan o‘tib, $x^2-y^2=16$ giperbolaga o‘tkazilgan urinma tenglamasini tuzing.

4.2.22. $C(1;-10)$ nuqtadan $\frac{x^2}{8}-\frac{y^2}{32}=1$ giperbolaga urinma o‘tkazilgan. C nuqtadan o‘tuvchi tenglamasini tuzing.
4.2.23. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning burchak koeffitsiyenti $k=\frac{2}{3}$ bo‘lgan vatariga qo‘shma diametrini aniqlang.
4.2.24. $\frac{x^2}{32}+\frac{y^2}{18}=1$ elipsning $M(4;3)$ nuqtasida o‘tkazilgan urinmasining tenglamasi tuzilsin.
4.2.25. $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning $N(10;4)$ nuqta orqali o‘tuvchi urunmalarining tenglamalarini tuzing.
4.2.26. $\frac{x^2}{16}+\frac{y^2}{9}=1$ ellipsning $x+y-1=0$ to‘g‘ri chiziqqa parallel bo‘lgan urinmalarini aniqlang.
4.2.27. $3x^2+8y^2=45$ ellips markazidan 3 birlik uzoqlikdan o‘tadigan urinmalarining tenglamalari tuzilsin.
4.2.28. $\frac{x^2}{6}+\frac{y^2}{3}=1$ ellipsga tashqi chizilgan kvadrat tomonlarining tenglamalarini tuzing.
4.2.29. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning ikkkita qo‘shma diametri orasidagi o‘tkir burchakning o‘zgarish chegaralarini aniqlang.
4.2.30. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning teng qo‘shma radiuslarini toping.
4.2.31. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipsning fokusi orqali o‘tib, uning qo‘shma diametrlariga parallel bo‘lgan ikki vatarining yig‘indisini toping.
4.2.32. Quyidagi tenglamalar giperbola hosil qilishini tekshiring va uning C markazi koordinatasini, yarim o‘qlarini, ekssentrisitetini, asimptota va direktrisa tenglamalarini toping:
1) $16x^2-9y^2-64x-54y-161=0$;
2) $9x^2-16y^2+90x+32y-367=0$;
3) $16x^2-9y^2-64x-18y+199=0$.
4.2.33. Giperbolaning bitta diametrini uchlariga o‘tkazilgan urinmalar parallel bo‘lishini isbotlang.
4.2.34. $y^2=4x$ parabolaning $M(9;6)$ nuqtasida o‘tkazilgan urinmasining tenglamasini tuzing.
4.2.35. $y^2=2px$ parabolaga o‘tkazilgan urinmaning $x-3y+9=0$ tenglamasi berilgan. Parabolaning tenglamasini tuzing.

4.2.36. Burchak koeffisiyenti k ning qanday qiymatlarida ushbu $y = ax + 2$ to‘g‘ri chiziq:
1) $y^2 = 4x$ parabolaga urinma bo‘ladi;
2) $y^2 = 4x$ parabola bilan kesishadi.
4.2.37. $y^2 = 2px$ parabolaga $M_1(x_1; y_1)$ nuqtadan o‘tuvchi urinma tenglamasini tuzing.
4.2.38. $y^2 = 2px$ parabolaga $M_1(x_1; y_1)$ nuqtadan o‘tib, $2x + 2y - -3 = 0$ to‘g‘ri chiziqqa parallel bo‘lgan urinma tenglamasini tuzing.
4.2.39. $y^2 = 16x$ parabolaga $M_1(x_1; y_1)$ nuqtadan o‘tib, $2x + 4y + +7 = 0$ to‘g‘ri chiziqga perpendikulyar bo‘lgan urinma tenglamasini tuzing.
4.2.40. $y^2 = 2px$ parabolaning o‘qiga 45^0 li burchak ostida og‘ishgan vatarlariga qo‘shma bo‘lgan diametri topilsin.

5-MAVZU: IKKINCHI TARTIBLI CHIZIQLARNING TENGLAMALARINI SODDALASHTIRISH.

Reja:
1. Markazli egri chiziqning tenglamasini soddalashtirish.
2. Markazli egri chiziqning kanonik tenglamasini tekshirish.
3. Markazsiz egri chiziqning tenglamasini soddalashtirish.
4. Umumiy tenglama bilan berilgan ikkinchi tartibli chiziqlarni aniqlash va sinflarga ajratish.

***Tayanch iboralar**:* ellips, giperbola, parabola, diametr, vatar, fokus, urinma, asimptota, direktrisa, invariant, mavhum ellips.

5.1. Markazli egri chiziqning tenglamasini soddalashtirish.

Ikkinchi tartibli egri chiziqning markazi koordinatalar boshida bo‘lgan holda uning tenglamasi

$$Ax_1^2 + 2Bx_1y_1 + Cy_1^2 + 2f(a,b) = 0, \qquad (5.1)$$

bunda

$$2f(a,b) = Aa^2 + 2Bab + Cb^2 + 2Da + 2Eb + F. \qquad (5.2)$$

O‘zgaruvchi koordinatalarni a, b orqali faraz qilganda,

$$f_a(a,b) + kf_b(a,b) = 0 \qquad (5.3)$$

bo‘ladi. Izlangan geometrik o‘rinning, ya’ni diametrning tenglamasi shundan iborat

$$f_a(a,b) = Aa + Bb + D, \qquad f_b(a,b) = Ba + Cb + E$$

bo‘ladi. (5.3) ga asosan markazli egri chiziqning a va b koordinatalari ushbu sistema bilan aniqlangan edi:

$$\begin{cases} Aa + Bb + D = 0, \\ Ba + Cb + E = 0, \end{cases} \qquad (5.4)$$

bulardan birinchisini a ga va ikkinchisini b ga ko‘paytirib, so‘ngra ularni qo‘shamiz. Bu holda

$$Aa^2 + 2Bab + Cb^2 + Da + Eb = 0$$

bo‘ladi. Buning ikkala tomoniga $Da + Eb + F$ ni qo‘shib, (5.2) ni e’tiborga olsak,

$$f(a,b) = Da + Eb + F \qquad (5.5)$$

bo‘ladi. Agar bundagi a va b ning o‘rniga ularning ifodalari qo‘yilsa:

$$2f(a,b) = \frac{D^2C + BDE + AE^2 - BDE}{B^2 - AC} + F =$$

$$= \frac{D^2C - 2BDE + AE^2 + B^2F - ACF}{B^2 - AC},$$

yoki

$$2f(a,b) = -\frac{\Delta}{B^2 - AC}. \qquad (5.6)$$

Shuning uchun (5.1) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$Ax_1^2 + 2Bx_1y_1 + Cy_1^2 = \frac{\Delta}{B^2 - AC}. \qquad (5.7)$$

Bu tenglamani yana soddalashtirish maqsadi bilan koordinata o‘qlarining yo‘nalishlarini o‘zgartiramiz, ya’ni biror, hozircha ma‘lum bo‘lmagan, ixtiyoriy burchakka aylantiramiz. Aylantirilgan burchak, ya’ni koordinata o‘qlarining yangi va eski yo‘nalishlar orasidagi burchak α faraz qilinsa va egri chiziqning yangi sistemaga nisbatan o‘zgaruvchi koordinatalari x va y faraz qilinsa, u holda almashtirish formulalari quyidagicha bo‘ladi:

$$\begin{cases} x_1 = xcos\alpha - ysin\alpha, \\ y_1 = xsin\alpha + ycos\alpha. \end{cases} \qquad (5.8)$$

Bularni (5.7) ga qo‘yamiz:

$$A(xcos\alpha - ysin\alpha)^2 + 2B(xcos\alpha - ysin\alpha)(xsin\alpha + ycos\alpha) +$$

$$+C(xsin\alpha + ycos\alpha)^2 = \frac{\Delta}{B^2 - AC},$$

yoki bundagi qavslarni ochib, x^2, xy va y^2 li hadlari to‘plab olinsa, uning ko‘rinishi quyidagicha bo‘ladi:

$$(Acos^2\alpha + 2Bcos\alpha \cdot sin\alpha + Csin^2\alpha)x^2 + 2(-Acos\alpha \cdot sin\alpha + +Bcos^2\alpha - Bsin^2\alpha + Csin\alpha \cdot cos\alpha)xy + (Asin^2\alpha - -2Bsin\alpha \cdot cos\alpha + Ccos^2\alpha)y^2 = \frac{\Delta}{B^2 - AC}. \quad (5.9)$$

Bu tenglamaning koeffitsiyentlarini quyidagicha ifoda qilamiz:

$$\begin{cases} A_1 = Acos^2\alpha + 2Bsin\alpha \cdot cos\alpha + Csin^2\alpha \\ B_1 = -Asin\alpha \cdot cos\alpha + Bcos^2\alpha - Bsin^2\alpha + Csin\alpha \cdot cos\alpha \\ C_1 = Asin^2\alpha - 2Bsin\alpha \cdot cos\alpha + Ccos^2\alpha \end{cases} \quad (5.10)$$

Bu holda (5.9) ning ko‘rinishi bunday bo‘ladi:

$$A_1x^2 + 2B_1xy + C_1y^2 = \frac{\Delta}{B^2 - AC}. \quad (5.11)$$

(5.10) dagi ifodalardan birinchisi bilan uchinchisini qo‘shsak:

$$A_1 + C_1 = A + C \quad (5.12)$$

va birinchisidan uchinchisini ayirsak:

$$A_1 - C_1 =$$
$$= A(cos^2\alpha - sin^2\alpha) + 4Bsin\alpha \cdot cos\alpha - C(cos^2\alpha - sin^2\alpha) =$$
$$= (A - C)(cos^2\alpha - sin^2\alpha) + 4Bsin\alpha \cdot cos\alpha;$$
$$cos^2\alpha - sin^2\alpha = cos2\alpha, \quad 2sin\alpha \cdot cos\alpha = sin2\alpha$$

bo‘lgani uchun

$$A_1 - C_1 = (A - C)cos2\alpha + 2Bsin2\alpha,$$
$$B_1 = B(cos^2\alpha - sin^2\alpha) - (A - C)sin\alpha \cdot cos\alpha, \quad (5.13)$$

yoki

$$B_1 = Bcos2\alpha - \frac{1}{2}(A - C)sin2\alpha, \quad (5.14)$$

yoki

$$4B_1^2 = 4B^2cos^22\alpha - 4B(A - C)sin2\alpha \cdot cos2\alpha + (A - C)^2sin^22\alpha, \quad (5.15)$$

$$(A_1 - C_1)^2 = (A - C)^2cos^22\alpha + 4B(A - C)sin2\alpha \cdot cos2\alpha + +4B^2sin^22\alpha \quad (5.16)$$

(5.15) va (5.16) ni qo‘shganda

$$(A_1 - C_1)^2 + 4B_1^2 = (A - C)^2 + 4B^2.$$

So‘ngi ifodadan (5.12) ning kvadratini ayirib olamiz:

$$(A_1 - C_1)^2 - (A_1 + C_1)^2 + 4B^2 = (A - C)^2 - (A + C)^2 + 4B^2,$$

yoki

$$-4A_1C_1 + 4B_1^2 = 4AC + 4B^2,$$

yoki

$$B_1^2 - A_1C_1 = B^2 - AC. \qquad (5.17)$$

Bajarilgan almashtirish muhim xossaga egadir. Haqiqatda, (5.7) tenglama, to‘g‘ri burchakli koordinatalarda bajarilgan (5.8) almashtirish natijasida (5.11) ga kelib, hamon o‘z ko‘rinishini saqladi. (5.7) tenglamaning chap tomonidagi x_1 va y_1 ga nisbatan tuzilgan bir jinsli ikkinchi darajali ko‘p hadli xuddi shunga o‘xshash x va y ga nisbatan tuzilgan (5.11) ning chap tomonidagi ko‘p hadiga aylanadi. Ikkinchi tomondan (5.12) va (5.17) ga muvofiq.

$$A + C \text{ va } B^2 - AC \qquad (5.18)$$

ifodalar forma va miqdor jihatdan o‘zini saqlab qoldi; umuman bunday xossaga ega bo‘lgan hadi kabi ifodalarni ***invariant*** deyiladi. Shuning uchun (5.18) ifodalari $Ax_1^2 + 2Bx_1y_1 + Cy_1^2$ bir jinsli ko‘p hadligining (5.8) almashtirish bo‘yicha invariant deyiladi. α burchagi hozirgacha bizda ixtiyoriy edi. Endi uning qiymatini shunday qilib aniqlaymizki, (5.11) tenglamaning (xy) li hadi yo‘q bo‘lsin. Bu esa (5.14) ga asosan

$$B_1 = Bcos2\alpha - \frac{1}{2}(A - C)sin2\alpha = 0$$

bo‘lganda, yoki

$$tg2\alpha = \frac{2B}{A - C} \qquad (5.19)$$

bo‘lgan holda mumkin. Bu chog‘da (5.11) tenglamaning ko‘rinishi bunday bo‘ladi:

$$A_1x^2 + C_1y^2 = \frac{\Delta}{B^2 - AC}. \qquad (5.20)$$

Ikkinchi tomondan $B_1 = 0$ bo‘lganda (5.17) ning ko‘rinishi bunday bo‘ladi:

$$-A_1C_1 = B^2 - AC,$$

yoki

$$A_1C_1 = AC - B^2 \qquad (5.21)$$

(5.12) bilan (5.21) ga asosan (5.20) tenglamaning A_1 va C_1 koeffitsiyentlari ushbu ikkinchi darajali tenglamaning ildizlaridan iborat:

$$t^2 - (A + C)t + (AC - B^2) = 0,$$

demak,

$$A_1 = \frac{A + C + \sqrt{(A + C^2) - 4(AC - B^2)}}{2} =$$

$$= \frac{A + C + \sqrt{A^2 + 2AC + C^2 - 4AC + 4B^2}}{2} =$$

$$= \frac{A + C + \sqrt{A^2 - 2AC + C^2 + 4B^2}}{2} = \frac{A + C + \sqrt{(A - C)^2 + 4B^2}}{2};$$

$$C_1 = \frac{A + C - \sqrt{(A - C)^2 + 4B^2}}{2}.$$

Shuning bilan natijada markazli egri chiziqning eng sodda yoki kanonik tenglamasining koeffitsiyentlari ushbu formulalar bilan aniqlanadi:

$$\begin{cases} A_1 = \frac{1}{2}(A + C + \sqrt{(A - C)^2 + 4B^2}) \\ C_1 = \frac{1}{2}(A + C - \sqrt{(A - C)^2 + 4B^2}. \end{cases} \quad (5.22)$$

5.2. Markazli egri chiziqning kanonik tenglamasini tekshirish.

Muvofiq markazli egri chiziqning ***kanonik tenglamasi*** bo‘lsa,

$$A_1x^2 + C_1y^2 = \frac{\Delta}{M} \quad (5.23)$$

bunda

$$M = B^2 - AC. \quad (5.24)$$

Endi (5.23) tenglamaning qanday egri chiziq ifoda qilishini tekshiramiz. U tenglamaning ikkala tomonini $\frac{\Delta}{M}$ ga bo‘lganda

$$\frac{A_1M}{\Delta}x^2 + \frac{C_1M}{\Delta}y^2 = 1 \quad (5.25)$$

bo‘ladi. Bu tenglamaning geometrik ma’nosi uning koeffitsiyentlariga bog‘liqdir. Shuning uchun ularning ustida turlicha faraz qilishga to‘g‘ri keladi. $M \neq 0$ bo‘lgani uchun uning ustida ikki xil faraz qilish mumkin: 1) $M < 0$ va 2) $M > 0$. Eng avval birinchi holni tekshiramiz, ya’ni

$$M = B^2 - AC < 0 \quad (5.26)$$

bo‘lsin. (5.12) va (5.21) tenglamalarni (5.26) ga asosan

$$AC > B^2$$

bu esa A va C ishoralarning bir xilligini ko‘rsatadi. (5.26) ga muvofiq (5.21) dan $A_1C_1 > 0$, bu esa A_1 va C_1 ishoralarning bir xilligini ko‘rsatadi. Shuning uchun $A_1 + C_1$ va $A + C$ yig‘indilari ham bir xil ishorali bo‘ladi. Ikkinchi tomondan (5.12) ga asosan bu yig‘indilar o‘zaro teng bo‘lgani uchun: A, C, A_1 va C_1 koeffitsiyentlarining ishoralari bir xil bo‘ladi. Shuning uchun ulardan birining ishorasiga diqqat qilinsa kifoya bo‘ladi. Masalan, A ni olganda, agar:

a) $A\cdot\Delta < 0$ bo‘lsa, ya’ni A va Δ ning ishoralari har xil bo‘lsa, u holda (5.23) ga asosan

$$\frac{A_1M}{\Delta} > 0, \qquad \frac{C_1M}{\Delta} > 0.$$

Shuning uchun

$$\frac{A_1M}{\Delta} = \frac{1}{a^2}, \qquad \frac{C_1M}{\Delta} = \frac{1}{b^2}$$

faraz qilinsa, (5.25) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \tag{5.27}$$

Bu esa yarim o‘qlari a va b dan iborat bo‘lgan ellipsni ifoda qiladi,

b) $A\cdot\Delta > 0$ bo‘lsa, ya’ni A va Δ ning ishoralari bir xil bo‘lsa, u holda (5.26) ga asosan

$$\frac{A_1M}{\Delta} < 0, \qquad \frac{C_1M}{\Delta} < 0.$$

Shuning uchun bu holda

$$\frac{A_1M}{\Delta} = -\frac{1}{a^2}, \qquad \frac{C_1M}{\Delta} = -\frac{1}{b^2}$$

faraz qilinsa, (5.25) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \ \ yoki \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = -1. \tag{5.28}$$

Bu tenglamada x va y hech qanday haqiqiy qiymatga ega bo‘la olmaydi. Shuning uchun ***mavhim ellipsni*** ifoda qiladi.

Endi M ni musbat faraz qilamiz:

$$M = B^2 - AC > 0. \tag{5.29}$$

Bu holda (5.21) ga asosan $A_1C_1 < 0$, ya’ni A_1 va C_1 ning ishoralari har xil bo‘ladi. Shuning uchun bu holda

$$\frac{A_1M}{\Delta} = \pm\frac{1}{a^2}, \qquad \frac{C_1M}{\Delta} = \pm\frac{1}{b^2}$$

faraz qilish mumkin. Bu holda (5.25) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{\pm a^2} + \frac{y^2}{\pm b^2} = 1 \ \ yoki \ \frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1.$$

Bu esa yarim o‘qlari a va b dan iborat bo‘lgan giperbolani ifoda qiladi.

5.3. Markazsiz egri chiziqning tenglamasini soddalashtirish.

Egri chiziqning markazi cheksiz uzoqda bo‘lgan holda

$$M = B^2 - AC = 0 \text{ yoki } AC = B^2 \qquad (5.30)$$

bo‘ladi. Ikkinchi tartibli egri chiziqning umumiy tenglamasini olib uning ikkila tomonini A ga ko‘paytiramiz:

$$A^2x^2 + 2ABxy + ACy^2 + A(2Dx + 2Ey + F) = 0$$

yoki (5.30) ga asosan:

$$(Ax + By)^2 + A(2Dx + 2Ey + F) = 0. \qquad (5.31)$$

Tenglamani soddalashtirish maqsadi bilan koordinata o‘qlarining yo‘nalishlarini o‘zgartiramiz, masalan, uni biror α burchakka aylantiramiz. Bu holda almashtirish formulalari quyidagicha bo‘ladi:

$$\begin{cases} x = x_1 cos\alpha - y_1 sin\alpha \\ y = x_1 sin\alpha + y_1 cos\alpha. \end{cases} \qquad (5.32)$$

Bularni (5.31) formulaga qo‘yilsa:

$$[A(x_1 cos\alpha - y_1 sin\alpha) + B(x_1 sin\alpha + y_1 cos\alpha)]^2 +$$
$$+A[2D(x_1 cos\alpha - y_1 sin\alpha) + 2E(x_1 sin\alpha + y_1 cos\alpha) + F] = 0$$

yoki

$$[(Acos\alpha + Bsin\alpha)x_1 + (Bcos\alpha - Asin\alpha)y_1]^2 +$$
$$+A[2(Dcos\alpha + Esin\alpha)x_1 + 2(Ecos\alpha - Dsin\alpha)y_1 + F] = 0.$$
$$(5.33)$$

Hozirgacha α ixtiyoriy burchak edi. Endi uning qiymatini shunday aniqlaymizki,

$$Acos\alpha + Bsin\alpha = 0$$

yoki

$$tg\alpha = -\frac{A}{B} \qquad (5.34)$$

bo‘lsin. Buni e’tiborga olib,

$$\begin{cases} N = (Bcos\alpha - Asin\alpha)^2 \\ P = A(Dcos\alpha + Esin\alpha) \\ Q = A(Ecos\alpha - Dsin\alpha) \\ \qquad R = AF \end{cases} \qquad (5.35)$$

faraz qilinsa, (5.33) tenglamaning ko‘rinishi bunday bo‘ladi:

$$Ny_1^2 + 2Px_1 + 2Qy_1 + R = 0 \qquad (5.36)$$

(5.34) ga asosan $tg\alpha$ ma’lum bo‘lgani uchun uning yordami bilan hamma vaqt (5.35) dagi $sin\alpha$ va $cos\alpha$ ni aniqlash mumkin. Demak (5.36) ning hamma koeffitsiyentlari ma’lum bo‘ladi.

(5.36) tenglamani yana soddalashtirish maqsadi bilan koordinatalar boshini biror (a, b) nuqtaga ko‘chiramiz. Bu holda almashtirish formulalari

$$x_1 = x + a, \quad y_1 = y + b$$

bo‘ladi va (5.36) ning ko‘rinishi

$$N(y+b)^2 + 2P(x+a) + 2Q(y+b) + R = 0$$

yoki

$$Ny^2 + 2(Nb+Q)y + 2Px + (Nb^2 + 2Pa + 2Qb + R) = 0 \quad (5.37)$$

bo‘ladi. Nuqtaning a va b koordinatalariga shunday qiymat tayin qilamizki,

$$Nb + Q = 0, \quad Nb^2 + 2Pa + 2Qb + R = 0$$

bo‘lsin. Bu esa

$$b = -\frac{Q}{N} \;\; va \;\; \frac{Q^2}{N} + 2Pa - \frac{2Q^2}{N} + Q = 0$$

yoki

$$2PNa - Q^2 + NR = 0 \;\; yoki \;\; a = \frac{Q^2 - NR}{2PNB}$$

bo‘lgan holda mumkin. Bu chog‘da (5.32) ning ko‘rinishi bunday bo‘ladi:

$$Ny^2 + 2Px = 0 \;\; yoki \;\; y^2 = -2\frac{P}{N}x, \quad (5.38)$$

yoki

$$-\frac{P}{N} = p$$

faraz qilinsa,

$$y^2 = 2px \quad (5.39)$$

bo‘ladi va bu ***parabolani*** ifoda qiladi.

p ning qiymatini aniqlash uchun (5.34) dan $sin\alpha$ va $cos\alpha$ ning qiymatlarini aniqlashga to‘g‘ri keladi. Buning uchun (5.34) ni

$$\frac{sin\alpha}{cos\alpha} = -\frac{A}{B} \quad yoki \quad \frac{sin\alpha}{-A} = \frac{cos\alpha}{B}$$

kabi yozib, undan ushbu hosila proportsiyani tuzamiz:

$$\frac{sin\alpha}{-A} = \frac{cos\alpha}{B} = \frac{\sqrt{sin^2\alpha + cos^2\alpha}}{\sqrt{A^2 + B^2}} = \frac{1}{\sqrt{A^2 + B^2}}.$$

Demak

$$sin\alpha = \frac{-A}{\sqrt{A^2 + B^2}}, \quad cos\alpha = \frac{B}{\sqrt{A^2 + B^2}}. \quad (5.40)$$

(5.36) ga muvofiq

$$P = \frac{ABD - A^2E}{\sqrt{A^2+B^2}}, N = \left(\frac{A^2+B^2}{\sqrt{A^2+B^2}}\right)^2 = A^2+B^2.$$

Demak

$$p = -\frac{P}{N} = \frac{A(AE-BD)}{(A^2+B^2)\sqrt{A^2+B^2}} = \sqrt{\frac{A^2(AE-BD)^2}{(A^2+B^2)^3}} =$$

$$= \sqrt{\frac{A^2(A^2E^2 - 2ABDE + B^2D^2)}{(A^2+B^2)^3}} =$$

$$= \sqrt{\frac{A^2(A^2E^2 - 2ABDE + ACD^2)}{(A^2+AC)^3}} =$$

$$= \sqrt{-\frac{(-AE^2+2BDE-CD^2)}{(A+C)^3}} = \sqrt{-\frac{\Delta}{(A+C)^3}};$$

chunki $B^2 - AC = 0$ bo‘lganda $\Delta = -AE^2 + 2BDE - CD^2$ bo‘ladi. Natijada

$$p = \sqrt{-\frac{\Delta}{(A+C)^3}}. \quad (5.41)$$

$B^2 = AC$ bo‘lgani uchun A va C ning ishoralari bir xil bo‘ladi. A ning ishorasini hamma vaqt musbat qilish mumkin. Shuning uchun $(A+C)$ ni musbat faraz qilib bo‘ladi. Ikkinchi tomondan

$$\Delta = -AE^2 + 2BDE - CD^2 = -(AE^2 + CD^2 - 2BDE) =$$

$$= -\left(AE^2 + CD^2 - 2DE\sqrt{AC}\right) = -\left(E\sqrt{A} - D\sqrt{C}\right)^2 < 0.$$

Shuning uchun parabolaning diskriminanti hamma vaqt manfiy bo‘ladi va radikal ostida musbat son bo‘ladi. Demak, $p -$ hamma vaqt mavjud va musbat sondan iborat.

Shuning bilan, tekshirishimizning natijasini ushbu jadval bilan tasvirlash mumkin:

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0,$$

$$\Delta = \begin{vmatrix} A & B & D \\ B & C & E \\ D & E & F \end{vmatrix},$$

$$M = B^2 - AC.$$

	$M < 0$		$M > 0$	$M = 0$
$\Delta \neq 0$	$A \cdot \Delta < 0$	Ellips	Giperbola	Parabola
	$A \cdot \Delta > 0$	Mavhum ellips		
$\Delta = 0$		Ikkita bir – birini kesuvchi mavhum to‘g‘ri chiziq	Ikkita bir – birini kesuvchi haqiqiy to‘g‘ri chiziq	Ikkita parallel to‘g‘ri chiziq

5.4. Umumiy tenglama bilan berilgan ikkinchi tartibli chiziqlarni aniqlash va sinflarga ajratish.

$$\begin{cases} x' = x + a \\ y' = y + a \end{cases}$$

parallel ko‘chirish formulasi.

$$\begin{cases} x = x'cos\alpha - y'sin\alpha \\ y = x'sin\alpha + y'cos\alpha \end{cases}$$

burish formulasi.

$$\begin{cases} x = x'cos\alpha - y'sin\alpha + a \\ y = y'cos\alpha + x'sin\alpha + b \end{cases}$$

parallel ko‘chirish va burish birgalikda harakat deyiladi.

$$a_{11}x^2 + a_{22}y^2 + 2a_{10}x + 2a_{20}y + 2a_{12}xy + a_{00} = 0$$

shu ifoda bilan berilgan tenglama ***ikkinchi tartibli chiziqning umumiy tenglamasi*** deyiladi.

Umumiy tenglama bilan berilgan ikkinchi tartibli chiziqni qanday chiziq ekanligini aniqlash uchun quyidagi ishlarni bajaramiz.

$$\begin{cases} x' = x + a \\ y' = y + a \end{cases}$$

$$a_{11}(x' + a)^2 + a_{22}(y' + b)^2 + 2a_{10}(x' + a) + 2a_{20}(y' + b) +$$
$$+2a_{12}(x' + a)(y' + b) + a_{00} = 0 \Longrightarrow$$

$$a_{11}(x'^2 + 2ax' + a^2) + a_{22}(y'^2 + 2by' + b^2) + (2a_{10}x' + 2aa_{10}) +$$
$$+2a_{20}y' + 2a_{20}b + 2a_{12}(x'y' + ay' + bx' + ab) + a_{00} = 0 \Longrightarrow$$

$$a_{11}x'^2 + 2aa_{11}x' + a^2a_{11} + a_{22}y'^2 + 2y'ba_{22} + a_{22}b^2 +$$
$$+2a_{10}x' + 2aa_{10} + 2a_{20}y' + 2a_{20}b + 2a_{12}x'y' +$$
$$+2a_{12}ay' + 2a_{12}bx' + 2a_{12}ab + a_{00} = 0$$

$$\begin{cases} 2a_{11}a + 2a_{12}b = -2a_{10} \\ 2a_{22}b + 2a_{12}a = -2a_{20} \end{cases}$$

$$a_{11}{x'}^2 + a_{22}{y'}^2 + A_{00} = 0 \Rightarrow \begin{cases} x = x''cos\alpha - y''sin\alpha \\ y = x''sin\alpha + y''cos\alpha \end{cases}$$

kelib chiqadi.

$$a_{11}(x''cos\alpha - y''sin\alpha)^2 + a_{22}(x''sin\alpha + y''cos\alpha)^2 +$$
$$+2a_{12}(x''cos\alpha - y''sin\alpha)(x''sin\alpha + y''cos\alpha) + A_{00} = 0$$
$$a_{11}({x''}^2cos^2\alpha - 2a_{11}x''cos\alpha y''sin\alpha + {y''}^2sin^2\alpha) +$$
$$+a_{22}({x''}^2sin^2\alpha + 2x''sin\alpha y''cos\alpha + {y''}^2cos^2\alpha) + A_{00} = 0 \Rightarrow$$
$$a_{11}{x''}^2cos^2\alpha - 2a_{11}x''cos\alpha y''sin\alpha + a_{11}{y''}^2sin^2\alpha +$$
$$+a_{22}{y''}^2cos^2\alpha + 2a_{13}{x''}^2sin\alpha cos\alpha + 2a_{12}{x''}^2{y''}^2cos^2\alpha -$$
$$-2a_{12}x''y''sin^2\alpha - 2a_{12}{y''}^2sin\alpha cos\alpha + A_{00} = 0 \Rightarrow$$
$$(a_{11}cos^2\alpha + a_{22}sin^2\alpha + 2a_{12}sin\alpha cos\alpha){x''}^2 +$$
$$+(a_{11}sin^2\alpha + a_{22}cos^2\alpha - 2a_{12}sin\alpha cos\alpha){y''}^2 +$$
$$+(-2a_{11}sin\alpha cos\alpha + 2a_{22}sin\alpha cos\alpha + 2a_{12}cos^2\alpha$$
$$- 2a_{12}sin^2\alpha)x''y'' + A_{00} = 0$$
$$(a_{22} - a_{11})sin2\alpha + 2a_{12}cos2\alpha = 0 \Rightarrow$$
$$tg2\alpha = \frac{2a_{12}}{a_{11} - a_{22}} \qquad (5.42)$$

$A_{11}{x'}^2 + A_{22}{y'}^2 + A_{00} = 0$ tenglamani xususiy hollarini qaraymiz:

1) $A_{11} > 0,\ A_{22} > 0, A_{00} < 0$ bo‘lsa, ellips;
2) $A_{11} < 0,\ A_{22} < 0, A_{00} > 0$ bo‘lsa, ellips;
3) $A_{11}\ A_{22} < 0, A_{00} > 0$ bo‘lsa, giperbola;
4) $A_{11}\ A_{22} < 0, A_{00} < 0$ bo‘lsa, giperbola;
5) $A_{11}\ A_{22} < 0, A_{00} = 0$ bo‘lsa, nuqta;
6) $A_{11} = 0,\ \ A_{22}\, A_{00} < 0,\ A_{00} > 0$ bo‘lsa, parallel to‘g‘ri chiziqlar hosil bo‘ladi.

1-Misol. Quyidagi tenglamaning geometrik ma’nosini tekshirib, uning kanonik tenglamasi tuzilsin:

$$5x^2 + 4xy + 8y^2 - 32x - 56y + 80 = 0.$$

Yechish: Egri chiziqning jinsini aniqlash uchun Δ va M ni hisoblashga to‘g‘ri keladi:

$$\Delta = \begin{vmatrix} 5 & 2 & -16 \\ 2 & 8 & -28 \\ -16 & -28 & 80 \end{vmatrix} = -1296, \qquad A \cdot \Delta < 0.$$

$$M = B^2 - AC = 4 - 5 \cdot 8 = -36 < 0.$$

Demak, berilgan tenglama haqiqiy ellipsdan iborat. Uning kanonik tenglamasining ko‘rinishi:

$$A_1 x^2 + C_1 y^2 = \frac{\Delta}{M},$$

$$A_1 = \frac{1}{2}\left[A + C + \sqrt{(A-C)^2 + 4B^2}\right] =$$

$$= \frac{1}{2}\left[5 + 8 + \sqrt{(5-8)^2 + 4 \cdot 2^2}\right] = 9;$$

$$C_1 = \frac{1}{2}\left[A + C - \sqrt{(A-C)^2 + 4B^2}\right] =$$

$$= \frac{1}{2}\left[5 + 8 - \sqrt{(5-8)^2 + 4 \cdot 2^2}\right] = 4;$$

$$\frac{\Delta}{M} = \frac{-1296}{-36} = 36.$$

Demak, ellipsning kanonik tenglamasi

$$9x^2 + 4y^2 = 36$$

yoki

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

2-Misol. Ox o‘qi parabolaning simmetriya o‘qi bo‘lib, uning uchi koordinatalar boshida yotadi. Parabola uchidan fokusigacha bo‘lgan masofa 4 birlikka teng. Parabola va uning direktrisasi tenglamasini toping.

Yechish: Dastlab, masala shartiga asosan, parabolaning p parametrini topamiz:

$$|OF| = 4 \implies p/2 = 4 \implies p = 8.$$

Unda, (5.39) formulaga asosan, parabola tenglamasini topamiz:

$$y^2 = 2px \implies y^2 = 2 \cdot 8x = 16x.$$

Bu yerdan direktrisa tenglamasi $x = -p/2 \implies x = -4$ ekanligini ko‘ramiz.

Shuni ta’kidlab o‘tish kerakki, $y = ax^2 + bx + c$ $(a \neq 0)$ kvadrat uchhadning grafigi uchi koordinatalari

$$x_0 = -\frac{b}{2a}, \; y_0 = \frac{4ac - b^2}{4a}$$

bo‘lgan $M_0(x_0; y_0)$ nuqtada, simmetriya o‘qi esa Oy o‘qiga parallel va $x = -b/2a$ tenglamaga ega bo‘lgan vertikal to‘g‘ri chiziqdan tashkil topgan paraboladan iboratdir. Agar $a > 0$ bo‘lsa, parabola yuqoriga, $a < 0$ bo‘lsa, pastga yo‘nalgan bo‘ladi.

Mustaqil yechish uchun topshiriqlar.

5.1.1. Quyidagi giperbolalarning tenglamalari sodda shaklga keltirilsin:
1) $9x^2 - 25y^2 - 18x - 100y - 316 = 0$;
2) $5x^2 - 6y^2 + 10x - 12y - 31 = 0$;
3) $x^2 - 4y^2 + 6x + 5 = 0$.
Markazlarining koordinatalari va o‘qlari topilsin.

5.1.2. Quyidagi tenglamalar bilan qanday egri chiziqlar berilganligi tekshirilsin:
1) $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$;
2) $x^2 - 2xy - 2y^2 - 4x - 6y + 3 = 0$;
3) $x^2 - 2xy + y^2 - 4x - 6y + 3 = 0$;
4) $x^2 - 2xy + 2y^2 - 4x - 6y + 29 = 0$;
5) $x^2 - 2xy - 2y^2 - 4x - 6y - \frac{13}{3} = 0$.

5.1.3. Quyidagi egri chiziqlarning turlari aniqlansin:
1) $x^2 + 6xy + y^2 + 6x + 2y - 1 = 0$;
2) $3x^2 - 2xy + 3y^2 + 4x + 4y - 4 = 0$;
3) $x^2 - 4xy + 3y^2 + 2x - 2y = 0$;
4) $y^2 + 5xy - 14x^2 = 0$;
5) $x^2 - xy - y^2 - x - y = 0$.

5.1.4. Berilgan tenglamalarning chap tomonlarini ko‘paytuvchilarga ajratishdan foydalanib, tenglamalarning geometrik ma’nosi ko‘rsatilsin:
1) $xy - bx - ay + ab = 0$;
2) $x^2 - 2xy + 5x = 0$;
3) $x^2 - 4xy + 4y^2 = 0$;
4) $9x^2 + 30xy + 25y^2 = 0$;
5) $4x^2 - 12xy + 9y^2 - 25 = 0$.

5.1.5. $y^2 - xy - 5x + 7y + 10 = 0$ tenglamaning ikki (qo‘sh) to‘g‘ri chiziqni ifodalashi tekshirilsin va bu to‘g‘ri chiziqlardan har birining tenglamasi topilsin.

5.1.6. Quyidagi tenglama bilan berilgan ikkita to‘g‘ri chiziqdan har birining tenglamasi topilsin:

1) $21x^2 + xy - 10y^2 = 0$;
2) $x^2 + 2xy + y^2 + 2x + 2y - 4 = 0$;
3) $y^2 - 4xy - 5y^2 + 5x - y = 0$;
4) $4x^2 - 4xy + y^2 + 12x - 6y + 9 = 0$.

5.1.7. Egri chiziqlar tekshirilsin:
1) $2x^2 + 3xy - 5y^2 = 0$;
2) $x^2 + 4xy + 4y^2 = 0$;
3) $10x^2 - 7xy + y^2 = 0$;
4) $5x^2 - 4xy + y^2 = 0$.

5.1.8. Invariantlardan foydalanib, quyidagi egri chiziq tenglamalari sodda shaklga keltirilsin:
1) $x^2 + 2xy - y^2 + 8x + 4y - 8 = 0$;
2) $7x^2 - 24xy - 38x + 24y + 175 = 0$;
3) $5x^2 + 8xy + 5y^2 - 18x - 18y + 9 = 0$;
4) $5x^2 + 12xy - 22x - 12y - 19 = 0$;
5) $6xy + 8y^2 - 12x - 26y + 11 = 0$.
Bu egri chiziqlarning hammasi to‘g‘ri burchakli koordinatalar sistemasiga nisbatan berilgan.

5.1.9. Invariantlardan foydalanib, quyidagi parabolalarning tenglamalari soddalashtirilsin:
1) $x^2 - 2xy + y^2 - 10x - 6y + 25 = 0$;
2) $4x^2 - 4xy + y^2 - 2x - 14y + 7 = 0$;
3) $x^2 - 2xy + y^2 - x - 2y + 3 = 0$;
4) $4x^2 - 4xy + y^2 - x - 2 = 0$.
$\omega = \frac{\pi}{2}$ bo‘lgan hol uchun.

5.1.10. Quyidagi egri chiziqlarning tenglamalari soddalashtirilsin:
1) $x^2 - 3xy + y^2 + 1 = 0, \quad \omega = 60^0$;
2) $2x^2 + 2y^2 - 2x - 6y + 1 = 0, \quad \omega = 60^0$;
3) $4x^2 - 4xy + y^2 - 4x - 4y + 7 = 0, \quad \omega = 120^0$.

5.1.11. Invariantlardan foydalanib, $8y^2 + 6xy - 12x - 26y + 11 = 0$ giperbolaning asimptotalariga nisbatan tenglamasi yozilsin. $\omega = 90^0$.

5.1.12. To‘g‘ri burchakli koordinatalar sistemasiga nisbatan quyidagi tenglamalar bilan berilgan giperbolalarning asimptotalariga nisbatan yozilgan tenglamasi topilsin:
1) $2x^2 + 3xy - 2y^2 - 8x - 11y = 0$;
2) $4x^2 + 2xy - y^2 + 6x + 2y + 3 = 0$;

3) $y^2 - 2xy - 4x - 2y - 2 = 0$.

5.1.13. Biror to‘g‘ri burchakli koordinatalar sistemasiga nisbatan egri chiziq $5x^2 + 2xy - 22x - 12y - 19 = 0$ tenglama bilan ifodalanadi. Bu egri chiziqning o‘z uchiga nisbatan tenglamasi topilsin.

5.1.14. Quyidagi tenglamalarning har biri ellipsni ifodalasa, uning markazi bo‘lgan C nuqtaning koordinatasi, yarim o‘qi, ekssentrisiteti va direktrisasi tenglamalarini tuzing:

1) $5x^2 + 9y^2 - 30x + 18y + 9 = 0$;
2) $16x^2 + 25y^2 + 32x - 100y - 284 = 0$;
3) $4x^2 + 3y^2 - 8x + 12y - 32 = 0$.

5.1.15. Quyidagi tenglamalar giperbola hosil qilishini tekshirib va uning markazi bo‘lgan C nuqtaning koordinatasini, yarim o‘qlarini, ekssentrisitetini, asimptota va direktrisa tenglamalarini tuzing:

1) $16x^2 - 9y^2 - 64x - 54y - 161 = 0$;
2) $9x^2 - 16y^2 + 90x + 32y - 367 = 0$;
3) $16x^2 - 9y^2 - 64x - 18y + 199 = 0$.

5.1.16. Quyidagi chiziqlardan qaysilari: 1) yagona markazga; 2) ko‘p markazlarga; 3) markazga ega emasligini aniqlang.

1) $3x^2 - 4xy - 2y^2 + 3x - 12y - 7 = 0$;
2) $4x^2 + 5xy + 3y^2 - x + 9y - 12 = 0$;
3) $4x^2 - 4xy + y^2 - 6x + 8y + 13 = 0$;
4) $4x^2 - 4xy + y^2 - 12x + 6y - 11 = 0$;
5) $x^2 - 2xy + 4y^2 + 5x - 7y + 12 = 0$;
6) $x^2 - 2xy + y^2 - 6x + 6y - 3 = 0$;
7) $x^2 - 20xy + 25y^2 - 14x + 2y - 15 = 0$;
8) $4x^2 - 6xy - 9y^2 + 3x - 7y + 12 = 0$.

5.1.17. Quyidagi berilgan chiziqlar markazga ega bo‘lsa, ularning markaziy nuqtalarini toping:

1) $3x^2 + 5xy + y^2 - 8x - 11y - 7 = 0$;
2) $5x^2 + 4xy + 2y^2 + 20x + 20y - 18 = 0$;
3) $9x^2 - 4xy - 7y^2 - 12 = 0$;
4) $2x^2 - 6xy + 5y^2 + 22x - 36y + 11 = 0$.

5.1.18. Quyidagi har bir chiziqning ko‘p markazli bo‘lishini tekshirib, ularning har biri uchun geometrik markazini aniqlaydigan tenglamasini tuzing:

1) $x^2 - 6xy + 9y^2 - 12x + 36y + 20 = 0$;
2) $4x^2 + 4xy + y^2 - 8x - 4y - 21 = 0$;

3) $25x^2 - 10xy + y^2 + 40x - 8y + 7 = 0$.

5.1.19. Quyidagi tenglamalar markaziy chiziqni ifodalashini tekshirib, ularning har birini koordinata boshiga ko‘chiruvchi tenglamasini tuzing:

1) $3x^2 - 6xy + 2y^2 - 4x + 2y + 1 = 0$;

2) $6x^2 + 4xy + y^2 + 4x - 2y + 2 = 0$;

3) $4x^2 + 6xy + y^2 - 10x - 10 = 0$;

4) $4x^2 + 2xy + 6y^2 + 6x - 10y + 9 = 0$.

5.1.20. *m* va *n* ning qanday qiymatlarida

$$mx^2 + 12xy + 9y^2 + 4x + ny - 13 = 0$$

tenglama quyidagilarni aniqlaydi:

а) markaziy chiziqni;

б) markazsiz chiziqni;

в) ko‘p markazli chiziqlarni.

5.1.21. Parallel ko‘chirish yo‘li bilan quyidagi tenglamalarning har birining turini aniqlab, sodda holga keltiring. Qanday geometrik shaklni ifodalashini toping. Eski va yangi koordinatalar sistemasida grafigini chizing.

1) $4x^2 + 9y^2 - 40x + 36y + 100 = 0$;

2) $9x^2 - 16y^2 - 54x - 64y - 127 = 0$;

3) $9x^2 + 4y^2 + 18x - 8y + 49 = 0$;

5.1.22. Quyidagi berilgan tenglamalarning har birini eng oddiy shaklga keltirib, ularning turini, qanday geometrik shaklni tasvirlashini, grafiklarning eski va yangi koordinata o‘qlariga nisbatan joylashishini aniqlang:

1) $32x^2 + 52xy - 7y^2 + 180 = 0$;

2) $5x^2 - 6xy + 5y^2 - 32 = 0$;

3) $17x^2 - 12xy + 8y^2 = 0$;

4) $5x^2 + 24xy - 5y^2 = 0$;

5) $5x^2 - 6xy + 5y^2 + 8 = 0$.

5.1.23. Quyidagi berilgan tenglamalarning har birini eng oddiy shaklga keltirib, ularning turini, qanday geometrik shaklni tasvirlashini, grafiklarning eski va yangi koordinata o‘qlariga nisbatan joylashishini aniqlang:

1) $14x^2 + 24xy + 21y^2 - 4x + 18y - 139 = 0$;

2) $11x^2 - 20xy - 4y^2 - 20x - 8y + 1 = 0$;

3) $7x^2 + 60xy + 32y^2 - 14x - 60y + 7 = 0$;

4) $50x^2 - 8xy + 35y^2 + 100x - 8y + 67 = 0$;

5.1.24. Quyidagi tenglamalarni koordinatalar sistemasini almashtirmasdan, har biri ellipsni ifodalashini va uning yarim o‘qlardagi qiymatlarini toping:

1) $41x^2 + 24xy + 9y^2 + 24x + 18y - 36 = 0$;
2) $8x^2 + 4xy + 5y^2 + 16x + 4y - 28 + 9 = 0$;
3) $13x^2 + 18xy + 37y^2 - 26x - 18y + 3 = 0$;
4) $13x^2 + 10xy + 13y^2 + 46x + 62y + 13 = 0$.

5.1.25. Koordinatalar sistemasini almashtirmasdan quyidagi tenglamalar bitta nuqtani ifodalashini isbotlang.

1) $5x^2 - 6xy + 2y^2 - 2x + 2 = 0$;
2) $x^2 + 2xy + 2y^2 + 6y + 9 = 0$;
3) $5x^2 + 4xy + y^2 - 6x - 2y + 2 = 0$;
4) $x^2 - 6xy + 10y^2 + 10x - 32y + 26 = 0$.

5.1.26. Koordinatalar sistemasini almashtirmasdan, har biri giperbolani ifodalasa, uning yarim o‘qlarining qiymatini toping:

1) $4x^2 + 24xy + 11y^2 + 64x + 42y + 51 = 0$;
2) $12x^2 + 26xy + 12y^2 - 52x - 48y + 73 = 0$;
3) $3x^2 + 4xy - 12x + 16 = 0$;
4) $x^2 - 6xy - 7y^2 + 10x - 30y + 23 = 0$.

5.1.27. Koordinatalar sistemasini almashtirmasdan, quyidagi tenglamalarning har biri kesishgan to‘g‘ri chiziqlar juftligini (degenerat giperbola) belgilashini aniqlang va ularning tenglamalarini toping:

1) $3x^2 + 4xy + y^2 - 2x - 1 = 0$;
2) $3x^2 - 6xy + 8y^2 - 4y - 4 = 0$;
3) $x^2 - 4xy + 3y^2 = 0$;
4) $x^2 + 4xy + 3y^2 - 6x - 12y + 9 = 0$.

5.1.28. Quyidagi tenglamalarning har biri parabolik ekanligini aniqlang; ularning har birini eng oddiy shaklga keltiring; ular qanday geometrik tasvirlarni belgilashlarini aniqlang:

1) $9x^2 + 24xy + 16y^2 - 18x + 226y + 209 = 0$;
2) $x^2 - 2xy + y^2 - 12x + 12y - 14 = 0$;
3) $4x^2 + 12xy + 9y^2 - 4x - 6y + 1 = 0$.

5.1.29. Koordinatalar sistemasini almashtirmasdan, quyidagi tenglamalarning har biri parabolani aniqlaganligini aniqlang va ushbu parabola parametrini toping:

1) $9x^2 + 24xy + 16y^2 - 120x + 90y = 0$;

2) $9x^2 - 24xy + 16y^2 - 54x - 178y + 181 = 0$;
3) $x^2 - 2xy + y^2 + 6x - 14y + 29 = 0$;
4) $9x^2 - 6xy + y^2 - 50x + 50y - 275 = 0$.

5.1.30. Koordinatalar sistemasini almashtirmasdan, quyidagi tenglamalarning har biri bir juft parallel to‘g‘ri chiziqni belgilashini aniqlang va ularning tenglamalarini toping:
1) $4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0$;
2) $4x^2 - 12xy + 9y^2 + 20x - 30y - 11 = 0$;
3) $25x^2 - 10xy + y^2 + 10x - 2y - 15 = 0$.

6-MAVZU: IKKINCHI TARTIBLI SIRTLAR

Reja:
1. Ellipsoid va sfera.
2. Fazoda ba’zi sirtlarning tenglamalari va nomlanishi.

Tayanch iboralar: transtendent sirt, ellipsoid, siqma ellipsoid, cho‘ziq ellipsoid, diametral tekislik, bir pallali giperboloid , ikki pallali giperboloid , sfera, konus, elliptik silindr, giperbolik silindr, parabolik silindr, elliptik paraboloid, giperbolik paraboloid .

6.1. Ellipsoid va sfera.

Sirtlar, ularning Dekart koordinatalariga nisbatan ifoda qilingan tenglamalarga qarab, tekislikdagi chiziqlar kabi, algebraik va transtendent sirtlarga bo‘linadi. Shuning uchun algebraik sirt deb, shunday sirtga aytiladiki, agarda uni

$$f(x; y; z) = 0$$

ko‘rinishidagi tenglama bilan ifodalash mumkin bo‘lsa va $f(x; y; z)$ esa x, y, z ga nisbatan polinom(ko‘p hadli) bo‘lsa, algebraik bo‘lmagan hamma sirtlarni ***transtendent sirtlar*** deyiladi.

Algebraik sirtlar, o‘z navbatida, turli tartibli sirtlarga bo‘linadi. Agarda $f(x; y; z)$ polinomning darajasi n bo‘lsa, unday sirtlarni $\boldsymbol{n} -$ ***tartibli sirt*** deyiladi.

Dekart o‘zgaruvchi x, y, z koordinatalariga nisbatan ikkinchi darajali algebraik tenglama bilan ifoda qilingan sirt ***ikkinchi tartibli sirt deyiladi.*** Shuning uchun ikkinchi tartibli sirt ifoda qiladigan ikkinchi darajali algebraik tenglamaning umumiy ko‘rinishi quyidagi ko‘rinishda bo‘ladi:

$$A_1x^2+A_2y^2+A_3z^2+B_1xy+B_2xz+B_3yz+C_1x+C_2y+$$
$$+C_3z+F=0,$$

bunda $A_1, A_2, A_3, B_1, B_2, \dots, F$ koeffitsiyentlar haqiqiy o‘zgarmas sonlardan iborat bo‘lib, xususiy holda ulardan ba’zilari nolga teng bo‘lishi mumkin. Bu tenglamaning umumiyligiga xalal bermay uni quyidagi ko‘rinishda yozish mumkin:

$$A_1x^2+A_2y^2+A_3z^2+2B_1yz+2B_2xz+2B_3xy+$$
$$+2C_1x+2C_2y+2C_3z+F=0. \quad (6.1)$$

Tenglamani ushbu ko‘rinishda yozsak, uning bilan bog‘langan amallarni bajarish birmuncha qulay bo‘ladi.

Koordinatalar sistemasini alamashtirish yordamida (6.1) tenglamani soddalashtirib, uni

$$A_1x^2+A_2y^2+A_3z^2+F=0 \quad (6.2)$$

yoki

$$A_1x^2+A_2y^2+2C_3z=0 \quad (6.3)$$

shaklga keltirish mumkin.

(6.2) tenglama bilan ifoda qilingan sirt ikkinchi tartibli ***markazli sirt*** deyiladi va (6.3) tenglama bilan ifoda qilingan sirt ikkinchi tartibli ***markazsiz***(yoki markazi cheksizlikdagi) ***sirt*** deyiladi.

Faraz qilaylik, ikkinchi tartibli markazli sirtning eng sodda tenglamasi berilgan bo‘lsin:

$$A_1x^2+A_2y^2+A_3z^2+F=0 \quad (6.4)$$

va bundagi ozod had bo‘lgan F ning ishorasi qolgan koeffitsiyentlarining ishorasiga teskari bo‘lsin. Tenglamaning F koeffitsiyentini o‘ng tomonga o‘tkazib, so‘ngra uning ikkala tomonini $(-F)$ ga bo‘lamiz:

$$A_1x^2+A_2y^2+A_3z^2=-F,$$
$$\frac{A_1x^2}{-F}+\frac{A_2y^2}{-F}+\frac{A_3z^2}{-F}=1,$$

yoki

$$\frac{x^2}{-\frac{F}{A_1}}+\frac{y^2}{-\frac{F}{A_2}}+\frac{z^2}{-\frac{F}{A_3}}=-1. \quad (6.5)$$

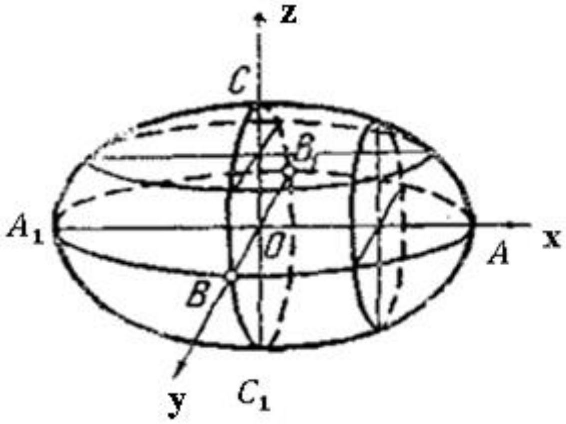
(6.4) tenglamaning koeffitsiyentlari to‘g‘risida qilingan farazga muvofiq F ning ishorasi qolgan koeffitsiyentlarning ishorasiga teskari bo‘lgani uchun, (6.5) tenglamaning chap tomonidagi har bir kasrning maxraji musbat bo‘ladi. Shuning uchun ulardan birinchisini a^2, ikkinchisini b^2 va uchinchisini c^2 deb faqaz qilamiz:

$$-\frac{F}{A_1} = a^2,\ -\frac{F}{A_2} = b^2,\ -\frac{F}{A_3} = c^2, \qquad (6.6)$$

demak, (6.5) tenglamaning ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \qquad (6.7)$$

Bu tenglama bilan ifoda qilingan sirt ***ellipsoid*** deyiladi.



6.1.1-chizma

Tenglamaning tuzilishiga qaraganda uning chap tomonidagi har bir kasrning qiymati birdan katta bo‘la olmaydi, ya’ni

$$\frac{x^2}{a^2} \leq 1, \qquad \frac{y^2}{b^2} \leq 1, \qquad \frac{x^2}{c^2} \leq 1,$$

yoki

$$x^2 \leq a^2, \qquad y^2 \leq b^2, \quad z^2 \leq c^2,$$

demak,

$$|x| \leq a, \quad |y| \leq b, \quad |z| \leq c.$$

Endi ellipsoidning shaklini tekshiramiz. Buning uchun eng avval uning koordinata o‘qlari bilan uchrashgan nuqtalarini topamiz. Agar (6.7) tenglamada $y = 0$, $z = 0$ faraz qilinsa, $x = \pm a$ bo‘ladi, ya’ni absissa o‘qi ellipsoidni koordinatalar boshiga nisbatan simmetrik bo‘lgan $A(a; 0; 0)$ va $A_1(-a; 0; 0)$ nuqtalarda kesib o‘tadi. Shunga o‘xshash $x = 0$, $z = 0$ faraz qilinsa, $y = \pm b$ bo‘ladi, ya’ni ordinata o‘qi ellipsoidni koordinatalar boshiga nisbatan simmetrik bo‘lgan $B(0; b; 0)$ va $B_1(0; -b; 0)$ nuqtalarda kesib o‘tadi; $x = 0$, $y = 0$ faraz qilinsa, $z = \pm c$ bo‘ladi, ya’ni applikata o‘qi ellipsoidni koordinatalar boshiga nisbatan simmetrik bo‘lgan $C(0;0;c)$ va $C_1(0;0;-c)$ nuqtalarda kesib o‘tadi.

Aniqlangan nuqtalardan $A-$ ellipsoidning yOz tekislikdan eng uzoqlashgan nuqtasi bo‘ladi; shunga o‘xshash qolgan nuqtalar ham tegishli koordinata tekisliklaridan eng uzoqlashgan nuqtalardan iborat. Shuning uchun ularni ellipsoidning boshlari deyiladi va har ikki nuqtalarning orasidagi $2a$, $2b$, $2c$ masofalar ellipsoidning ***o‘qlari*** deyiladi. Ellipsoidning o‘qlari to‘g‘risida qo‘shimcha shart bo‘lmagan holda $a > b > c$ faraz qilinadi. Tekshirishdan chiqarilgan natijalarga qaraganda ellipsoid yopiq sirtdan iborat, chunki u

$$x = \pm a, \quad y = \pm b, \quad z = \pm c$$

tekisliklardan yasalgan parallelepipedning ichida bo‘ladi.

Endi ellipsoidning koordinata tekisliklari bilan kesilishidan hosil bo‘lgan shakllarni tekshiramiz. Masalan, xOy tekisligi bilan kesish uchun $z = 0$ faraz qilishga to‘g‘ri keladi va bu holda (6.7) ning ko‘rinishi ushbu ko‘rinishida bo‘ladi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \tag{6.8}$$

Shunga o‘xshash $y = 0$ faraz qilinsa,

$$\frac{x^2}{a^2}+\frac{y^2}{c^2}=1 \tag{6.9}$$

va $x = 0$ faraz qilinsa,

$$\frac{x^2}{b^2}+\frac{y^2}{c^2}=1. \tag{6.10}$$

(6.8), (6.9), (6.10) tenglamalardan har biri ellipsni ifoda qiladi. Demak, ellipsoidning koordinata tekisliklari bilan kesimlari ellipslardan iborat. Bular ellipsoidning ***bosh kesimlari*** deyiladi.

Endi ellipsoidni koordinata tekisliklariga parallel bo‘lgan tekisliklar bilan kesib ko‘ramiz. Masalan, xOy tekislikka parallel bo‘lgan tekislikning tenglamasini birgalikda yechishga to‘g‘ri keladi.

$z = k$ ni (6.7) ga qo‘yilsa:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{k^2}{c^2}=1,$$

yoki

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1-\frac{k^2}{c^2},$$

yoki

$$\frac{x^2}{\frac{a^2(c^2-k^2)}{c^2}}+\frac{y^2}{\frac{b^2(c^2-k^2)}{c^2}}=1,$$

yoki

$$\frac{a^2(c^2-k^2)}{c^2}=a_1^2, \qquad \frac{b^2(c^2-k^2)}{c^2}=b_1^2 \qquad (6.11)$$

faraz qilinsa, tenglamani ko‘rinishi quyidagicha bo‘ladi:

$$\frac{x^2}{a_1^2}+\frac{y^2}{b_1^2}=1. \qquad (6.12)$$

Bu tenglama ellipsni ifoda qiladi. Biroq, bu ellipsning haqiqiy bo‘lishi uchun $|k|\le c$ bo‘lishi lozim, chunki (6.11) dagi tengliklarga qaraganda $|k|>c$ bo‘lgan holda a_1 va b_1 mavhum bo‘ladi. Shunga o‘xshash ellipsoidni yOz va xOz tekisliklarga parallel bo‘lgan tekislik bilan kesgan holda ham hamon shu kabi natija kelib chiqadi, ya’ni ellips hosil bo‘ladi.

Ellipsoidning o‘qlaridan ikkitasi o‘zaro teng bo‘lganda, unday ellipsoid ***aylanma ellipsoid*** deyiladi. Masalan, ellipsoidning (6.7) tenglamasida $a=b>c$ faraz qilinsa, u tenglamaning ko‘rinishi

$$\frac{x^2+y^2}{a^2}+\frac{z^2}{c^2}=1 \qquad (6.13)$$

bo‘ladi va bu ellipsoid ***siqma ellipsoid*** deyiladi, chunki

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$$

ellipsning kichik o‘qi atrofida aylanishidan hosil bo‘ladi.

Agar (6.13) da $z=0$ deb faraz qilinsa,

$$x^2+y^2=a^2$$

bo‘ladi, bu esa aylanani ifoda qiladi. Demak, (6.13) aylana ellipsoidning xOy tekisligi bilan kesimi aylanadan iborat. Shunga o‘xshash, xOy tekisligiga parallel bo‘lgan tekislik bilan (6.13) ni kesganda yana aylana hosil bo‘ladi. Agar (6.7) tenglamada $a>b=c$ faraz qilinsa, u tenglamaning ko‘rinishi

$$\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1 \qquad (6.14)$$

bo‘ladi va bu ellipsoid ***cho‘ziq ellipsoid*** deyiladi, chunki u

$$\frac{x^2}{a^2}+\frac{z^2}{b^2}=1$$

ellipsning katta o‘qi atrofida aylanishidan hosil bo‘ladi. Agar (6.14) da $x = 0$ faraz qilinsa, $y^2 + z^2 = b^2$ bo‘ladi, ya’ni cho‘ziq ellipsoidning yOz tekisligiga parallel bo‘lgan tekislik bilan (6.14) ni kesganda, yana aylana hosil bo‘ladi.

Ellipsoidning o‘qlari o‘zaro teng bo‘lgan holda ya’ni $a = b = c$ bo‘lganda (6.7) tenglamaning ko‘rinishi

$$x^2 + y^2 + z^2 = a^2 \qquad (6.15)$$

bo‘ladi. Bu tenglama markazi koordinatalar boshida bo‘lgan radiusi a ga teng bo‘lgan ***sferani*** ifoda etadi.

6.2. Fazoda ba’zi sirtlarning tenglamalari va nomlanishi.

Endi bir nechta sirtlarning tenglamalarini va nomlarini keltirib o‘tamiz:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 - \boldsymbol{bir\ pallali\ giperboloid}$;

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1 - \boldsymbol{ikki\ pallali\ giperboloid}$;

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 - \boldsymbol{konus}$;

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \boldsymbol{elliptik\ silindr}$;

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 - \boldsymbol{giperbolik\ silindr}$;

$y^2 = 2px - \boldsymbol{parabolik\ silindr}$;

$\frac{x^2}{p} + \frac{y^2}{q} = 2z - \boldsymbol{elliptik\ paraboloid}$; $(p > 0, q > 0)$.

$\frac{x^2}{p} - \frac{y^2}{q} = 2z - - \boldsymbol{giperbolik\ paraboloid}$; $(p > 0,\ q > 0)$.

Bu sirtlar ham ellipsoid kabi tahlil qilinadi. Tahlilni o‘quvchiga qoldiramiz.

Mustaqil yechish uchun topshiriqlar.

6.1.1 - 6.1.38 mashqlarda berilgan sirtlarning:

1) koordinata o‘qlari bilan kesishgan nuqtalarini;

2) koordinata tekisliklari bilan kesishgan nuqtalarining geometrik o‘rnini;

3) koordinata tekisliklariga parallel tekisliklar bilan kesishgan nuqtalarining geometrik o‘rnini aniqlang.

6.1.1. $\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{4}=1$ tenglama bilan berilgan sirt.

6.1.2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{b^2}=1$ tenglama bilan berilgan sirt.

6.1.3. $\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{c^2}=1$ tenglama bilan berilgan sirt.

6.1.4. $\frac{x^2}{25}+\frac{y^2}{25}+\frac{z^2}{121}=1$ tenglama bilan berilgan sirt.

6.1.5. $\frac{x^2}{49}+\frac{y^2}{16}+\frac{z^2}{16}=1$ tenglama bilan berilgan sirt.

6.1.6. $x^2+y^2+z^2=25$ tenglama bilan berilgan sirt.

6.1.7. $x^2+y^2+z^2=25$ tenglama bilan berilgan sirt.

6.1.8. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ tenglama bilan berilgan sirt.

6.1.9. $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ tenglama bilan berilgan sirt.

6.1.10. $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ tenglama bilan berilgan sirt.

6.1.11. $\frac{x^2}{25}+\frac{y^2}{16}-\frac{z^2}{9}=1$ tenglama bilan berilgan sirt.

6.1.12. $\frac{x^2}{64}-\frac{y^2}{9}+\frac{z^2}{4}=1$ tenglama bilan berilgan sirt.

6.1.13. $\frac{x^2}{121}-\frac{y^2}{100}-\frac{z^2}{4}=-1$ tenglama bilan berilgan sirt.

6.1.14. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ tenglama bilan berilgan sirt.

6.1.15. $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$ tenglama bilan berilgan sirt.

6.1.16. $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ tenglama bilan berilgan sirt.

6.1.17. $\frac{x^2}{36}+\frac{y^2}{25}-\frac{z^2}{16}=-1$ (ikki pallali giperboloid) tenglama bilan berilgan sirt.

6.1.18. $\frac{x^2}{100}-\frac{y^2}{81}+\frac{z^2}{36}=-1$ tenglama bilan berilgan sirt.

6.1.19. $\frac{x^2}{81}-\frac{y^2}{49}-\frac{z^2}{16}=1$ tenglama bilan berilgan sirt.

6.1.20. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ (konus) tenglama bilan berilgan sirt.

6.1.21. $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$ tenglama bilan berilgan sirt.

6.1.22. $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ tenglama bilan berilgan sirt.

6.1.23. $\frac{x^2}{169}+\frac{y^2}{100}-\frac{z^2}{81}=0$ tenglama bilan berilgan sirt.
6.1.24. $x^2+y^2-z^2=0$ tenglama bilan berilgan sirt.
6.1.25. $\frac{x^2}{144}-\frac{y^2}{64}+\frac{z^2}{49}=0$ tenglama bilan berilgan sirt.
6.1.26. $\frac{x^2}{144}-\frac{y^2}{49}-\frac{z^2}{9}=0$ tenglama bilan berilgan sirt.
6.1.27. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (elliptik silindr) tenglama bilan berilgan sirt.
6.1.28. $\frac{x^2}{25}+\frac{y^2}{4}=1$ tenglama bilan berilgan sirt.
6.1.29. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (giperbolik silindr) tenglama bilan berilgan sirt.
6.1.30. $\frac{x^2}{36}-\frac{y^2}{16}=1$ tenglama bilan berilgan sirt.
6.1.31. $\frac{x^2}{25}-\frac{y^2}{9}=-1$ tenglama bilan berilgan sirt.
6.1.32. $y^2=2px$ (parabolik silindr) tenglama bilan berilgan sirt.
6.1.33. $y^2=8x$ tenglama bilan berilgan sirt.
6.1.34. $y^2=-2x$ tenglama bilan berilgan sirt.
6.1.35. $\frac{x^2}{p}+\frac{y^2}{q}=2z$ (elliptik paraboloid) tenglama bilan berilgan sirt.
6.1.36. $\frac{x^2}{12}+\frac{y^2}{8}=2z$ tenglama bilan berilgan sirt.
6.1.37. $\frac{x^2}{p}-\frac{y^2}{q}=2z$ (giperbolik paraboloid) tenglama bilan berilgan sirt.
6.1.38. $\frac{x^2}{18}-\frac{y^2}{6}=2z$ tenglama bilan berilgan sirt.

6.1.39 - 6.1.40 mashqlarda tenglama bilan berilgan sirtlarni simmetriklikka tekshiring.

6.1.39. 1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{b^2}=1$; 2) $\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{c^2}=1$; 3) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$; 4) $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$; 5) $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$; 6) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$; 7) $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$; 8) $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$; 9) $\frac{x^2}{4}+\frac{y^2}{25}-\frac{z^2}{36}=-1$;

10) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$; 11) $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$; 12) $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$; 13) $x^2+y^2+z^2=a^2$

6.1.40. 1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; 2) $\frac{x^2}{a^2}-\frac{y^2}{a^2}=1$; 3) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$;

4) $y^2 = 2px;$ 5) $\frac{x^2}{p} + \frac{y^2}{q} = 2z$; 6) $\frac{x^2}{p} - \frac{y^2}{q} = 2z$;

7) $\frac{x^2}{81} + \frac{y^2}{49} = 1;$ 8) $\frac{x^2}{25} - \frac{y^2}{16} = 1;$ 9) $\frac{x^2}{49} - \frac{y^2}{4} = -1;$

10) $y^2 = -4x;$ 11) $\frac{x^2}{12} + \frac{y^2}{8} = 6$; 12) $\frac{x^2}{16} - \frac{y^2}{10} = 8$

7-MAVZU: IKKINCHI TARTIBLI SIRTLARNING TO'G'RI CHIZIQLI YASOVCHILARI

Reja:

1. Ikkinchi tartibli sirtlarning to'g'ri chiziqli yasovchilari.

2. Ellipsoid va sferaning urinma tekislik tenglamalari.

Tayanch iboralar: bir pallali giperboloid, ikki pallali giperboloid, sfera, konus, elliptik silindr, giperbolik silindr, parabolik silindr, elliptik paraboloid, giperbolik paraboloid .

7.1. Ikkinchi tartibli sirtlarning to'g'ri chiziqli yasovchilari.

Konus va silindrlar to'g'ri chiziqli yasovchilarni o'z ichiga olgan birdan – bir sirtlardan emas. Ma'lum bo'lishicha, bir pallali giperboloid bilan giperbolik paraboloid ham shu xossaga ega ekan.

Haqiqatdan ham, ushbu

$$\boldsymbol{z = \lambda\left(\frac{x}{a} + \frac{y}{b}\right), \qquad 1 = \frac{1}{\lambda}\left(\frac{x}{a} - \frac{y}{b}\right)} \qquad \mathbf{(7.1)}$$

tenglamalar bilan berilgan har bir g_λ to'g'ri chiziq giperbolik paraboloid:

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2} \qquad (7.2)$$

ni ustida yotadi, chunki (7.1) tenglamalarni qanoatlantiruvchi, (7.2) tenglamani qanoatlantiradi, (7.2) esa (7.1) dan hadma – had ko'paytirish natijasida hosil qilinadi.

Giperbolik paraboloid ustida g_λ oiladan tashqari to'g'ri chiziqlarning yana bitta $g_\lambda{}'$ oilasi joylashadi:

$$z = \lambda\left(\frac{x}{a} + \frac{y}{b}\right), \qquad 1 = \frac{1}{\lambda}\left(\frac{x}{a} - \frac{y}{b}\right).$$

Shuning singari ***bir pallali giperboloid:***

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}-1=0.$$

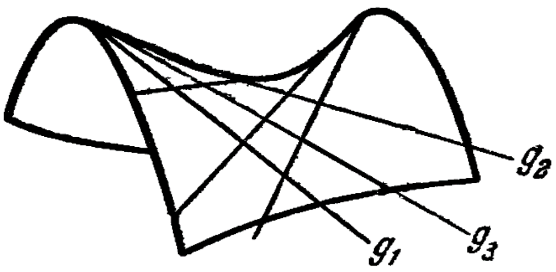
ustida to‘g‘ri chiziqli yasovchilarning ikkita oilasi joylashadi:

$$g_\lambda:\ \frac{x}{a}-\frac{z}{c}=\lambda\left(1-\frac{y}{b}\right), \qquad \frac{x}{a}+\frac{z}{c}=\frac{1}{\lambda}\left(1+\frac{y}{b}\right);$$

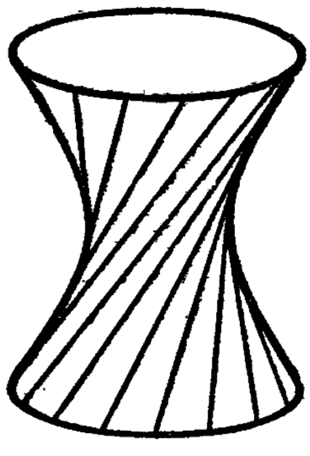
$$g_\lambda{}':\ \frac{x}{a}-\frac{z}{c}=\lambda\left(1+\frac{y}{b}\right), \qquad \frac{x}{a}+\frac{z}{c}=\frac{1}{\lambda}\left(1-\frac{y}{b}\right).$$

Ikkila holda (giperbolik paraboloid va bir pallali giperboloid) ham bitta oilaga qarashli to‘g‘ri chiziqli yasovchilar kesishmaydi, turli oilaga qarashli to‘g‘ri chiziqlar esa kesishadi.

Giperbolik paraboloid bilan bir pallali giperboloidda to‘g‘ri chiziqli yasovchilarning mavjudligi bu sirtlarni hosil qilishning yangi usulini berish imkoniyatini tug‘diradi; bir oilaga qarashli uchta to‘g‘ri chiziqli yasovchini olamiz: g_1, g_2, g_3. Bunday holda ikkinchi oilaga tegishli har bir to‘g‘ri chiziqli yasovchi g yuqoridagi g_1, g_2, g_3 ni kesadi. Demak, sirt berilgan uchta to‘g‘ri chiziqni kesadigan to‘g‘ri chiziqlardan tashkil topadi(7.1.1-chizma).



7.1.1-chizma



7.1.2-chizma

Bir pallali aylanma giperboloid masalasiga kelganda, uning istalgan to‘g‘ri chiziqli yasovchisining sirt o‘qi atrofida aylantirish natijasida ham hosil bo‘lishini ta’kidlab o‘tamiz(7.1.2-chizma).

Ikkinchi tartibli boshqa sirtlarda ham to‘g‘ri chiziqli yasovchilarning mavjud bo‘lishini pirovardida aytib o‘taylik, biroq bu sirtlarda ular – mavhum. Masalan,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0$$

ellipsoid ustida mavhum to‘g‘ri chiziqlarning

$$g_\lambda: \frac{x}{a} + i\frac{z}{c} = \lambda\left(1 - \frac{y}{b}\right), \qquad \frac{x}{a} - i\frac{z}{c} = \frac{1}{\lambda}\left(1 + \frac{y}{b}\right);$$

$$g_\lambda{}': \frac{x}{a} + i\frac{z}{c} = \lambda\left(1 + \frac{y}{b}\right), \qquad \frac{x}{a} - i\frac{z}{c} = \frac{1}{\lambda}\left(1 - \frac{y}{b}\right).$$

ikkita oilasi joylashdi.

7.2. Ellipsoid va sferaning urinma tekislik tenglamalari.

1.Fazoda

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad (7.3)$$

tenglama bilan berilgan ellipsoidni biror ixtiyoriy tekislik bilan kesib ko‘ramiz. Faraz qilaylik, bu tekislikning tenglamasi

$$Ax + By + Cz + D = 0 \qquad (7.4)$$

bo‘lsin. Bu tenglama bilan (7.3) tenglama birgalikda izlangan kesimni ifoda qiladi. Agar bu tenglamalardan z chiqarilsa, izlangan kesimning xOy tekislikdagi proyeksiyasi hosil bo‘ladi. (7.4) dan ($C \neq 0$):

$$Z = -\frac{Ax + By + D}{C},$$

buni ellipsoidning (7.3) tenglamasiga qo‘ysak:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{(Ax + By + D)^2}{C^2c^2} = 1,$$

yoki qavslarni ochib, quyidagi ko‘rinishda ham yozish mumkin:

$$\left(\frac{C^2}{a^2} + \frac{A^2}{c^2}\right)x^2 + \left(\frac{C^2}{b^2} + \frac{B^2}{c^2}\right)y^2 + 2\frac{AB}{c^2}xy + 2\frac{AD}{c^2}x +$$

$$+2\frac{BD}{c^2}y + \frac{D^2}{c^2} - C^2 = 0,$$

yoki

$$\frac{C^2}{a^2} + \frac{A^2}{c^2} = A_1, \quad \frac{C^2}{b^2} + \frac{B^2}{c^2} = C_1,$$

$$\frac{AB}{c^2} = B_1, \quad \frac{AD}{c^2} = D_1, \quad \frac{BD}{c^2} = E_1, \quad \frac{D^2}{c^2} - C^2 = F_1$$

faraz qilinsa:

$$A_1x^2 + 2B_1xy + C_1y^2 + 2D_1x + 2E_1y + F_1 = 0. \qquad (7.5)$$

Bu tenglama xOy tekislikda ikkinchi tartibli chiziqni ifoda qiladi. Bu chiziqning jinsini tekshirish uchun $M = B_1^2 - A_1C_1$ va

$$\Delta = \begin{vmatrix} A_1 & B_1 & D_1 \\ B_1 & C_1 & E_1 \\ D_1 & E_1 & F_1 \end{vmatrix}$$

tuzishga to‘g‘ri keladi. Bizda

$$M = \frac{A^2B^2}{c^2} - \frac{(C^2c^2 + A^2a^2)(C^2c^2 + B^2b^2)}{a^2b^2c^4} =$$

$$= -(A^2a^2 + B^2b^2 + C^2c^2)\frac{C^2}{a^2b^2c^4} < 0; \qquad (7.6)$$

Δ ni tuzganda uning ifodasi quyidagicha bo‘ladi:

$$\Delta = -\frac{A^2a^2 + B^2b^2 + C^2c^2 - D^2}{a^2b^2c^4} \cdot C^4 \qquad (7.7)$$

(7.6) ga qaraganda hamma vaqt $M < 0$, lekin (7.7) ga qaraganda Δ ning noldan kichik yoki nolga teng bo‘lishi mumkin. Bunga qarab (7.5) tenglama haqiqiy ellipsni yoki mavhum ellipsni yoki nuqtani ifoda qiladi. (7.7) ning tuzilishiga qaraganda Δ ning miqdori o‘z navbatida ushbu ifodaga bog‘liq:

$$a^2A^2 + b^2B^2 + c^2C^2 - D^2. \qquad (7.8)$$

Agar bu ifoda musbat bo‘lsa, $\Delta < 0$ bo‘ladi va bu holda izlanayotgan kesim haqiqiy ellipsdan iborat bo‘ladi; shunga o‘xshash agar (7.8) manfiy bo‘lsa, $\Delta > 0$ bo‘ladi va bu holda izlangan kesim mavhum ellipsdan iborat bo‘ladi, agarda (7.8) nolga teng bo‘lsa, bu holda $\Delta = 0$ bo‘ladi va izlangan kesim nuqtaga aylanadi.

Agarda tekislik ellipsoidni kessa, ellips hosil bo‘ladi, yoki tekislik bilan ellipsoidning umumiy nuqtasi bo‘lmaydi, yoki ikkalasining umumiy nuqtasi bo‘ladi.

Tekislik bilan ellipsoidning bir umumiy nuqtasi bo‘lganda, ya’ni tekislik ellipsoidga urunma bo‘lganda

$$a^2A^2 + b^2B^2 + c^2C^2 - D^2 = 0 \qquad (7.9)$$

bo‘ladi. Bu munosabatga asoslanib, ellipsoidga urinma bo‘lgan tekislikning tenglamasini tuzish mumkin. Haqiqatdan ham (7.9) ni ushbu ko‘rinishda yozish mumkin:

$$\left(-\frac{a^2A}{D}\right)^2 : a^2 + \left(-\frac{b^2B}{D}\right)^2 : b^2 + \left(-\frac{c^2C}{D}\right)^2 : c^2 = 1,$$

ya’ni koordinatalari

$$x_1 = -\frac{a^2A}{D}, \quad y_1 = -\frac{b^2B}{D}, \quad z_1 = -\frac{c^2C}{D} \qquad (7.10)$$

bo‘lgan nuqta ellipsoid tenglamasini qanoatlantiradi. Ikkinchi tomondan (7.9) ta’minlanganda $(x_1; y_1; z_1)$ nuqtaning koordinatalari (7.4) tekislikning tenglamasini ham qanoatlantiradi. Demak, $(x_1; y_1; z_1)$ nuqta (7.4) tenglamaning ellipsoidga urinish nuqtasi bo‘ladi. Ellipsoidga urinma bo‘lgan (7.4) tekislikning koeffitsiyentlari (7.10) dan aniqlanadi:

$$A = -\frac{Dx_1}{a^2}, \quad B = -\frac{Dy_1}{b^2}, \quad C = -\frac{Dz_1}{c^2},$$

natijada, ellipsoidning $(x_1; y_1; z_1)$ nuqtasidan o‘tgan urinma tekislikning tenglamasi quyidagicha bo‘ladi:

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} + \frac{zz_1}{c^2} = 1. \quad (7.11)$$

2.Fazoda

$$x^2 + y^2 + z^2 = R^2 \quad (7.12)$$

tenglama bilan berilgan sferani biror ixtiyoriy tekislik bilan kesib ko‘ramiz. Faraz qilaylik, bu tekislikning tenglamasi

$$Ax + By + Cz + D = 0 \quad (7.13)$$

bo‘lsin. Bu tenglama bilan (7.12) tenglama birgalikda izlangan kesimni ifoda qiladi. Agar bu tenglamalardan z chiqarilsa, izlangan kesimning xOy tekislikdagi proyeksiyasi hosil bo‘ladi. (7.13) dan $(C \neq 0)$:

$$Z = -\frac{Ax + By + D}{C},$$

buni sferaning (7.12) tenglamasiga qo‘ysak:

$$x^2 + y^2 + \frac{(Ax + By + D)^2}{C^2} = R^2,$$

yoki qavslarni ochib, quyidagi ko‘rinishda ham yozish mumkin:

$$(A^2 + C^2)x^2 + (B^2 + C^2)y^2 + 2ABxy + 2ADx +$$
$$+2BDy + D^2 - C^2R^2 = 0,$$

yoki

$$A^2 + C^2 = A_1, \ B^2 + C^2 = C_1,$$
$$AB = B_1, \ AD = D_1, \ BD = E_1, \ D^2 - C^2R^2 = F_1$$

faraz qilinsa:

$$A_1x^2 + 2B_1xy + C_1y^2 + 2D_1x + 2E_1y + F_1 = 0. \quad (7.14)$$

Bu tenglama xOy tekislikda ikkinchi tartibli chiziqni ifoda qiladi. Bu chiziqning jinsini tekshirish uchun $M = B_1^2 - A_1C_1$ va

$$\Delta = \begin{vmatrix} A_1 & B_1 & D_1 \\ B_1 & C_1 & E_1 \\ D_1 & E_1 & F_1 \end{vmatrix}$$

tuzishga to‘g‘ri keladi. Bizda

$$M = A^2B^2 - (A^2 + C^2)(B^2 + C^2) =$$
$$= -(A^2C^2 + B^2C^2 + C^4) < 0; \qquad (7.15)$$

Δ ni tuzganda uning ifodasi quyidagicha bo‘ladi:

$$\Delta = -(A^2R^2 + B^2R^2 + C^2R^2 - D^2)C^4 \qquad (7.16)$$

(7.15) ga qaraganda hamma vaqt $M < 0$, lekin (7.16) ga qaraganda Δ ning noldan kichik yoki nolga teng bo‘lishi mumkin.

(7.16) ning tuzilishiga qaraganda Δ ning miqdori o‘z navbatida ushbu ifodaga bog‘liq:

$$A^2R^2 + B^2R^2 + C^2R^2 - D^2. \qquad (7.17)$$

Agar bu ifoda $\Delta < 0$ va $\Delta > 0$ bo‘lsa, izlanayotgan kesim aylanadan iborat bo‘ladi; agarda (7.17) nolga teng bo‘lsa, bu holda $\Delta = 0$ bo‘ladi va izlangan kesim nuqtaga aylanadi.

Agarda tekislik sferani kessa, aylana hosil bo‘ladi, yoki tekislik bilan sferaning umumiy nuqtasi bo‘lmaydi, yoki ikkalasining umumiy nuqtasi bo‘ladi.

Tekislik bilan sferaning bir umumiy nuqtasi bo‘lganda, ya’ni tekislik sferaga urunma bo‘lganda

$$A^2R^2 + B^2R^2 + C^2R^2 - D^2 = 0 \qquad (7.18)$$

bo‘ladi. Bu munosabatga asoslanib, sferaga urinma bo‘lgan tekislikning tenglamasini tuzish mumkin. Haqiqatdan ham (7.18) ni ushbu ko‘rinishda yozish mumkin:

$$\left(-\frac{R^2A}{D}\right)^2 : R^2 + \left(-\frac{R^2B}{D}\right)^2 : R^2 + \left(-\frac{R^2C}{D}\right)^2 : R^2 = 1,$$

ya’ni koordinatalari

$$x_1 = -\frac{R^2A}{D}, \quad y_1 = -\frac{R^2B}{D}, \quad z_1 = -\frac{R^2C}{D} \qquad (7.19)$$

bo‘lgan nuqta sfera tenglamasini qanoatlantiradi. Ikkinchi tomondan (7.18) ta’minlanganda $(x_1; y_1; z_1)$ nuqtaning koordinatalari (7.13) tekislikning tenglamasini ham qanoatlantiradi. Demak, $(x_1; y_1; z_1)$ nuqta (7.13) tenglamaning sferaga urinish nuqtasi bo‘ladi. Sferaga urinma bo‘lgan (7.13) tekislikning koeffitsiyentlari (7.19) dan aniqlanadi:

$$A = -\frac{Dx_1}{R^2}, \quad B = -\frac{Dy_1}{R^2}, \quad C = -\frac{Dz_1}{R^2},$$

natijada, sferaning $(x_1; y_1; z_1)$ nuqtasidan o‘tgan urinma tekislikning tenglamasi quyidagicha bo‘ladi:

$$xx_1 + yy_1 + zz_1 = R^2. \qquad (7.20)$$

Mustaqil yechish uchun topshiriqlar.

7.1.1. $\frac{x^2}{p} + \frac{y^2}{q} = 2z \ (p > 0, \ q > 0)$ sirt to‘g‘ri chiziqli yasovchilarining Oxz tekisligidagi proyeksiyalari $x^2 = 2pz$ parabolaga urinishini isbotlang.

7.1.2. $x^2 - y^2 = 2z$ giperbolik paraboloid ikkita o‘zaro perpendikulyar yasovchilari kesishish nuqtalarining geometrik o‘rni topilsin.

7.1.3. Quyidagi sfera markazining koordinatalari va radiusi aniqlansin.

1) $x^2 + y^2 + z^2 - 12x + 4y - 6z = 0$;

2) $x^2 + y^2 + z^2 + 8x = 0$;

3) $x^2 + y^2 + z^2 - 2x + 4y - 6z - 22 = 0$;

4) $x^2 + y^2 + z^2 - 6z - 7 = 0$.

7.1.4. $A(3; 0; 4)$, $B(3; 5; 0)$, $C(3; 4; 4)$, $D(5; 4; 6)$ nuqtalarning $(x-1)^2 + (y+2)^2 + (z-1)^2 = 49$ sferaga nisbatan vaziyati aniqlansin.

7.1.5. $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sferaning ushbu $x = x_0 + lt, \ y = y_0 + mt, \ z = z_0 + nt$ to‘g‘ri chiziqqa qo‘shma bo‘lgan diametrial tekisligining tenglamasi tuzilsin.

7.1.6. Ushbu $(x-1)^2 + (y-4)^2 + (z+1)^2 = 25$ sferaning $M(3; 5; 1)$ nuqtada teng ikkiga bo‘linadigan vatarlarining geometrik o‘rni topilsin.

7.1.7. $x^2 + y^2 + z^2 - R^2 = 0$ sferaning $N(x_0; y_0; z_0)$ nuqtadan o‘tuvchi vatarlari o‘rtalarining geometrik o‘rni topilsin.

7.1.8. $x^2 + y^2 + z^2 - R^2 = 0$ sferaning $M(-R; 0; 0)$ nuqtadan o‘tuvchi vatarlari o‘rtalarining geometrik o‘rni topilsin.

7.1.9. $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sferaning $K(x_0; y_0; z_0)$ nuqtadan o‘tuvchi vatarlari o‘rtalarining geometrik o‘rni topilsin.

7.1.10. $x^2 + y^2 = 9, \ z = 0$ va $z = 2$ aylanalardan o‘tuvchi sfera tenglamasi tuzilsin.

7.1.11. Koordinatalar boshidan va $(x+1)^2+(y-2)^2+(z+2)^2=$ $=49,\ 2x+2y-z+4=0$ aylanadan o‘tadigan sfera tenglamasi tuzilsin.

7.1.12. $A(1;-2;0)$ nuqtadan va $(x+1)^2+(y-2)^2+(z-2)^2=$ $=49,\ 2x+2y-z+4=0$ aylanadan o‘tuvchi sfera tenglamasi tuzilsin.

7.1.13. Quyidagi tekisliklarning ushbu $(x-1)^2+(y-2)^2+$ $+(z-4)^2=25$ sferaga nisbatan vaziyati aniqlansin.

1) $2x+2y+z+2=0$;

2) $2x+2y+z+5=0$;

3) $2x+2y+z+11=0$.

7.1.14. $(x-1)^2+(y+3)^2+(z-2)^2=49$ sferaga $N(7;-1;5)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.

7.1.15. $(x+2)^2+(y-1)^2+z^2=169$ sferaga $K(-5;-3;12)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.

7.1.16. $(x-a)^2+(y-b)^2+(z-c)^2=R^2$ sferaga $N_0(x_0;y_0;z_0)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.

7.1.17. $x^2+y^2+z^2=R^2$ sferaga $M_0(x_0;y_0;z_0)$ nuqtada o‘tkazilgan urinma tekislik tenglamasi tuzilsin.

7.1.18. Qanday zaruriy va yetarli shart bajarilganda $Ax+By+Cz+$ $+D=0$ tekislik $x^2+y^2+z^2=R^2$ sferaga urinadi? Bu shart bajarilgan deb urinish nuqtasining koordinatalari topilsin.

7.1.19. $x^2+y^2+z^2-3x+6y+2z-5=0,\ x-2y-2z+1=0$ aylanadan o‘tuvchi va $2x+2y+z-7=0$ tekislikka urinadigan sfera tenglamasi tuzilsin.

7.1.20. $x^2+y^2-11=0,\ z=0$ aylanadan o‘tuvchi va $x+y+z-$ $-5=0$ tekislikka urinadigan sfera tenglamasi tuzilsin.

7.1.21. Koordinatalar boshi bilan $N(1;1;1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqda $(x-2)^2+(y-5)^2+z^2=1$,
$(x-4)^2+(y-3)^2+(z-6)^2=2$ sferalarga o‘tkazilgan urinma uzunliklari bir-biriga teng bo‘lgan nuqta topilsin.

7.1.22. $x=x_0+lt,\quad y=y_0+mt,\quad z=z_0+nt$ to‘g‘ri chiziq $(x-a)^2+(y-b)^2+(z-c)^2=R^2$ sferaga urinishi uchun qanday shartlarning bajarilishi zarur va yetarli?

7.1.23. Ikkita ayqash

$x=x_1+lt,\ y=y_1+mt,\ z=z_1+nt;$

$x=x_2+lt,\ y=y_2+mt,\ z=z_2+nt.$

to‘g‘ri chiziqqa urinuvchi sferalar markazlarining geometrik o‘rni topilsin, bu yerda ${l_1}^2 + {m_1}^2 + {n_1}^2 = 1$ va ${l_2}^2 + {n_2}^2 + {n_2}^2 = 1$ deb faraz qilinadi.

7.1.24. Quyida ko‘rsatilgan hollarning har birida:

1) berilgan nuqtadan o‘tuvchi;
2) berilgan ikki nuqtadan o‘tuvchi;
3) berilgan uch nuqtadan o‘tuvchi;
4) berilgan to‘g‘ri chiziqqa urinuvchi;
5) berilgan tekislikka urinuvchi;
6) berilgan tekislikka urinuvchi va belgili radiusga ega;
7) berilgan tekislikdagi markazga ega bo‘lgan;
8) berilgan aylanadagi markazga ega bo‘lgan;
9) berilgan aylana orqali o‘tuvchi

sferalar to‘plami nechta parametrga bog‘liq?

7.1.25. $x = x_1 + lt,\ y = y_1 + mt,\ z = z_1 + nt$ to‘g‘ri chiziq orqali ushbu $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sferaga urinma tekisliklar o‘tkazilsin.

7.1.26. Qanday zaruriy va yetarli shartlarda $x = x_1 + lt, y = y_1 + mt,$ $z = z_1 + nt$ to‘g‘ri chiziq bilan $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sfera

1) kesishmaydi?
2) kesishadi?

7.1.27. $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$ sferaning $Ax + By +$ $+Cz + D = 0$ tekislikka parallel bo‘lgan urinma tekisliklari tenglamalari tuzilsin.

7.1.28. $\frac{x^2}{27} + \frac{y^2}{12} + \frac{z^2}{75} = 1$ ellipsoidning $N(3; 2; 5)$ nuqtasidagi urinma tekisligi tenglamasi tuzilsin.

7.1.29. $\frac{x^2}{49} + \frac{y^2}{18} + \frac{z^2}{50} = 1$ ellipsoidning $A(0; -3; 5)$ nuqtasidagi urinma tekisligi tenglamasi tuzilsin.

7.1.30. $Ax + By + Cz + D = 0$ tekislikning $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoidga urinishi uchun zaruriy va yetarli shart topilsin.

7.1.31. $Ax + By + Cz + D = 0$ tekislikning $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoid bilan kesishishi uchun qanday shartning bajarilishi zarur va yetarli?

7.1.32. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning markazidan uning urinma tekisligiga tushurilgan perpendikulyarlar asoslarining geometrik o‘rni topilsin.
7.1.33. $2x+3y-z+1=0$ tenglama bilan berilgan tekislik va $\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{9}=1$ tenglama bilan berilgan ellipsoidning kesishishidan hosil bo‘lgan chiziqni aniqlang.
7.1.34. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning $Ax+By+Cz+D=0$ tekislik bilan kesishish chizig‘ining markazi topilsin.
7.1.35. $\frac{x^2}{25}+\frac{y^2}{16}+\frac{z^2}{9}=1$ ellipsoidning markazidan barcha nuqtalarida unga o‘tkazilgan urinma tekisliklargacha bo‘lgan masofalar d ga teng bo‘ladigan nuqtalarning geometrik o‘rni topilsin.
7.1.36. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning markazidan barcha nuqtalarida unga o‘tkazilgan urinma tekisliklargacha bo‘lgan masofalar d ga teng bo‘ladigan nuqtalarning geometrik o‘rni topilsin.
7.1.37. Oxy tekislikka va $x^2+y^2+z^2=a^2$ sferaga urinadigan sferalar markazlarining geometrik o‘rni topilsin.
7.1.38.O‘qlari koordinata o‘qlari bilan ustma-ust tushuvchi,Oxz va Oyz tekisliklarni mos ravishda $y=0,\ \frac{x^2}{25}+\frac{z^2}{16}=1,\ x=0,\ \frac{y^2}{9}+\frac{z^2}{16}=1$ chiziqlar bo‘ylab kesib o‘tuvchi ellipsoid tenglamasi tuzilsin.
7.1.39. O‘qlari koordinata o‘qlaridan iborat, $z=0,\ \frac{x^2}{9}+\frac{y^2}{16}=1$ ellips va $N(1;2;\sqrt{23})$ nuqta orqali o‘tuvchi ellipsoid tenglamasi tuzilsin.
7.1.40. O‘qlari koordinata o‘qlaridan iborat bo‘lgan va x^2+y^2+ $+z^2=9, x=z$ aylanadan hamda $N(3;1;1)$ nuqtadan o‘tgan ellipsoid tenglamasi tuzilsin.

8-MAVZU: IKKINCHI TARTIBLI SIRTLAR UMUMIY TENGLAMALARINI SODDALASHTIRISH. MARKAZIY VA NAMARKAZIY SIRT TENGLAMALARINI KANONIK KO‘RINISHGA KELTIRISH

Reja:

1. Ikkinchi tartibli sirtlar umumiy tenglamalarini soddalashtirish.
2. Markaziy sirtning tenglamasini kanonik ko‘rinishga keltirish.
3. Nomarkaziy sirt tenglamasini kanonik ko‘rinishga keltirish.

***Tayanch iboralar**:* markaziy sirt, nomarkaziy sirt, invariant, parallel ko‘chirish, xarakteristik tenglama, mavhum parallel tekislik.

8.1. Ikkinchi tartibli sirtlar umumiy tenglamalarini soddalashtirish.

Ikkinchi tartibli sirtning tenglamasi

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{23}yz + 3a_{31}zx + \\ +2a_1x + 2a_2y + 2a_3z + a = 0 \qquad (8.1)$$

ko‘rinishga ega. Bu tenglama to‘g‘ri burchakli koordinatalar sistemasiga nisbatan berilgan bo‘lsa, quyidagi ifodalar to‘g‘ri burchakli dekart koordinatalari sistemasini parallel ko‘chirish va burishga nisbatan invariantlari hisoblanadi:

$$I_1 = a_{11} + a_{22} + a_{33}, \quad I_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix},$$

$$I_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \quad K_4 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_1 \\ a_{21} & a_{22} & a_{23} & a_2 \\ a_{31} & a_{32} & a_{33} & a_3 \\ a_1 & a_2 & a_3 & a \end{vmatrix}.$$

Yariminvariant nomini olgan quyidagi ikki ifoda, to‘g‘ri burchakli dekart koordinatalar sistemasini burishga nisbatan invariantlardir.

$$K_3 = \begin{vmatrix} a_{11} & a_{12} & a_1 \\ a_{21} & a_{22} & a_2 \\ a_1 & a_2 & a \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} & a_1 \\ a_{31} & a_{33} & a_3 \\ a_1 & a_3 & a \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} & a_2 \\ a_{32} & a_{33} & a_3 \\ a_2 & a_3 & a \end{vmatrix},$$

$$K_2 = \begin{vmatrix} a_{11} & a_1 \\ a_1 & a \end{vmatrix} + \begin{vmatrix} a_{22} & a_2 \\ a_2 & a \end{vmatrix} + \begin{vmatrix} a_{33} & a_3 \\ a_3 & a \end{vmatrix}$$

$I_3 = 0, K_4 = 0$ holda K_3 yariminvariant ayni vaqtda burishga nisbatan ham invariant bo‘ladi, $I_3 = 0,\ K_4 = 0,\ I_2 = 0,\ K_3 = 0$ holda esa K_2 yariminvariant parallel ko‘chirishga nisbatan invariant bo‘ladi.

I. $I_3 \neq 1$ holda ikkinchi tartibli sirt tenglamasini to‘g‘ri burchakli koordinatalar sistemasini parallel ko‘chirish va burish natijasida quyidagi ko‘rinishga keltirish mumkin:

$$\lambda_1 X^2 + \lambda_2 Y^2 + \lambda_3 Z^2 + \frac{K_4}{I_3} = 0 \qquad (8.2)$$

bu yerda, $\lambda_1, \lambda_2, \lambda_3 -$ quyidagi xarakteristik tenglamaning ildizlaridir:

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0 \qquad (8.3)$$

yoki

$$\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0.$$

1^0. Agar $\lambda_1, \lambda_2, \lambda_3$ bir xil ishorali, $\frac{K_4}{I_3}$ esa ularga teskari ishorada bo‘lsa, u holda (8.2) tenglama ellipsoidni aniqlaydi. $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ deb hisoblab, (8.2) tenglamani

$$\frac{x^2}{-\frac{K_4}{\lambda_1 I_3}} + \frac{y^2}{-\frac{K_4}{\lambda_2 I_3}} + \frac{z^2}{-\frac{K_4}{\lambda_3 I_3}} = 1$$

ko‘rinishda yozib olamiz. Bunda ellipsoidni yarim o‘qlarini

$$a = \sqrt{-\frac{K_4}{\lambda_1 I_3}}, \quad b = \sqrt{-\frac{K_4}{\lambda_2 I_3}}, \quad c = \sqrt{-\frac{K_4}{\lambda_3 I_3}}$$

ko‘rinishda yoza olamiz va $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ qilingan farazga ko‘ra $a \geq b \geq c$ munosabatlar o‘rinli bo‘ladi.

2^0. $\lambda_1, \lambda_2, \lambda_3$, $\frac{K_4}{I_3}$ bir xil ishorali bo‘lsa, u holda (8.2) tenglama mavhum ellipsoidni aniqlaydi: $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ deb hisoblagan holda uni $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$ ko‘rinishga keltiramiz, bunda: $a = \sqrt{\frac{K_4}{\lambda_1 I_3}}$, $b = \sqrt{\frac{K_4}{\lambda_2 I_3}}$, $c = \sqrt{\frac{K_4}{\lambda_3 I_3}}$ qilingan $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ farazga ko‘ra $a \geq b \geq c$ ekanligiga ishonch hosil qilamiz.

3^0. $\lambda_1, \lambda_2, \lambda_3$ sonlar bir xil ishorali, va $K_4 = 0$ bo‘lsa, u holda (8.2) tenglama mavhum konusni aniqlaydi. $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$ deb hisoblagan holda uni $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ ko‘rinishga keltiramiz, bunda:

$$a = \sqrt{\frac{1}{|\lambda_1|}}, \quad b = \sqrt{\frac{1}{|\lambda_2|}}, \quad c = \sqrt{\frac{1}{|\lambda_3|}}$$

va shu bilan birga $a \geq b \geq c$.

4^0. Agar (8.3) xarakteristik tenglama ildizlarining ikkitasi bir xil ishorali, uchinchi ildizi bilan $\frac{K_4}{I_3}$ ularga teskari ishorali bo‘lsa, (8.2) tenglama bir pallali giperboloidni aniqlaydi. Bu holda xarakteristik tenglamaning bir xil ishorali ildizlarini λ_1 va λ_2 deb belgilab va $|\lambda_1| < |\lambda_2|$ deb faraz qilib (8.2) tenglamani yoki

$$\frac{x^2}{-\frac{K_4}{\lambda_1 I_3}} + \frac{y^2}{-\frac{K_4}{\lambda_2 I_3}} + \frac{z^2}{-\frac{K_4}{\lambda_3 I_3}} = 1$$

ko‘rinishda yozib olamiz.

Bu yerda: $a = \sqrt{-\frac{K_4}{\lambda_1 I_3}}$, $b = \sqrt{-\frac{K_4}{\lambda_2 I_3}}$, $c = \sqrt{-\frac{K_4}{\lambda_3 I_3}}$, $a \geq b$.

5^0. Xarakteristik tenglamaning ikki ildizi va $\frac{K_4}{I_3}$ ozod hadi bir xil ishorali, xarakteristik tenglamaning uchinchi ildizi esa ularga teskari ishorali bo‘lsa, (8.2) tenglama ikki pallali giperboloidni aniqlaydi. Bu holda xarakteristik tenglamaning bir xil ishorali ildizlari λ_1 va λ_2 ni olib $|\lambda_1| \leq |\lambda_2|$ deb hisoblasak, (8.3) tenglamani

$$\frac{x^2}{\frac{K_4}{\lambda_1 I_3}} + \frac{y^2}{\frac{K_4}{\lambda_2 I_3}} - \frac{z^2}{-\frac{K_4}{\lambda_3 I_3}} = -1 \ yoki \ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

ko‘rinishida yozamiz, bunda: $a \geq b$.

6^0. Xarakteristik tenglamaning ikkita ildizi bir xil ishorali, uchinchi ildizi ularga teskari va $K_4 = 0$ bo‘lsa, u holda (8.2) tenglama konusni aniqlaydi. λ_1 va λ_2 sonlar bir xil ishorali ildizlar va $|\lambda_1| \leq |\lambda_2|$ deb hisoblanganda (8.2) tenglamani

$$\frac{x^2}{\frac{1}{|\lambda_1|}} + \frac{y^2}{\frac{1}{|\lambda_2|}} - \frac{z^2}{\frac{1}{|\lambda_3|}} = 0 \ yoki \ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

ko‘rinishga keltiramiz. Bu yerda $a \geq b$ bo‘lib:

$$a = \sqrt{\frac{1}{|\lambda_1|}}, \quad b = \sqrt{\frac{1}{|\lambda_2|}}, \qquad c = \sqrt{\frac{1}{|\lambda_3|}}$$

Xarakteristik tenglamadagi musbat ildizlar soni uning koeffitsiyentlari orasidagi ishoralar almashuvlari soniga teng bo‘ladi (Dekart qoidasi).

II. $I_3 = 0,\ K_4 \neq 0$ bo‘lsa, u holda to‘g‘ri burchakli koordinatalar sistemasini parallel ko‘chirish va burish natijasida ikkinchi tartibli sirt tenglamasini

$$\lambda_1 x^2 + \lambda_2 y^2 \pm 2\sqrt{-\frac{K_4}{I_2}}z = 0 \qquad (8.4)$$

ko‘rinishga keltirish mumkin. Bu tenglamada λ_1 va λ_2 xarakteristik tenglamaning noldan farqli bo‘lgan ildizlari.

7^0. λ_1, λ_2 sonlar bir xil ishorali bo‘lsa, u holda (8.4) tenglama elliptik paraboloidni aniqlaydi. $|\lambda_1| \leq |\lambda_2|$ deb hisoblab (8.4) tenglamani

$$\frac{x^2}{\pm\frac{1}{\lambda_1}\sqrt{-\frac{K_4}{I_2}}} + \frac{y^2}{\pm\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}}} = 2z$$

ko‘rinishda yoza olamiz.

$$\pm\frac{1}{\lambda_1}\sqrt{-\frac{K_4}{I_2}} = p, \qquad \pm\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}} = q$$

deb faraz qilib, ushbu tenglamani hosil qilamiz:

$$\frac{x^2}{p} + \frac{y^2}{q} = 2z$$

bunda: $p \geq q \geq 0$.

8^0. λ_1, λ_2 sonlar har xil ishorali bo‘lsa, (8.4) tenglama giperbolik paraboloidni aniqlaydi. λ_1 musbat, λ_2 manfiy ildiz deb olib, $\sqrt{-\frac{K_4}{I_2}}$ radikal oldidagi ishoradan minusini olib, (8.4) tenglamani

$$\frac{x^2}{\frac{1}{\lambda_1}\sqrt{-\frac{K_4}{I_2}}} - \frac{y^2}{-\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}}} = 2z \ \ yoki \ \ \frac{x^2}{p} - \frac{y^2}{q} = 2z$$

ko‘rinishda yozamiz, bu yerda:

$$p = \frac{1}{\lambda_1}\sqrt{\frac{K_4}{I_2}}, \qquad q = -\frac{1}{\lambda_2}\sqrt{-\frac{K_4}{I_2}}$$

III. $I_3 = 0,\ K_4 = 0,\ I_2 \neq 0$ bo‘lsa, to‘g‘ri burchakli koordinatalar sistemasini burish va parallel ko‘chirish natijasida ikkinchi tartibli sirt tenglamasini

$$\lambda_1 x^2 + \lambda_2 y^2 + \frac{K_3}{I_2} = 0 \qquad (8.5)$$

ko‘rinishga keltirish mumkin. Bu yerda: λ_1, λ_2 sonlar xarakteristik tenglamaning noldan farqli ildizlari.

9^0. λ_1, λ_2 sonlar bir xil ishorali, $\frac{K_3}{I_2}$ esa ularga teskari ishorali bo‘lsa, (8.5) tenglama elliptik silindrni aniqlaydi. $|\lambda_1| \leq |\lambda_2|$ deb hisoblab, (8.5) tenglamani

$$\frac{x^2}{-\frac{K_3}{\lambda_1 I_2}} + \frac{y^2}{-\frac{K_3}{\lambda_2 I_2}} = 1 \quad yoki \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ko‘rinishda yozib olamiz, bu yerda: $a \geq b$ bo‘lib,

$$a = \sqrt{-\frac{K_3}{\lambda_1 I_2}}, \quad b = \sqrt{-\frac{K_3}{\lambda_2 I_2}}$$

10^0. λ_1, λ_2, $\frac{K_3}{I_2}$ sonlar bir xil ishorali bo‘lsa, (8.5) tenglama mavhum elliptik silindrni aniqlaydi. $|\lambda_1| \leq |\lambda_2|$ deb hisoblab, (8.5) tenglamani

$$\frac{x^2}{\frac{K_3}{\lambda_1 I_2}} + \frac{y^2}{\frac{K_3}{\lambda_2 I_2}} = -1 \quad yoki \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$$

ko‘rinishda yozamiz, bunda: $a \geq b$.

11^0. λ_1, λ_2 sonlar bir xil ishorali va $K_3 = 0$ bo‘lsa, u holda (8.5) tenglama kesishadigan ikkita mavhum tekisliklarni aniqlaydi. Bu holda (8.5) tenglamani

$$\frac{x^2}{\frac{1}{\lambda_1}} + \frac{y^2}{\frac{1}{\lambda_2}} = 0 \quad yoki \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$$

ko‘rinishda yozib olamiz, bunda $a \geq b$ bo‘lib,

$$a = \sqrt{\frac{1}{|\lambda_1|}}, \quad b = \sqrt{\frac{1}{|\lambda_2|}}.$$

12^0. λ_1, λ_2 sonlar har xil ishorali va $K_3 \neq 0$ bo‘lsa, (8.5) tenglama giperbolik silindrni aniqlaydi. λ_1 deb xarakteristik tenglamaning $\frac{K_3}{I_2}$ ning ishorasiga teskari ishorali ildizni olib, (8.5) tenglamani

$$\frac{x^2}{-\frac{K_3}{\lambda_1 I_2}} - \frac{y^2}{-\frac{K_3}{\lambda_2 I_2}} = 1 \quad yoki \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

ko‘rinishda yozib olamiz, bu yerda:

$$a = \sqrt{-\frac{K_3}{\lambda_1 I_2}}, \qquad b = \sqrt{\frac{K_3}{\lambda_2 I_2}}.$$

13^0. λ_1, λ_2 sonlar har xil ishorali va $K_3 = 0$ bo‘lsa, (8.5) tenglama kesishadigan ikkita tekislikni aniqlaydi. Xarakteristik tenglamaning musbat ildizini λ_1 deb olib, (8.5) tenglamani

$$\frac{x^2}{\frac{1}{\lambda_1}} - \frac{y^2}{\frac{1}{\lambda_2}} = 0 \quad yoki \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

ko‘rinishda yozamiz, bunda:

$$a = \sqrt{\frac{1}{\lambda_1}}, \quad b = \sqrt{-\frac{1}{\lambda_2}}.$$

IV. $I_3 = 0,\ K_4 = 0,\ I_2 = 0,\ K_3 \neq 0$ hol yuz bersa, to‘g‘ri burchakli koordinatalar sistemasini burish va parallel ko‘chirish natijasida ikkinchi tartibli sirt ternglamasini

$$\lambda_1 x^2 \pm 2\sqrt{-\frac{K_3}{I_1}}\, y = 0 \qquad (8.6)$$

ko‘rinishga keltirish mumkin, bu yerda: $\lambda_1 = I_1$ son xarakteristik tenglamaning noldan farqli bo lgan ildizi.

14^0. (8.6) tenglamani ushbu $x^2 = 2\sqrt{-\frac{K_3}{I_1}}\, y$ ko‘rinishda yozish ham mumkin. Bu tenglama parabolik silindrni aniqlaydi. Bu silindrni yasovchilariga perpendikulyar bo‘lgan tekislik bilan kesishish natijasida hosil bo‘lgan parabolaning parametrini ushbu

$$p = \sqrt{-\frac{K_3}{{I_1}^3}}$$

formuladan aniqlanadi.

V. $I_3 = 0,\ K_4 = 0,\ I_2 = 0,\ K_3 = 0$ holda, to‘g‘ri burchakli koordinatalar sistemasini burish natijasida ikkinchi tartibli sirt tenglamasini

$$\lambda_1 x^2 + \frac{K_2}{I_1} = 0 \quad yoki \quad I_1 x^2 + \frac{K_2}{I_1} = 0 \quad yoki$$

$$x^2 + \frac{K_2}{I_1{}^2} = 0 \qquad (8.7)$$

ko‘rinishga keltirish mumkin.

15^0. $K_2 < 0$ bo‘lsa, (8.7) tenglama ikkita parallel tekislikni aniqlaydi. Bu holda $\frac{K_2}{I_1{}^2} = -a^2$ deb tenglamani $x^2 - a^2 = 0$ ko‘rinishda yozib olamiz.

16^0. $K_2 > 0$ bo‘lsa, (8.7) tenglama ikkita mavhum parallel tekislikni aniqlaydi. $\frac{K_2}{I_1{}^2} = a^2$ deb uni $x^2 + a^2 = 0$ ko‘rinishda yozamiz.

17^0. Nihoyat, $K_2 = 0$ bo‘lsa, (8.7) tenglama ikkita ustma-ust tushuvchi tekislikni aniqlaydi. $x^2 = 0$.

Ikkinchi tartibli sirt aylanma sirt bo‘lishi uchun uning xarakteristik tenglamasi karrali ildizga ega bo‘lishi zarur va yetarlidir.

Kanonik tenglamasi ma’lum bo‘lgan sirt vaziyatini aniqlash uchun, kanonik sistemaning yangi koordinatalar boshi O' ni va shu bilan birga bu sistemaning yo‘naltiruvchi vektorlari koordinatalarini bilish kerak.

Kanonik koordinatalar sistemasi o‘qlari yo‘naltiruvchi vektorlari koordinatalari

$$\begin{cases} (a_{11} - \lambda)l + a_{12}m + a_{13}n = 0 \\ a_{21}l + (a_{22} - \lambda)m + a_{23}n = 0 \\ a_{31}l + a_{32}m + (a_{33} - \lambda)n = 0 \end{cases} \qquad (8.8)$$

tenglamalar sistemasidan aniqlandi, bunda $\lambda -$ xarakteristik tenglamaning ildizi. Aylanma sirtning joylashishini aniqlash uchun kanonik koordinatalar sistemasida yangi koordinata boshi O' ni va aylanish o‘qi yo‘naltiruvchi vektorining koordinatalarini bilish lozim. Yo‘naltiruvchi vektorining koordinatalari (8.8) sistemasidan aniqlanadi, bunda $\lambda -$ xarakteristik tenglamaning oddiy ildizi.

Sirt markazga ega bo‘lsa (yagona bo‘lishi shart emas), u holda kanonik sistemasining yangi koordinata boshi O' deb sirt markazi olinadi. Sirt markazining koordinatalari

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z + a_1 = 0 \\ a_{21}x + a_{22}y + a_{23}z + a_2 = 0 \\ a_{31}x + a_{32}y + a_{33}z + a_3 = 0 \end{cases} \qquad (8.9)$$

tenglamalar sistemasidan topiladi.

1^0. Uch o‘qli ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \qquad a > b > c.$$

Bu ellipsoid markazining koordinatalari (8.9) sistemadan topiladi. Katta o‘qi $(O'X)$ ning yo‘naltiruchi vektorining koordinatalari (8.8) tenglamadan topiladi, undagi son xarakteristik tenglamalarning modul jihatdan kichik bo‘lgan ildizi; o‘rta o‘q $(O'Y)$ning yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan topiladi, $\lambda -$ son xarakteristik tenglamaning modul jihatdan o‘rta bo‘lgan ildiz; kichik o‘q $(O'z)$ ning yo‘naltiruvchi vektorning koordinatalari ham (8.8) sistemadan topiladi, bunda $\lambda -$ xarakteristik tenglamaning modul jihatidan katta bo‘lgan ildizi.

2^0. Agar (8.1) tenglama nuqtani aniqlasa (mavhum konus), u holda bu nuqtaning koordinatalari (8.9) sistemadan topiladi.

3^0. Bir pallali giperboloidning kanonik tenglamasi:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \qquad a > b.$$

Bir pallali giperboloid markazining koordinatalari (8.9) sistemadan aniqlanadi.

λ_1, λ_2 sonlar xarakteristik tenglmaning bir xil ishorali ildizlari bo‘lib, bunda $|\lambda_1| < |\lambda_2|$ va λ_3 esa ishorasi λ_1 va λ_2 ildizlarning ishorasiga teskari ildiz bo‘lsin. Giperboloid $(O'Z)$ o‘qining yo‘naltiruvchi vektori koordinatalari (8.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_3^i$ bir pallali giperboloid bo‘g‘iz kesimning $(O'x)$ katta o‘qi yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_1$; bir pallali $\lambda = \lambda_1$ bo‘g‘iz kesimining $(O'y)$ kichik o‘qi yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan topiladi, bunda $\lambda = \lambda_2$.

4^0. Ikki pallali giperboloid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1, \qquad a > b.$$

Ikki pallali giperboloid markazining koordinatalari (8.9) sistemadan topiladi. λ_1, λ_2 sonlar xarakteristik tenglamaning bir xil ishorali ildizlari va $|\lambda_1| < |\lambda_2|$ bo‘lsin, $\lambda_3 -$ esa xarakteristik tenglamaning λ_1, λ_2 ildizlari ishorasiga teskari ishoraga ega bo‘lgan uchinchi ildizi bo‘lsin.

U holda giperboloid $(O'Z)$ o‘qining yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_3$; $O'X$ o‘qini (giperboloid o‘qiga perpendikulyar bo‘lgan o‘q bilan kesishi natijasida hosil bo‘lgan ellipsning katta o‘qi) yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan topiladi. Bunda $\lambda = \lambda_1$; $O'Y$ o‘qi yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan topiladi, bunda $\lambda = \lambda_2$.

5^0. Konus:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0, \quad a > b$$

Konus uchlarining koordinatalari (8.9) sistemadan aniqlanadi. λ_1, λ_2 sonlar xarakteristik tenglamaning bir xil ishorali ildizlari va $|\lambda_1| < |\lambda_2|$; $\lambda_3 -$ esa xarakteristik tenglamaning ishorasi λ_1, λ_2 ildizlar ishorasiga teskari ishorali ildizi bo‘lsin. U holda konusning $(O'z)$ o‘qi yo‘naltiruvchi vektorning koordinatalari (8.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_3$. $O'x$ o‘qi yo‘naltiruvchi vektorining koordinatalarini (8.8) sistemadan aniqlanadi, bunda $\lambda = \lambda_1$. $O'x$ o‘qi (ya‘ni konusning o‘qiga perpendikulyar bo‘lgan kesimda hosil qilingan ellipsning katta o‘qi) yo‘naltiruchi vektorning koordinatalari (8.8) sistemadan aniqlaydi, bunda $\lambda = \lambda_1$; $O'y$ o‘qi yo‘naltiruvchi vektorining koordinatalarini (8.8) sistemadan aniqlaymiz, bunda $\lambda = \lambda_2$.

II. 6^0. Elliptik parabolid:

$$\frac{x^2}{p} + \frac{y^2}{q} = 2z$$

kanonik sistemasining boshi, bu holda paraboloid uchidan iborat. Elliptik paraboloidning sirt botiqligi tomon yo‘nalgan o‘qining vektori ushbu munosabatdan aniqlanadi: $P\{I_1A_1, I_1A_2, I_1A_3\}$ bu yerda

$$A_1 = -\begin{vmatrix} a_{12} & a_{13} & a_1 \\ a_{22} & a_{23} & a_2 \\ a_{32} & a_{33} & a_3 \end{vmatrix}; \quad A_2 = \begin{vmatrix} a_{11} & a_{13} & a_1 \\ a_{21} & a_{23} & a_2 \\ a_{31} & a_{33} & a_3 \end{vmatrix};$$

$$A_3 = -\begin{vmatrix} a_{11} & a_{12} & a_1 \\ a_{21} & a_{22} & a_2 \\ a_{31} & a_{32} & a_3 \end{vmatrix}.$$

Bu yerdagi A_1, A_2, A_3 sonlar K_4 determinantdagi a_1, a_2, a_3 elementlarining algebraik to‘ldiruvchilarini bildiradi.

λ_1, λ_2 xarakteristik tenglamaning noldan farqli ildizlari va $|\lambda_1| < |\lambda_2|$ bo‘lsin, bu holda $O'X$ o‘qining (ya’ni elliptik paraboloidni o‘ziga perpendikulyar bo‘lgan tekislik bilan kesishishdan hosil bo‘lgan ellips katta o‘qi) yo‘naltiruvchi vektorining koordinatalari $\lambda = \lambda_1$ holda (8.8) sistemadan aniqlanadi, $O'Y$ o‘qining yo‘naltiruvchi vektorini koordinatalari esa $\lambda = \lambda_2$ holda (8.8) sistemadan aniqlanadi. Elliptik paraboloidni uchi ushbu

$$\begin{cases} \dfrac{a_{11}x + a_{12}y + a_{13}z + a_1}{A_1} = \dfrac{a_{21}x + a_{22}y + a_{23}z + a_2}{A_2} = \dfrac{a_{31}x + a_{32}y + a_{33}z + a_3}{A_3} \\ a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{23}yz + 2a_{31}zx + 2a_1x + 2a_2y + 2a_3z + a = 0 \end{cases}$$

(8.10) tenglamalar sistemasidan topiladi.

7^0. Giperbolik paraboloid:

$$\frac{x^2}{p} - \frac{y^2}{q} = 2z.$$

Bu holda kanonik sistemaning boshi paraboloid uchidan iborat. Giperbolik paraboloidning $(O'XZ)$ tekislik bilan kesishishi natijasida hosil bo‘lgan katta parametrli bosh kesimning botiqlik tomonga yo‘nalgan parabola o‘qining yo‘naltiruvchi vektori ushbu koordinatalarga ega bo‘ladi:

$$\{I_1A_1, I_1A_2, I_1A_3\}$$

bu yerda A_1, A_2, A_3 sonlar K_4 determinantning a_1, a_2, a_3 elementlarining algebraik to‘ldiruvchilaridir, λ_1, λ_2 sonlar xarakteristik tenglamaning ildizlari bo‘lib, $|\lambda_1| < |\lambda_2|$. U holda $O'X$ o‘qining yo‘naltiruvchi vektorining koordinatalari (ya’ni paraboloid uchidan o‘tuvchi to‘g‘ri chiziqli yasovchilar orasidagi o‘tkir burchak bissektrisalari) (8.8) sistemadan $\lambda = \lambda_1$ deb aniqlanadi: $O'Y$ o‘qni yo‘naltiruvchi vektorining koordinatalari (8.8)sistemadan $\lambda = \lambda_2$ deb aniqlanadi. Giperbolik paraboloidning uchi (8.10) sistemadan aniqlanadi.

Agar giperbolik paraboloid uchun $\lambda = -\lambda_2$ tenglik o‘rinli bo‘lsa, tegishli tenglama ushbu $x^2 - y^2 = 2pZ$ ko‘rinishni qabul qiladi.
Bu holda paraboloidning $O'XZ$, $O'YZ$ tekisliklar bilan kesimida hosil qilingan parabolalar bir xil parametrga ega. Bunda parabola o‘qining yo‘nalishi $\{A_1, A_2, A_3\}$ vektor orqali aniqlanadi.

III. 8^0. Elliptik silindr. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a\neq b$ bo‘lganda, elliptik silindrning joylashishini aniqlash uchun uning o‘qini va silindr o‘qiga perpendikulyar kesimidagi katta va kichik o‘qlarining yo‘naltiruvchi vektorlarini bilish kerak.

Silindr o‘qi (8.9) tenglamalar yordamida topiladi (ulardan chiziqli erklilarini olish kerak). λ_1, λ_2 sonlar xarakteristik tenglamaning noldan farqli ildizlari va $|\lambda_1| < |\lambda_2|$ bo‘lsin.

U holda $O'X$ o‘qi (silindr o‘qiga perpendikulyar kesimida hosil bo‘lgan katta o‘qi) yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan topiladi, bunda $\lambda = \lambda_1$; $O'Y$ o‘qi yo‘naltiruvchi vektorining koordinatalari (8.8) sistemadan aniqlanib, bunda $\lambda = \lambda_2$ farazda $\lambda_1 = \lambda_2$

$$x^2 + y^2 = a^2$$

silindr hosil qilinadi va uning joylashishini aniqlash uchun o‘qini bilish yetarli.

9^0. Giperbolik silindr.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

Giperbolik silindrning joylashishini bilish uchun uning o‘qini va o‘qiga perpendikulyar kesimining haqiqiy va mavhum o‘qlarining yo‘naltiruvchi vektorlarini bilish kerak. $\lambda_1,\ \lambda_2$ sonlar xarakteristik tenglamaning noldan farqli ildizlari, va λ_1 deb ishorasi $\frac{K_3}{I_2}$ ishorasiga teskari bo‘lgan ildiz belgilangan. U holda $O'X$ o‘qni yo‘naltiruvchi vektorlarini koordinatalari (silindrni o‘qqa perpendikulyar kesimini haqiqiy o‘qi) (8.8) tenglamalardan ($\lambda = \lambda_1$ holda) topiladi. $O'Y$ o‘qni (mavhum o‘qni) yo‘naltiruvchi vektorini koordinatalari esa $\lambda = \lambda_2$ holda (8.8) tenglamalardan topiladi.

1-misol. Koordinatalarning to‘g‘ri burchakli sistemasiga nisbatan

$$x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y + 2z = 0$$

tenglama bilan berilgan sirt ko‘rinishi va uning joylashishi aniqlansin.

Yechish.

$I_3 = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{vmatrix} = -36 \neq 0,$ sirt yagona simmetriya markazga ega. So‘ngra

$$K_4 = \begin{vmatrix} 1 & 1 & 3 & -1 \\ 1 & 5 & 1 & 3 \\ 3 & 1 & 1 & 1 \\ -1 & 3 & 1 & 0 \end{vmatrix} = 36 > 0: \quad I_1 = 1 + 5 + 1 = 7; \; I_1 I_3 < 0$$

ekanidan, berilgan sirt bir pallali giperboloidligi kelib chiqadi. $I_2 -$ ni topamiz:

$$I_2 = \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} = 0.$$

Xarakteristik tenglamani tuzamiz va yechamiz:

$$\lambda^3 - 7\lambda^2 + 36 = 0; \quad \lambda_1 = 3, \;\; \lambda_2 = 6, \quad \lambda_3 = -2.$$

Sodda tenglamasi

$$3x^2 + 6y^2 - 2z^2 + \frac{36}{-36} = 0 \; yoki \; 3x^2 + 6y^2 - 2z^2 - 1 = 0 \; yoki$$

$$\frac{x^2}{\left(\frac{1}{\sqrt{3}}\right)^2} + \frac{y^2}{\left(\frac{1}{\sqrt{6}}\right)^2} - \frac{z^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

ko‘rinishga ega ekan, bu yerda $a = \frac{1}{\sqrt{3}}, \; b = \frac{1}{\sqrt{6}}, \; c = \frac{1}{\sqrt{2}}$.

Sirt markazini

$$\begin{cases} x + y + 3z - 1 = 0 \\ x + 5y + z + 3 = 0 \\ 3x + y + z + 1 = 0 \end{cases}$$

sistemani yechib topamiz, bundan $C\left(-\frac{1}{3}; -\frac{2}{3}; \frac{2}{3}\right)$.

2-misol. To‘g‘ri burchakli koordinatalar tenglamalar sistemasiga nisbatan

$$5x^2 + 2y^2 + 5z^2 - 4xy - 2xz - 4yz + 10x - 4y - 2z + 4 = 0$$

tenglama bilan berilgan sirtning ko‘rinishi va joylashishi aniqlansin.

Yechish.

$$I_3 = \begin{vmatrix} 5 & -2 & -1 \\ -2 & 2 & -2 \\ -1 & -2 & 5 \end{vmatrix} = 0, \quad K_4 = \begin{vmatrix} 5 & -2 & -1 & 5 \\ -2 & 2 & -2 & -2 \\ -1 & -2 & 5 & -1 \\ 5 & -2 & -1 & 4 \end{vmatrix} = 0,$$

$$I_2 = \begin{vmatrix} 5 & -2 \\ -2 & 2 \end{vmatrix} + \begin{vmatrix} 5 & -1 \\ -1 & 5 \end{vmatrix} + \begin{vmatrix} 2 & -2 \\ -2 & 5 \end{vmatrix} = 36,$$

$$K_3 = \begin{vmatrix} 5 & -2 & 5 \\ -2 & 2 & -2 \\ 5 & -2 & 4 \end{vmatrix} + \begin{vmatrix} 5 & -1 & 5 \\ -1 & 5 & -1 \\ 5 & -1 & 4 \end{vmatrix} + \begin{vmatrix} 2 & -2 & -2 \\ -2 & 5 & -1 \\ -2 & -1 & 4 \end{vmatrix} = 36,$$

$$I_1 = 5 + 2 + 5 = 12. \quad I_3 = K_4 = 0, \quad I_2 > 0, \quad I_1 K_3 < 0$$

bo‘lgani uchun berilgan tenglama elliptik silindrni aniqlaydi. Xarakteristik $\lambda^3 - 12\lambda^2 + 36\lambda = 0$ tenglama ildizlari: $\lambda_1 = \lambda_2 = 6$, $\lambda_3 = 0$. Sodda tenglamasi $6x^2 + 6y^2 - \frac{36}{36} = 0$ yoki $x^2 + y^2 = \frac{1}{6}$ ko‘rinishga ega. Bu tenglama radiusi $\frac{1}{\sqrt{6}}$ ga teng aylanma silindrni aniqlaydi. Silindrning o‘qi ushbu

$$\begin{cases} 5x - 2y - z + 5 = 0 \\ -2x + 2y - 2z - 2 = 0 \\ -x - 2y + 5z - 1 = 0 \end{cases}$$

tenglamalar sistemasidan topiladi, ammo bu sistemadagi ikkita tenglamani olish kifoya.

Mustaqil yechish uchun topshiriqlar.

8.1.1 - 8.1.2 mashqlarni Lagranj usulidan foydalanib, quyidagi tenglamalar ikkita tekislikka ajraluvchi sirtni aniqlashini isbotlang, va bu tekisliklarni toping.

8.1.1. 1) $y^2 + 2xy + 4xz + 2yz - 4x - 2y = 0$;
2) $x^2 + 4y^2 + 9z^2 - 4xy + 6xz - 12yz - x + 2y - 3z - 6 = 0$;
3) $3x^2 - 4y^2 + 3z^2 + 4xy + 10xz - 4yz + 6x - 20y - 14z - 24 = 0$.

8.1.2. 1) $5x^2 + 4y^2 + 3z^2 + 9xy + 8xz + 7yz + 7x + 6y + 5z + 2 = 0$;
2) $4x^2 + 49y^2 + z^2 - 28xy + 4xz - 14yz + 8x - 28y + 4z + 3 = 0$;
3) $16x^2 + 9y^2 + 100z^2 + 24xy + 80xz + 60yz + 56x + 42y + 140z + 49 = 0$.

8.1.3 - 8.1.6 mashqlarni Lagranj usulidan foydalanib, tenglamalarni kvadratlar yig‘indisi shakliga keltirib, quyidagi sirtlarning ko‘rinishi aniqlansin:

8.1.3. 1) $4x^2 + 6y^2 + 4z^2 + 4xz - 8y - 4z + 3 = 0$;
2) $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y - 10z = 0$;
3) $x^2 + y^2 - 3z^2 - 2xy - 6xz - 6yz + 2x + 2y + 4z = 0$.

8.1.4. 1) $x^2 - 2y^2 + z^2 + 4xy - 8xz - 4yz - 14x - 4y + 14z +$
$+16 = 0$;
2) $2x^2 + y^2 + 2z^2 - 2xy - 2yz + x - 4y - 3z + 2 = 0$;
3) $x^2 - 2y^2 + z^2 + 4xy - 10xz + 4yz + x + y - z = 0$.
8.1.5. 1) $2x^2 + y^2 + 2z^2 - 2xy - 2yz + 4x - 2y = 0$;
2) $x^2 - 2y^2 + z^2 + 4xy - 10xz + 4yz + 2x + 4y - 10z - 1 = 0$;
3) $x^2 + y^2 + 4z^2 + 2xy + 4xz + 4yz - 6z + 1 = 0$.
8.1.6. 1) $4xy + 2x + 4y - 6z - 3 = 0$;
2) $xy + xz + yz + 2x + 2y - 2z = 0$.
8.1.7 - 8.1.12 mashqlarni parallel ko‘chirish va burish almashtirishlari yoki hadlarni gruppalash yordamida quyidagi sirtlarning ko‘rinishi va joylashishi aniqlansin.
8.1.7. 1) $z = 2x^2 - 4y^2 - 6x + 8y + 1$;
2) $z = x^2 + 3y^2 - 6y + 1$;
3) $x^2 + 2y^2 - 3z^2 + 2x + 4y - 6z = 0$.
8.1.8. 1) $x^2 + 2xy + y^2 - z^2 = 0$;
2) $z^2 = 3x + 4y + 5$;
3) $z = x^2 + 2xy + y^2 + 1$.
8.1.9. 1) $z^2 = x^2 + 2xy + y^2 + 1$;
2) $x^2 + 4y^2 + 9z^2 - 6x + 8y - 18z - 14 = 0$;
3) $2xy + z^2 - 2z + 1 = 0$.
8.1.10. 1) $x^2 + y^2 - z^2 - 2xy + 2z - 1 = 0$;
2) $x^2 + 4y^2 - z^2 - 10x - 16y + 6z + 16 = 0$;
3) $2xy + 2x + 2y + 2z - 1 = 0$.
8.1.11. 1) $3x^2 + 6x - 8y + 6z - 7 = 0$;
2) $x^2 + y^2 + 2z^2 + 2xy + 4z = 0$;
3) $3x^2 + 3y^2 + 3z^2 - 6x + 4y - 1 = 0$.
8.1.12. 1) $3x^2 + 3y^2 - 6x + 4y - 1 = 0$;
2) $3x^2 + 3y^2 - 3z^2 - 6x + 4y + 4z + 3 = 0$;
3) $4x^2 - y^2 - 4x + 4y - 3 = 0$.
8.1.13 - 8.1.24 mashqlardagi sirtlarning kanonik tenglamasi va joylashishini aniqlansin.
8.1.13.$x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y + 2z = 0$.
8.1.14. $2x^2 + y^2 + 2z^2 - 2xy + 2yz + 4x - 2y = 0$.
8.1.15. $x^2 + y^2 + 4z^2 + 2xy + 4xz + 4yz - 6z + 1 = 0$.
8.1.16. $4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4zx + 4x - 6y + 2z - 5 =$
$= 0$.

8.1.17. $7x^2 + 6y^2 + 5z^2 - 4xy - 4yz - 6x - 24y + 18z + 30 = 0.$
8.1.18. $2x^2 + 2y^2 - 5z^2 + 2xy - 2x - 4y - 4z + 2 = 0.$
8.1.19. $x^2 - 2y^2 + z^2 + 4xy - 8xz - 4yz - 14x - 4y + 14z +$
$+16 = 0.$
8.1.20. $2x^2 + 2y^2 + 3z^2 + 4xy + 2xz + 2yz - 4x + 6y - 2z + 3 =$
$= 0.$
8.1.21. $2x^2 + 5y^2 + 2z^2 - 2xy + 2yz - 4xz + 2x - 10y - 2z -$
$-1 = 0.$
8.1.22. $x^2 + 5y^2 + z^2 + 2xy + 6xz + 2yz - 2x + 6y + 2z = 0.$
8.1.23. $5x^2 + 2y^2 + 5z^2 - 4xy - 2xz - 4yz + 10x - 4y - 2z +$
$+4 = 0.$
8.1.24. $x^2 - 2y^2 + z^2 + 4xy - 10xz + 4yz + 2x + 4y - 10z - 1 =$
$= 0.$
8.1.25. $5x^2 - y^2 + z^2 + 4xy + 6xz + 2x + 4y + 6z - 8 = 0.$
8.1.26. $2x^2 + 10y^2 - 2z^2 + 12xy + 8yz + 12x + 4y + 8z - 1 = 0.$
8.1.27. Ikkinchi tartibli sirtning tenglamasi elliptik silindrni aniqlaydi. Uning
1) ozod hadini o‘zgartirsak;
2) birinchi darajali koordinatalari oldidagi koeffitsiyentlarini o‘zgartirsak sirtda qanday o‘zgarish bo‘ladi?
8.1.28. Yuqoridagi masala savollarini ikkinchi sirtning umumiy tenglamasi parabolik silindrni aniqlashini bilgan holda yeching.
8.1.29. λ va μ parametrni qanday qiymatlarida

$$x^2 - y^2 + 3z^2 + (\lambda x + \mu y)^2 - 1 = 0$$

tenglama doiraviy silindrni aniqlaydi?
8.1.30. $a(x^2 + 2yz) + b(y^2 + 2xz) + c(z^2 + 2xy) = 1$ tenglama bilan berilgan sirt aylanma sirt bo‘lishi uchun qanday shart bajarilishi kerak?

9-MAVZU: AFFIN VA ORTOGONAL ALMASHTIRISHLAR, XOSSALARI. IZOMETRIK ALMASHTIRISHLAR. HARAKAT.

Reja:

1. $\boldsymbol{n}$ ***o'lchamli vektorli yevklid fazosi.***
2. Affin almashtirishlar.
3. Harakat.

Tayanch iboralar: invariant, parallel ko'chirish, ortogonal, ekvivalentlik, izomorfizm, ortonormallangan bazis, reper, ikkinchi tur harakat, harakatlar gruppasi, kongruent.

9.1. $\boldsymbol{n}$ o'lchamli vektorli yevklid fazosi.

Ta'rif. *V_n vektor fazoning ixtiyoriy ikki $\vec{a}, \vec{b}$ vektoriga ularning skalyar ko'paytmasi deb atalgan haqiqiy son mos qo'yilgan bo'lib (vektor ko'paytmani $\vec{a} \cdot \vec{b}$ bilan belgilaymiz), quyidagi to'rtta aksioma bajarilsa, bunday fazo n o'lchamli vektorli yevklid fazosi deb ataladi va V_E kabi belgilanadi:*

$G_1)$ $\forall \vec{a}, \vec{b} \in V_n$ uchun $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$,
$G_2)$ $\forall \vec{a}, \vec{b}, \vec{c} \in V_n$ uchun $(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}$,
$G_3)$ $\forall \vec{a}, \vec{b} \in V_n$ va $\forall k \in \mathbb{R}$ uchun $k\vec{a} \cdot \vec{b} = k(\vec{a}\vec{b})$,
$G_4)$ $\forall \vec{a} \neq \bar{0} \in V_n$ uchun $\vec{a} \cdot \vec{a} > 0$.

Bu aksiomalar odatda vektorning ***skalyar ko'paytirish aksiomalari*** deb yuritiladi.

Yuqorida berilgan aksiomalardan kelib chiqadigan ba'zi natijalarini ko'ramiz.

1-natija. G_2 aksiomadagi komutativlik, assotsiativlik qonuni ikki qo'shiluvchi vektor uchun o'rinli bo'lsa, u istalgan $m \in \mathbb{N}$ sondagi qo'shiluvchlar uchun o'rinlidir, $(\vec{a}_1 + \vec{a}_2 + \ldots + \vec{a}_m) \cdot \vec{b} = \vec{a}_1\vec{b} + \vec{a}_2\vec{b} + \ldots + \vec{a}_m\vec{b}$ (ifodadagi barcha vektorlar V_E ga tegishli).

Haqiqatdan ham $\vec{a}_2 + \vec{a}_3 + \ldots + \vec{a}_m = \vec{b}_1$ desak, G_2 ga asosan $(\vec{a}_1 + \vec{b}_1)\vec{b} = \vec{a}_1\vec{b} + \vec{b}_1\vec{b}$, bu ifodaning ikkinchisi qo'shiluvchisidagi $\vec{b}_1$ ni $\vec{a}_2 + \vec{b}_2$ deb olsak, bunda $\vec{b}_2 = \vec{a}_3 + \vec{a}_1 + \ldots + \vec{a}_m$, u holda G_2 ni yana tadbiq qilsak, $\vec{a}_1\vec{b} + \vec{b}_1\vec{b} = \vec{a}_1\vec{b} + (\vec{a}_2 + \vec{b}_2)\vec{b} = \vec{a}_1\vec{b} + \vec{a}_2\vec{b} + \vec{b}_2\vec{b}$; endi shu ishni uchinchi qo'shiluvchi uchun takrorlaymiz va h.k.

$\vec{a}_1 + \vec{a}_2 + ... + \vec{a}_m$ ning soni chekli bo‘lgani uchun ma’lum qadamdan so‘ng izlangan tenglik hosil bo‘ladi.

2- natija. $\vec{0}$ vektorning har qanday vektor bilan skalyar ko‘paytmasi nolga tengdir, chunki G_3 ga asosan

$$(\vec{0} \cdot \vec{b}) = (0\vec{b} \cdot \vec{a}) = 0(\vec{b}\,\vec{a}) = 0.$$

3- natija. $\vec{a} \cdot \vec{a}$ skalyar ko‘paytma faqat $\vec{a} = 0$ bo‘lgandagina nolga tengdir, bu bevosita G_4 aksioma va 2 – natijadan kelib chiqadi.

Ta’rif. *V_E dagi $\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n$ bazis vektorlarining har biri birlik vektor bo‘lib, ularning istalgan ikkitasi o‘zaro ortogonal bo‘lsa, bunday vektorlar sistemasi ortonormallangan bazis (yoki dekart bazisi) deb ataladi, uni ham odatdagidek $\mathcal{B} = (\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$ deb belgilaymiz.*

Demak, ortonormallangan bazis uchun

$$\vec{e}_i\vec{e}_j = \begin{cases} 1, & agar\ i = j\ bo'lsa, \\ 0, & agar\ i \neq j\ bo'lsa, \end{cases} \tag{9.1}$$

bunda $i,\ j = 1,2, \dots, n$.

Endi ortonormal bazisda koordinatalari bilan berilgan ikki vektorning skalyar ko‘paytmasi, vektorning uzunligi, ikki vektor orasidagi burchakni hisoblash formulalarini topamiz.

Faraz qilaylik, dekart bazisida

$$\vec{a}\,(x_1, x_2, \dots, x_n) = x_1\vec{e}_1 + x_2\vec{e}_2 + ... + x_n\vec{e}_n,$$
$$\vec{b}\,(y_1, y_2, \dots, y_n) = y_1\vec{e}_1 + y_2\vec{e}_2 + ... + y_n\vec{e}_n.$$

bo‘lsin. U holda skalyar ko‘paytmasining xossalarini va (9.1) tenglikdan foydalanib,

$$\vec{a} \cdot \vec{b} = (x_1\vec{e}_1 + x_2\vec{e}_2 + ... + x_n\vec{e}_n)(y_1\vec{e}_1 + y_2\vec{e}_2 + ... + y_n\vec{e}_n) =$$
$$= x_1y_1(\vec{e}_1\vec{e}_1) + x_1y_2(\vec{e}_1\vec{e}_2) + ... + x_1y_n(\vec{e}_1\vec{e}_n) + x_2y_1(\vec{e}_2\vec{e}_1) +$$
$$+ x_2y_2(\vec{e}_2\vec{e}_2) + ... + x_2y_n(\vec{e}_2\vec{e}_n) + ... + x_ny_1(\vec{e}_n\vec{e}_1) + x_ny_2(\vec{e}_n\vec{e}_2) +$$
$$+ \ldots + x_ny_n(\vec{e}_n\vec{e}_n) = x_1y_1 + x_2y_2 + \ldots + x_ny_n,$$

ya’ni

$$\vec{a} \cdot \vec{b} = x_1y_1 + x_2y_2 + \ldots + x_ny_n \tag{9.2}$$

tenglikni hosil qilamiz.

Demak, V_E da ikki vektorning skalyar ko‘paytmasi shu vektorlar mos koordinatalari ko‘paytmalarining yig‘indisiga teng.

$$\vec{a} = \vec{b} \Rightarrow x_1 = y_1, x_2 = y_2, \dots, x_n = y_n \Rightarrow$$
$$\vec{a}\,\vec{a} = x_1^2 + x_2^2 + \ldots + x_n^2$$

yoki

$$\sqrt{\vec{a}\,\vec{a}} = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2},$$

vektor uzunligining ta’rifiga ko‘ra

$$|\vec{a}| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}. \qquad (9.3)$$

Demak, vektorning uzunligi va uning koordinatalari yig‘indisidan olingan arifmetik kvadrat ildizga teng.

(9.2) va (9.3) tengliklardan foydalanib, ikki vektor orasidagi burchakni hisoblash formulasini topamiz:

$$cos\varphi = \frac{x_1y_1 + x_2y_2 + \ldots + x_ny_n}{\sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}\,\sqrt{y_1^2 + y_2^2 + \ldots + y_n^2}} \qquad (9.4)$$

Ta’rif. *$\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ vektorlar sistemasida ixtiyoriy ikki vektor o‘zaro ortogonal bo‘lsa, bungay vektorlar sistemasi* ***ortogonal sistema*** *deb ataladi.*

1-misol. $\vec{a}(1;3;2;-1)$, $\vec{b}(5;1;-4;0)$, $\vec{c}(0;4;1;14)$ vektorlarning ortogonal sistemani hosil qilishini isbotlang.

Yechish. Yuqorida berilgan ta’rifdan foydalanib, $\vec{a}\vec{b}$, $\vec{a}\vec{c}$ va $\vec{b}\vec{c}$ larning skalyar ko‘paytmalarini hisoblaymiz:

$$\vec{a}\,\vec{b} = 1\cdot 5 + 3\cdot 1 + 2\cdot(-4) + (-1)\cdot 0 = 8 - 8 = 0,$$
$$\vec{a}\,\vec{c} = 1\cdot 0 + 3\cdot 4 + 2\cdot 1 + (-1)\cdot 14 = 14 - 14 = 0,$$
$$\vec{b}\,\vec{c} = 5\cdot 0 + 1\cdot 4 + (-4)\cdot 1 + 0\cdot 14 = 4 - 4 = 0.$$

Demak, $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar ortogonal sistema hosil qiladi.

9.2. Affin almashtirishlar.

Endi n o‘lchamli affin fazodagi almashtirishlar bilan tanishamiz. A_n vektor fazoda ikki $\mathcal{B} = (O, \vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n)$ va $\mathcal{B}' = \left(O', \overrightarrow{e_1'}, \overrightarrow{e_2'}, \ldots, \overrightarrow{e_n'}\right)$ reper berilgan bo‘lsin. Bu reperlar yordamida A_n ning nuqtalari orasida shunday f moslik o‘rnatamizki, ixtiyoriy $M \in A_n$ nuqta $\mathcal{B}$ reperda qanday koordinatalarga ega bo‘lsa, uning obrazi $M' = f(M)$ nuqta $\mathcal{B}'$ reperda xuddi shunday koordinatalarga ega bo‘lsin, ravshanki, bu moslik o‘zaro bir qiymatli bo‘lib, A_n ni o‘zini – o‘ziga o‘tkazadi. Demak, f biror almashtirishdir.

Ta’rif. *Yuqoridagicha aniqlangan f almashtirish A_n ni affin almashtirish deb ataladi.*

Bu ta’rifdan ko‘rinadiki, affin almashtirish bir juft affin reperlarning berilishi bilan to‘la aniqlanadi.

Endi affin almashtirishning ba'zi xossalari bilan tanishamiz.

1^0. f affin almashtirishda $\vec{a} \in A_n$ vektor shu fazoning biror $f(\vec{a}) = \overrightarrow{a'}$ yoki $\vec{b}$ vektorga teng vektoriga almashadi, $\vec{a} = \overrightarrow{MN}$ desak, M, N nuqtalarning obrazlari $f(M) = M'$, $f(N) = N'$ bo'lib, bu nuqtalar ham A_n ga tegishli bo'lgani uchun ularga mos kelgan $\overrightarrow{a'}$ vektor $f(\vec{a})$ bo'ladi. Xususiy holda nol vektor yana nol vektorga almashadi.

2^0. f affin almashtirishda $\vec{a}$ vektorning koordinatalari $\mathcal{B}$ da qanday bo'lsa, unga mos kelgan $\overrightarrow{a'}$ vektorning ham koordinatalari $\mathcal{B}'$ da xuddi shu sonlardan iborat bo'ladi. Bu xossa f ning ta'rifi va 1^0 dan bevosita kelib chiqadi.

3^0. f affin almashtirishda ikki vektorning yig'indisiga mos kelgan vektor qo'shiluvchi vektorlarga mos kelgan vektorlar yig'indisidan iborat, ya'ni $\vec{a} + \vec{b} = \vec{c} \Rightarrow f(\vec{c}) = f(\vec{a}) + f(\vec{b})$.

Bu xossaning isboti koordinatalari bilan berilgan vektorlarni qo'shish qoidasi va f ning ta'rifidan kelib chiqadi.

4^0. $k\vec{a}$ vektorga mos kelgan vektor $kf(\vec{a}) = k\overrightarrow{a'}$ vektordir.

Bu ikki $3^0, 4^0 -$ xossadan f almashtirishda $\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \ldots + \lambda_k\vec{a}_k$ vektorga $\lambda_1\overrightarrow{a_1'} + \lambda_2\overrightarrow{a_2'} + \ldots + \lambda_k\overrightarrow{a_k'}$ vektorning mos kelishi kelib chiqadi, ya'ni f almashtirish natijasida vektorlarning chiziqli kombinatsiyasi saqlanadi. Demak, chiziqli erkli vektorga yana chiziqli erkli vektorlar mos keladi. Bu xossalarni va ikki affin fazoning izomorfligi ta'rifini e'tiborga olsak, affin almashtirishning quyidagi ikkinchi ta'rifi kelib chiqadi.

Ta'rif. *A_n fazoni o'zini – o'ziga izomorf akslantiruvchi f* almashtirish *A_n dagi affin almashtirish deb ataladi.*

Ta'rif. *P nuqta MN kesmani λ nisbatda bo'lsa (ya'ni $\overrightarrow{MP} = \lambda\overrightarrow{PM}$ bo'lsa), u holda λ son M, P, N nuqtalarning oddiy nisbati deb atalib, uni odatdagidek $\lambda = (MN, P)$ ko'rinishda belgilanadi.*

5^0. f almashtirishda k o'lchovli Π_k tekislik yana k o'lchovli Π_k tekislikka almashadi, ya'ni tekislikning o'lchovi f uchun invariantdir.

6^0. f affin almashtirishda parallel tekisliklar yana parallel tekisliklarga o'tadi.

Bu xossa affin almashtirishning o'zaro bir qiymatli ekanligidan kelib chiqadi. Endi affin almashtirishning koordinatalaridagi ifodasini ko'ramiz.

A_n da $\mathcal{B} = (O, \vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$ reper berilgan bo‘lsin. $\forall M \in E_n$ ni olib, uning shu reperdagi koordinatalarini $x_1, x_2, \dots, x_n$, $f(M) = M'$ nuqtaning ham shu $\mathcal{B}$ reperga nisbatan koordinatalarini $x_1', x_2', \dots, x_n'$ ni bog‘lovchi munosabatlarni topamiz.

Faraz qilaylik, $f(O) = O'$, $f(\vec{e}_1) = \overrightarrow{e_1'}, \dots, f(\vec{e}_n) = \overrightarrow{e_n'}$ bo‘lib, bularning $\mathcal{B}$ ga nisbatan koordinatalari $O'(c_1, c_2, \dots, c_n)$, $\overrightarrow{e_1'}(c_{11}, c_{12}, \dots, c_{1n})$, $\overrightarrow{e_2'}(c_{21}, c_{22}, \dots, c_{2n})$, $\overrightarrow{e_n'}(c_{n1}, c_{n2}, \dots, c_{nn})$ bo‘lsin. U holda $\mathcal{B}' = \left(O', \overrightarrow{e_1'}, \overrightarrow{e_2'}, \dots, \overrightarrow{e_n'}\right)$ affin reper hosil bo‘lib, $f(\mathcal{B}) = \mathcal{B}'$ bo‘ladi. Xususiy holda $f\left(\overrightarrow{OM}\right) = \overrightarrow{O'M'}$. U holda M nuqtaning koordinatalari affin almashtirishning ta’rifiga ko‘ra $\mathcal{B}'$ reperga nisbatan $x_1, x_2, \dots, x_n$ bo‘lib,

$$\begin{aligned} x_1' &= c_{11}x_1 + c_{21}x_2 + \dots + c_{n1}x_n + c_1, \\ x_2' &= c_{12}x_1 + c_{22}x_2 + \dots + c_{n2}x_n + c_2, \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_n' &= c_{1n}x_1 + c_{2n}x_2 + \dots + c_{nn}x_n + c_n \end{aligned} \tag{9.5}$$

o‘rinli bo‘ladi. Bu (9.5) formulalar affin almashtirishning koordinatalardagi ifodasidir.

2-Misol. $M(1; 4; -5; 3; 2)$ nuqtadan $\Pi_4 = 3x_1 + x_2 + 2x_3 - -x_4 + x_5 - 3 = 0$ gipertekislikkacha bo‘lgan masofani toping.

Yechish. $\rho(M_0\Pi_{n-1}) = \frac{|a_1x_1^0 + a_2x_2^0 + \dots + a_nx_n^0 + a_0|}{\sqrt{a_1^2 + a_2^2 + \dots\dots + a_n^2}}$ formulaga asosan

$$\rho(M_0\Pi_4) = \frac{|3 \cdot 1 - 1 \cdot 4 + 2 \cdot (-5) + (-1) \cdot 3 + 2 \cdot 1 - 3|}{\sqrt{3^2 + 1^2 + 2^2 + (-1)^2 + 1^2}} =$$

$$= \frac{|-15|}{\sqrt{16}} = \frac{15}{4} = 3\frac{3}{4}.$$

9.3. Harakat.

E_n da ikkita $\mathcal{B} = (O, \vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$, $\mathcal{B}' = \left(O', \overrightarrow{e_1'}, \overrightarrow{e_2'}, \dots, \overrightarrow{e_n'}\right)$ dekart reperi berilgan bo‘lsin. E_n ning har bir M nuqtasini shu fazoning shunday M' nuqtasiga akslantiramizki, $\mathcal{B}$ reperda M nuqta qanday koordinatalarga ega bo‘lsa, $\mathcal{B}'$ reperda M' nuqta shunday koordinatalarga ega bo‘lsin. Bu yerda E_n nuqtalari yana shu fazo nuqtalariga mos qo‘yilib, bunday moslik o‘zaro bir qiymatlidir. Demak, E_n da almashtirish hosil qilindi, u E_n ning harakati deb ataladi. E_n dagi harakat ikkita dekart reperining berilishi bilan to‘liq aniqlanadi. Bu ta’rifni affin almashtirishning ta’rifi bilan taqqoslasak, harakat affin almashtirishning xususiy holi ekanligi ayon bo‘ladi. Shu sababli figuraning barcha affin xossalari harakatda saqlanib qoladi.

Harakat quyidagi xossalarga ega. Harakatda ikki nuqta orasidagi masofa saqlanadi. Haqiqatan, $\mathcal{B}$ reperdagi $M(x_1, x_2, \dots, x_n)$, $N(y_1, y_2, \dots, y_n)$ nuqtalarga harakat natijasida $\mathcal{B}'$ reperda mos kelgan M', N' nuqtalar ta’rifga asosan xuddi shunday koordinatalarga ega, ya’ni $M'(x_1, x_2, \dots, x_n)$, $N'(y_1, y_2, \dots, y_n)$ u holda

$$\rho(M, N) = \rho(M', N').$$

Masofa harakatning asosiy invariant hisoblanib, ba’zan harakat shu invariant orqali ta’riflanadi.

Teorema. *E_n ning biror f almashtirishida ikki nuqtasi orasidagi masofa saqlansa, bu almashtirish harakatdir.*

E_n da ixtiyoriy uchta O, A, B nuqtani olsak, yuqoridagi berilgan aksiomalarga asosan

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} \qquad (9.6)$$

yoki

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}.$$

Bu tenglikning chap va o‘ng tomonida turgan vektorlarni o‘zini – o‘ziga skalyar ko‘paytiraylik:

$$\overrightarrow{AB} \cdot \overrightarrow{AB} = (\overrightarrow{OB} - \overrightarrow{OA})(\overrightarrow{OB} - \overrightarrow{OA}) \Longrightarrow$$

$$\overrightarrow{AB^2} = \overrightarrow{OB^2} - 2(\overrightarrow{OB} \cdot \overrightarrow{OA}) + \overrightarrow{OA^2} \Longrightarrow$$

$$2(\overrightarrow{OA} \cdot \overrightarrow{OB}) = \overrightarrow{OA^2} + \overrightarrow{OB^2} - \overrightarrow{AB^2}. \qquad (9.7)$$

$f(O) = O'$, $f(A) = A'$, $f(B) = B'$ bo‘lsin, u holda O', A', B' nuqtalar uchun ham aksiomani tadbiq qilib va (9.6) singari tenglik yozib, tegishlicha ixchamlasak,

$$2\left(\overrightarrow{OB'} \cdot \overrightarrow{OA'}\right) = \overrightarrow{O'A'}^2 + \overrightarrow{O'B'}^2 - \overrightarrow{A'B'}^2 \qquad (9.8)$$

Lekin teorema shartiga ko‘ra $\rho(O, A) = \rho(O', A')$, $\rho(O, B) =$ $= \rho(O', B')$, $\rho(A, B) = \rho(A', B')$ bo‘lgani uchun (9.7) bilan (9.8) ning o‘ng tomonlarini taqqoslasak, ular o‘zaro tengdir, demak, chap tomonlari ham teng bo‘ladi:

$$\left(\overrightarrow{O'A'} \cdot \overrightarrow{O'B'}\right) = (\overrightarrow{OA} \cdot \overrightarrow{OB}). \qquad (9.9)$$

E_n da biror $\mathcal{B} = (O, \vec{e}_1, \vec{e}_2, \dots, \vec{e}_n)$ dekart reperini olaylik, u holda $\overrightarrow{OA_1} = \overrightarrow{e_1}$, $\overrightarrow{OA_2} = \overrightarrow{e_2}$, ..., $\overrightarrow{OA_n} = \overrightarrow{e_n}$ desak, $\mathcal{B}$ reperni quyidagicha yozish mumkin: $\mathcal{B} = (O, A_1, A_2, \dots, A_n)$. Shu reperni f bo‘yicha almashtirsak, $f(O) = O'$, $f(A_1) = A_1'$, ..., $(A_n) = A_n'$ bo‘lgani uchun bu nuqtalar sistemasi ham biror $\mathcal{B}' = (O', A_1', , \dots, A_n')$ reperni aniqlaydi. Bu reper ham dekart reperidan iboratdir, chunki:

1) almashtirishga asosan $\rho(O, A_i) = \rho(O', A_i')$, $(i = 1,2, \dots, n)$ ya’ni birlik vektor obrazi yana birlik vektordir;

2) (9.9) shartga asosan o‘zaro perpendikulyar vektorlar yana perpendikulyar vektorga o‘tadi.

E_n dagi ixtiyoriy M nuqtani olaylik, uning $\mathcal{B}$ dekart reperidagi koordinatalari $x_1, x_2, \dots, x_n$ bo‘lsin. M nuqtaga f almashtirishda mos kelgan M' nuqtaning shu reperdagi koordinatalari $y_1, y_2, \dots, y_n$ bo‘lsin. U holda

$$\overrightarrow{OM} \cdot \overrightarrow{OA_1} = \left|\overrightarrow{OM}\right| \cdot \left|\overrightarrow{OA_1}\right| cos\varphi = \left|\overrightarrow{OM}\right| \cdot \left|\overrightarrow{e_1}\right| cos\varphi =$$
$$= \left|\overrightarrow{OM}\right| cos\varphi = \left|\overrightarrow{OM_1}\right| = x_1$$

(bunda $\varphi = \left(\overrightarrow{OM}, ^\wedge \overrightarrow{OA_1}\right)$, $\overrightarrow{OM_1}$ vektor $\overrightarrow{OM}$ ning Ox o‘qdagi proyeksiyasi) bo‘lgani uchun

$$x_1 = \overrightarrow{OM} \cdot \overrightarrow{OA_1} \qquad (9.10)$$

shunga o‘xshash

$$\begin{cases} x_2 = \overrightarrow{OM} \cdot \overrightarrow{OA_2}, \\ x_3 = \overrightarrow{OM} \cdot \overrightarrow{OA_3}, \\ \dots \dots \dots \dots \dots \dots \dots \\ x_n = \overrightarrow{OM} \cdot \overrightarrow{OA_n}, \\ y_1 = \overrightarrow{O'M'} \cdot \overrightarrow{O'A_1'}, \\ y_2 = \overrightarrow{O'M'} \cdot \overrightarrow{O'A_2'}, \\ \dots \dots \dots \dots \dots \dots \dots \\ y_n = \overrightarrow{O'M'} \cdot \overrightarrow{O'A_n'} \end{cases} \qquad (9.11)$$

$$(9.9) - (9.11) \Rightarrow \begin{cases} x_1 = y_1, \\ x_2 = y_2, \\ \dots \dots \dots \\ x_n = y_n \end{cases} \Rightarrow f \text{ harakatdan iborat.}$$

Ortogonal matritsa deyiladi qachonki, uning determinant ± 1 ga teng, ya’ni (9.5) ning determinanti $\Delta = \pm 1$. Agar harakatning analitik ifodasida $\Delta = \pm 1$ bo‘lsa, bunday harakat, ***birinchi tur harakat***, deb ataladi, bu tur harakatda ikkita mos reper bir xil oreyentatsiyali bo‘ladi.

$\Delta = -1$ holda bunday harakat ***ikkinchi tur harakat*** deyilib, undagi mos reperlar har xil oriyentatsiyali.

E_n ning barcha harakatlari to‘plamini E bilan belgilaylik hamda $\forall f, g \in E$ ni olaylik; harakatda ikki nuqta orasidagi masofa o‘zgarmaganligi uchun ketma – ket bajarilgan ikki f, g harakat natijasida ham ikki nuqta orasidagi masofa o‘zgarmaydi, demak, $g \cdot f$ “ko‘paytma” harakat bo‘lib, E ga tegishlidir. f da ikki nuqta orasidagi masofa o‘zgarmagani uchun unga teskari f^{-1} da ham masofa o‘zgarmaydi, demak, $f^{-1} \in E$. Xullas, E_n ning barcha harakatlari to‘plami E gruppa hosil qiladi, u E_n ning ***harakatlar gruppasi*** deb ataladi. Harakat affin almashtirishning xususiy holi ekanligidan harakatlar gruppasi affin gruppaning qism gruppasi bo‘ladi. Demak, A ning barcha invariantlari E uchun ham invariant bo‘ladi, lekin buning teskarisi doimo to‘g‘ri bo‘lavermaydi; masalan, E ning invariantlaridan biri ikki nuqta orasidagi masofadir, bu esa A da invariant emas, shu nuqtai nazardan E_n dagi figura geometrik xossalar nuqtai nazardan A_n dagi figuraga nisbatan boyroqdir.

Endi Yevklid geometriyasiga quyidagicha ta’rif berish mumkin. Yevklid geometriyasi geometriyaning harakat natijasida figuraning o‘zgarmay qoladigan xossalarini o‘rganadigan bir bo‘limidir. O‘rta maktab geometriya kursida ikki va uch o‘lchovli (E_2, E_3) yevklid fazolari geometriyasi o‘rganiladi.

n o‘lchovli $(n > 3)$ yevklid geometriyasida ham o‘rta maktab geometriya kursida qaraladigan ba’zi tushunchalarni umumlashtirish mumkin. Masalan, kongruentlik tushunchasi E_n da quyidagicha kiritiladi: F, F' figuralardan birini ikkinchisiga o‘tkazuvchi harakat mavjud bo‘lsa, bu figuralar ***kongruent*** deb ataladi, yoki oddiy sferani umumlashtirib, E_n da gipersfera kiritiladi: E_n ning markaz deb atalgan C nuqtadan berilgan r masofada yotgan barcha nuqtalari to‘plami ***gipersfera*** deb ataladi.

Endi harakatlar gruppasining ba’zi qism gruppalari bilan tanishaylik.

1. I turdagi barcha harakatlar to‘plamini E_1 deb belgilasak, bu to‘plam gruppani hosil qiladi, chunki 1) E_1 ning har bir almashtirishida reper oriyentatsiyasi (demak, fazo oriyentatsiyasi) o‘zgarmaganligi uchun unga tegishli ikki harakatning kompozitsiyasi natijasida ham oriyentatsiya o‘zgarmaydi; 2) E_1 ning har bir harakatiga teskari harakat

ham oriyentatsiyani o‘zgartirmaydi, demak, E_1 ham E ning qism gruppasidir.

2. E_1 dagi barcha parallel ko‘chirishlar to‘plamini olaylik. Avvalo parallel ko‘chirishning harakat ekanligini isbotlaylik. M, N nuqtalar M', N' nuqtalarni $\vec{u}$ vektor bo‘yicha parallel ko‘chirishdan $\left(\overrightarrow{MM'} = \vec{u},\right.$ $\left.\overrightarrow{NN'} = \vec{u}\right)$ hosil qilingan bo‘lsa, $\left|\overrightarrow{MN}\right| = \left|\overrightarrow{M'N'}\right| \Longrightarrow \rho(M, N) =$ $= \rho(M', N')$. Demak, parallel ko‘chirish harakatdir. U holda bunday harakatlarning to‘plami ham E ning qismidir.

3. E ning shunday harakatlari to‘plamini qaraymizki, bu harakatlar natijasida E ning biror O nuqtasi o‘z – o‘ziga o‘tsin, bunday xossaga ega bo‘lgan harakatlarni E_n ni O nuqta atrofida ***burish*** deyiladi, bu to‘plamni E_0 deb belgilasak, E_0 ning gruppa hosil qilishini ko‘rsatish osondir (buni ko‘rsatishni o‘quvchiga havola qilamiz); demak, E_0 ham E ning qism gruppasidir.

3-misol. $E = R^3$ Yevklid fazosida $\overrightarrow{b_1}(1;0;0)$, $\overrightarrow{b_2}(1;1;0)$, $\overrightarrow{b_3}(1;1;1)$ vektorlar sistemasiga ortogonallashtirish jarayonini qo‘llang.

Yechish. Ma’lumki, R^n fazoda n ta vektordan iborat sistemaning chiziqli erkli bo‘lishi uchun bu vektorlarning koordinatalaridan tuzilgan determinantning noldan farqli bo‘lishi zarur va yetarlidir. Berilgan vektorlar uchun bu determinant

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1 \neq 0$$

bo‘lganligi sababli, ular chiziqli erklidir. Endi bu elementlarga ortogonallashtirish jarayonini qo‘llaymiz. $\overrightarrow{c_1} = \overrightarrow{b_1} = (1;0;0)$ deb olsak, $|\overrightarrow{c_1}| = \sqrt{1^2 + 0^2 + 0^2} = 1$ bo‘ladi. $\overrightarrow{c_2}$ elementni $\overrightarrow{c_2} = \overrightarrow{b_2} - a_{21}\overrightarrow{c_1}$ ko‘rinishda olib, a_{21} koeffitsiyentni $(\overrightarrow{c_2}, \overrightarrow{c_1}) = 0$ ortogonallik shartini qanoatlantiradigan qilib tanlaymiz:

$$0 = (\overrightarrow{c_2}, \overrightarrow{c_1}) = \left(\overrightarrow{b_2}, \overrightarrow{c_1}\right) - a_{21}(\overrightarrow{c_2}, \overrightarrow{c_1}) \quad \text{yoki}$$

$$a_{21} = \frac{\left(\overrightarrow{b_2}, \overrightarrow{c_1}\right)}{|\overrightarrow{c_1}|^2} = \frac{1}{1} = 1.$$

U holda

$$c_2 = (1;1;0) - (1;0;0) = (0;1;0), \quad |\overrightarrow{c_2}| = 1,$$

bo‘ladi. $\overrightarrow{c_3}$ vektorni quyidagi ko‘rinishda izlaymiz:

$$\overrightarrow{c_3} = \overrightarrow{b_3} - a_{31}\overrightarrow{c_1} - a_{32}\overrightarrow{c_2}. \qquad (9.12)$$

Bunda a_{31}, a_{32} koeffitsiyentlar, ortogonallik shartlaridan, ya'ni

$$(\overrightarrow{c_3}, \overrightarrow{c_1}) = (\overrightarrow{c_3}, \overrightarrow{c_2}) = 0 \qquad (9.13)$$

shartlardan topiladi. Buning uchun (9.12) ni $\overrightarrow{c_1}$ va $\overrightarrow{c_2}$ ga skalyar ko'paytirib, (9.13) shartlardan foydalansak, a_{31}, a_{32} koeffitsiyentlarga nisbatan chiziqli tenglamalar sistemasi hosil bo'ladi. Bu tenglamaning yechimi:

$$a_{31} = \frac{(\overrightarrow{b_3}, \overrightarrow{c_1})}{|\overrightarrow{c_1}|^2} = \frac{1\cdot 1 + 1\cdot 0 + 0\cdot 1}{1} = 1,$$

$$a_{32} = \frac{(\overrightarrow{b_3}, \overrightarrow{c_2})}{|\overrightarrow{c_2}|^2} = \frac{1}{1} = 1.$$

Demak, $\overrightarrow{c_3} = (1;1;1) - (1;0;0) - (0;1;0) = (0;0;1), \quad |\overrightarrow{c_3}| = 1.$ Hosil bo'lgan $\overrightarrow{c_1}$, $\overrightarrow{c_2}$, $\overrightarrow{c_3}$ vektorlar sistemasi ortonormaldir.

Mustaqil yechish uchun topshiriqlar.

9.1.1 – 9.1.9 - misollarda keltirilgan vektorlarning chiziqli erkliligini tekshiring va ortogonallashtirish jarayonini qo'llab, ortonormal sistema hosil qiling.

9.1.1. $E = \mathbb{R}^2$, $\vec{x}(0;1)$, $\vec{y}(1;0)$.

9.1.2. $E = \mathbb{R}^2$, $\vec{x}(1;0)$, $\vec{y}(0;1)$.

9.1.3. $E = \mathbb{R}^3$, $\vec{x}(0;0;1)$, $\vec{y}(0;1;1)$, $\vec{z}(1;1;1)$.

9.1.4. $E = \mathbb{R}^3$, $\vec{x}(1;1;1)$, $\vec{y}(0;1;1)$, $\vec{z}(0;0;1)$.

9.1.5. $E = \mathbb{R}^3$, $\vec{x}(1;1;0)$, $\vec{y}(2;0;-1)$, $\vec{z}(0;-1;1)$.

9.1.6. $E = \mathbb{R}^3$, $\vec{x}(0;2;-1)$, $\vec{y}(1;-1;1)$, $\vec{z}(1;0;0)$.

9.1.7. $E = \mathbb{R}^3$, $\vec{x}(-1;0;0)$, $\vec{y}(0;-1;1)$, $\vec{z}(2;0;-1)$.

9.1.8. $E = \mathbb{R}^4$, $\vec{x}(0;1;-1;1)$, $\vec{y}(1;-1;1;0)$, $\vec{z}(1;0;0;1)$.

9.1.9. $E = \mathbb{R}^4$, $\vec{x}(-1;0;0;1)$, $\vec{y}(0;-1;1;0)$, $\vec{z}(2;0;-1;1)$.

9.1.10. Chiziqli fazoda mos ravishda $\overrightarrow{a_1}(1;1;0;0)$, $\overrightarrow{a_2}(0;1;1;0)$, $\overrightarrow{a_3}(0;0;1;1)$ va $\overrightarrow{b_1}(1;0;1;0)$, $\overrightarrow{b_2}(0;2;1;1)$, $\overrightarrow{b_3}(1;2;1;2)$ bazislarga ega V_1 va V_2 qism fazolar yig'indisi va kesishmasining bazisini toping.

9.1.11. Chiziqli fazoda mos ravishda $\overrightarrow{a_1}(1;2;0;1)$, $\overrightarrow{a_1}(1;1;1;0)$ va $\overrightarrow{b_1}(1;0;1;0)$, $\overrightarrow{b_2}(1;3;0;1)$ bazislarga ega V_1 va V_2 qism fazolar yig'indisi va kesishmasining bazisini toping.

9.1.12. To'g'ri chiziq va gipertekislik mos ravishda $x_1 = 8\text{t}$, $x_2 = 4\text{t}$, $x_3 = 3\text{t}$, $x_4 = -3\text{t}$ va $2x_1 + 2x_2 - x_3 + x_4 = 0$ tenglamalar bilan berilgan. Berilgan $\vec{x}(1;2;3;4)$ vektorni to'g'ri chiziqqa tegishli $\vec{y}$ vektor va gipertekislikka tegishli $\vec{z}$ vektorlarning yig‘indisi кo‘rinishida ifodalang.

9.1.13. Tekislikning $(-1;1;0;1;5)$, $(2;-1;3;4;0)$, $(1;2;7;6;1)$ nuqtalardan o‘tishi ma’lum bo‘lsa, uning parametrik va umumiy tenglamalari tuzilsin.

9.1.14. Umumiy tenglamasi bilan berilgan

$$\begin{cases} 5x_1 + 6x_2 - 2x_3 + 7x_4 + 4x_5 - 3 = 0 \\ 2x_1 + 3x_2 - x_3 + 4x_4 + 2x_5 - 6 = 0 \end{cases}$$

tekislikning parametrik tenglamasini yozing.

9.1.15. Birinchi to‘g‘ri chiziq $(1;0;-2;1)$ nuqta va $\vec{a}(1;2;-1;-3)$ vektor bilan, ikkinchi tekislik esa $(0;1;1;-1)$ nuqta va $\vec{b}(2;3;-2;-4)$ vektor bilan aniqlangan bo‘lsa, ularni o‘z ichiga oluvchi eng kichik o‘lchamli tekislik tenglamasini yozing.

9.1.16. Ikkita $x_1 = 1 + \mathrm{t}$, $x_2 = 2 + \mathrm{t}$, $x_3 = 3 + \mathrm{t}$, $x_4 = 4 + \mathrm{t}$, $x_1 = 0$, $x_4 - 3 = 0$, $x_2 - x_3 + 1 = 0$ to‘g‘ri chiziq o‘z ichiga oluvchi eng kichik o‘lchamli tekislik tenglamasini yozing.

9.1.17. To‘rt o‘lchamli fazoda

$$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 - 3 = 0 \\ x_1 + 4x_2 + 5x_3 + 2x_4 - 2 = 0 \\ 2x_1 + 9x_2 + 8x_3 + 3x_4 - 7 = 0 \end{cases}$$

sistema bilan berilgan tekislik va $5x_1 + 7x_2 + 9x_3 + 2x_4 - 20 = 0$ to‘g‘ri chiziqning o‘zaro vaziyatini aniqlang.

9.1.18. To‘rt o‘lchamli fazoda $5x_1 + 9x_3 + 2x_4 - 20 = 0$, $x_2 = 0$ tekislik va

$$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 - 3 = 0 \\ x_1 + 4x_2 + 5x_3 + 2x_4 - 2 = 0 \\ 2x_1 + 9x_2 + 8x_3 + 3x_4 - 7 = 0 \end{cases}$$

to‘g‘ri chiziq berilgan. Ularning o‘zaro vaziyatini aniqlang.

9.1.19. To‘g‘ri chiziq va tekislik mos ravishda $x_1 = 1 + \mathrm{t}$, $x_2 = 2 + 2\mathrm{t}$, $x_3 = 3 + 3\mathrm{t}$, $x_4 = 4 + 4\mathrm{t}$ va $x_1 + x_2 + 1 = 0$, $x_3 - x_4 = 0$ tenglamalar bilan berilgan. Ularning kesishmasligini ko‘rsating va to‘g‘ri chiziqqa parallel bo‘lib berilgan tekislikdan o‘tuvchi eng kichik o‘lchamli tekislik tenglamasini yozing.

9.1.20. Ortonormal bazisga nisbatan uchta $\vec{a}(1;2;2;1)$, $\vec{b}(1;1;-5;3)$, $\vec{c}(3;2;8;-7)$ vektorlar berilgan. Berilgan vektorlarga tortilgan qism fazoning bazisini toping va uni fazoning bazisigacha to‘ldiring.

9.1.21. Besh o‘lchamli fazoda ortonormal bazisga nisbatan $x_1 - x_2 - -2x_3 + 4 = 0$ gipertekislik berilgan. Birinchi to‘rttasi berilgan gipertekislikda yotuvchi yangi bazisni toping.

9.1.22. Gipertekislik $2x_1 - 2x_2 - x_3 + x_4 = 0$ va $\vec{x}(2;0;4;6)$ vektor berilgan. $\vec{x}$ vektorni berilgan gipertekislikka tegishli $\vec{y}$ vektor va shu gipertekislikka orthogonal $\vec{z}$ vektorlarning yig‘indisi ko‘rinishida ifodalang.

9.1.23. Yevklid fazosi V da $\overrightarrow{x_1}, \overrightarrow{x_2}$ vektorlar, V ning qism fazosi V' da $\overrightarrow{y_1}, \overrightarrow{y_2}$ vektorlar va V' ga ortogonal bo‘lgan $\overrightarrow{z_1}, \overrightarrow{z_2}$ vektorlar berilgan. Agar, $\overrightarrow{x_2} - \overrightarrow{x_1}$ V' fazoga tegishli bo‘lsa, $\overrightarrow{z_1} = \overrightarrow{z_2}$ munosabat o‘rinli bo‘lishini isbotlang.

9.1.24. O‘zining $M(x_0, y_0)$ bazislari bilan berilgan qism fazoga $\vec{a}(4;-1;3;4)$ vektorning ortogonal proeksiyasini toping.

9.1.25. Tenglamalar sistemasi

$$\begin{cases} 2x_1 + x_2 + x_3 + 3x_4 = 0 \\ 3x_1 + 2x_2 + 2x_3 + x_4 = 0 \end{cases}$$

bilan berilgan qism fazoga $\vec{a}(7;-4;-1;2)$ vektorning ortogonal proyeksiyasini toping.

9.1.26. To‘rt o‘lchamli fazoda $x_1 = x_2,\ x_3 = x_4,\ x_2 = 2x_3$ to‘g‘ri chiziq va $3x_1 - 2x_2 + x_4 = 0,\ x_2 + x_3 = 0$ tekislik orasidagi burchak topilsin.

9.1.27. To‘rt o‘lchamli yevklid fazosida $\vec{a}(1;1;1;1)$, $\vec{b}(1;-1;1;-1)$ vektorlarga hamda $\vec{c}(2;2;1;0)$, $\vec{d}(1;-2;2;0)$ vektorlarga qurilgan qism fazolar orasidagi burchak topilsin.

9.1.28. Berilgan $M(5;1;0;8)$ nuqtadan $A(1;2;3;4)$, $B(2;3;4;5)$, $C(2;2;3;7)$ nuqtalardan o‘tuvchi tekislikka tushirilgan perpendikulyarning uzunligi va asosi topilsin.

9.1.29. Berilgan $M(4;2;-5;1)$ nuqtadan

$$\begin{cases} 2x_1 - 2x_2 + x_3 + 2x_4 = 9 \\ 2x_1 - 4x_2 + 2x_3 + 3x_4 = 12 \end{cases}$$

tekislikka tushirilgan perpendikulyarning uzunligi va asosi topilsin.

9.1.30. Berilgan $A(1;1;1;1)$, $B(2;2;0;0)$, $C(1;2;0;1)$ nuqtalardan o‘tuvchi tekislik va $D(1;1;1;2)$, $E(1;1;2;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning o‘zaro vaziyatini aniqlang, ularning umumiy perpendikulyarining tenglamasini yozing va uzunligini toping.

TEST SINOVI

1. Quyida berilgan aylana tenglamasidan aylana markazi va radiusi topilsin.

$x^2 + y^2 - 6y = 0$

A) $R = 3, \ (0; 3)$; C) $R = 9, \ (0; 3)$;
B) $R = 3, \ (0; -3)$; D) $R = 9, \ (0; -3)$.

2. Ellipsning yarim o‘qlarini toping: $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

A) ± 3 va ± 5 C) 3 va 5
B) ± 5 va ± 3 D) 5 va 3

3. Quyidagilardan qaysi biri ellips tenglamasini ifodalaydi?

A) $x^2 + 25y^2 = 4$ C) $x^2 - 16y^2 = 16$
B) $x^2 + 9y^2 = 0$ D) $x^2 - y^2 = 1$

4. Radiusi $R = 5$, markazi $(2; -4)$ nuqtada bo‘lgan aylana tenglamasini toping.

A) $x^2 + y^2 + 4x + 8y + 5 = 0$ C) $x^2 + y^2 - 4x - 8y + 5 = 0$
B) $x^2 + y^2 + 4x - 8y - 5 = 0$ D) $x^2 + y^2 - 4x + 8y - 5 = 0$

5. Ellips fokuslarining koordinatalarini toping: $\frac{x^2}{64} + \frac{y^2}{100} = 1$.

A) $(\pm 6; 0)$ C) $(6; 0)$ B) $(0; \pm 6)$ D) $(0; -6)$

6. Quyidagi ellips tenglamasining ekssentrisitetini aniqlang:

$x^2 + 4y^2 = 1$

A) $\varepsilon = \sqrt{3}$ C) $\varepsilon = \frac{\sqrt{3}}{2}$
B) $\varepsilon = \pm\sqrt{3}$ D) $\varepsilon = \pm\frac{\sqrt{3}}{2}$

7. Direktrisalari $x = \pm\frac{7}{2}$ ekssentrisiteti $\varepsilon = \frac{2}{\sqrt{7}}$ bo‘lgan ellips tenglamasi topilsin.

A) $7x^2 + 3y^2 = 21$ C) $3x^2 + 7y^2 = 1$
B) $7x^2 + 3y^2 = 1$ D) $3x^2 + 7y^2 = 21$

8. Quyidagi ellips tenglamasining fokuslari orasidagi masofani aniqlang: $\frac{x^2}{25} + \frac{y^2}{169} = 1$

A) 8 B) 12 C)18 D) 24

9. Yarim o‘qlari 2 va 5 bo‘lgan ellips tenglamasini ko‘rsating.

A) $\frac{x^2}{4} + \frac{y^2}{25} = 1$ C) $\frac{x^2}{2} + \frac{y^2}{5} = 1$
B) $\frac{x^2}{5} + \frac{y^2}{2} = 1$ D) $\frac{x^2}{25} + \frac{y^2}{4} = 1$

10. Tekislikda berilgan nuqtadan bir xil uzoqlikdagi nuqtalarning geometrik o‘rniga …… . . deyiladi.
A) ellips C) shar
B) aylana D) giperbola
11. Parabola tenglamasining umumiy ko‘rinishini ko‘rsating.
A) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ C) $y^2 = 2px$
B) $x^2+y^2 = R^2$ D) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
12. $y^2 = 8x$ parabolaning direktrisasini toping.
A) $y = 2$ C) $x = 2$
B) $y = -2$ D) $x = -2$
13. Direktrisasi $y = -6$ bo‘lgan parabola tenglamasini aniqlang.
A) $y^2 = 24x$ C) $y^2 = -24x$
B) $x^2 = 24y$ D) $x^2 = -24y$
14. $y^2 = 12x$ parabola tenglamasining fokusi nimaga teng?
A) $F(0;3)$ C) $F(3;0)$
B) $F(0;-3)$ D) $F(-3;0)$
15. Agar $F(-5;0)$ fokus va direktrisa 0tenglamasi $x = 5$ bo‘lsa, parabola tenglamasini tuzing.
A) $y^2 = -20x$ C) $y^2 = -10x$
B) $y^2 = 20x$ D) $y^2 = 10x$
16. Quyidagi nuqtalardan qaysilari $y^2 = 18x$ parabolaga tegishli?
A) $A(2;6)$ C) $C(1;18)$
B) $B(2;36)$ D) $D(-1;18)$
17. Ushbu nuqtalar tegishli bo‘lgan parabola tenglamasi toping? $A(-7;7), B(-1;\sqrt{7})$.
A) $y^2 = -6x+7$ C) $y^2 = -2x+5$
B) $y^2 = -7x$ D) $y^2 = -\sqrt{7}x$
18. Parabolaning fokusidan direktrisasigacha bo‘lgan masofa 4 ga teng. Uning kanonik tenglamasini tuzing.
A) $y^2 = 16x$ C) $y^2 = -8x$
B) $y^2 = -16x$ D) $y^2 = 8x$
19. $y^2 = 20x$ parabola tenglamasi berilgan. Fokal radiusi 10 ga teng bo‘ladigan M nuqtani toping.
A) $(8;-11), (8;11)$ C) $(11;-8), (11;8)$
B) $(-11;8), (11;8)$ D) $(-8;11), (8;11)$

20. $x^2 = 10y$ parabola $(5;7)$ nuqtadan o‘tganda ushbu nuqtada fokal radius topilsin.

A) $\sqrt{41}$
C) $\sqrt{26}$
B) $\sqrt{53}$
D) $\sqrt{50}$

21. Giperbola tenglamasi uchun qaysi shart bajarilganda teng yonli giperbola deyiladi?

A) $a \neq b$
C) $a > b$
B) $a < b$
D) $a = b$

22. Haqiqiy o‘qi 10, mavhum o‘qi 8 ga teng bo‘lgan giperbola tenglamasi topilsin.

A) $\frac{x^2}{64} - \frac{y^2}{100} = 1$
C) $\frac{x^2}{10} - \frac{y^2}{8} = 1$
B) $\frac{x^2}{100} - \frac{y^2}{64} = 1$
D) $\frac{x^2}{8} - \frac{y^2}{10} = 1$

23.Fokuslari orasidagi masofa $2c = 8$, ekssentrisiteti $\varepsilon = \frac{4}{3}$ bo‘lgan giperbolaning kanonik tenglamasini aniqlang.

A) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
C) $\frac{x^2}{7} - \frac{y^2}{9} = 1$
B) $\frac{x^2}{9} - \frac{y^2}{7} = 1$
D) $\frac{x^2}{9} + \frac{y^2}{7} = 1$

24. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaning ixtiyoriy nuqtasidan uning ikki asimptotasigacha bo‘lgan masofalar ko‘paytmasi har doim …….. ga teng bo‘ladi.

A) $\frac{ab}{a^2+b^2}$
C) $\frac{a^2b^2}{a^2+b^2}$
B) $\frac{a^2b^2}{a+b}$
D) $\frac{ab}{a+b}$

25. $\frac{x^2}{9} - \frac{y^2}{64} = 1$ giperbola tenglamasi berilgan bo‘lsa, uning yarim o‘qlari topilsin.

A) 3 va 8
C) -8 va -3
B) 8 va 3
D) -3 va -8

26. Yarim o‘qlari $a = 6$, $b = 4$ bo‘lgan giperbolaning ekssentrisiteti nimaga teng?

A) $\frac{\sqrt{13}}{2}$
C) $-\frac{\sqrt{13}}{3}$
B) $-\frac{\sqrt{13}}{2}$
D) $\frac{\sqrt{13}}{3}$

27. $\frac{x^2}{7} - \frac{y^2}{15} = 1$ giperbolaning asimptotalarini aniqlang.

A) $y = \pm\sqrt{\frac{7}{15}}x$ C) $y = \pm\sqrt{\frac{15}{7}}x$

B) $x = \sqrt{\frac{7}{15}}y$ D) $x = \pm\sqrt{\frac{15}{7}}y$

28. Asimptotalari $y = \pm\frac{4}{3}x$, fokuslari orasidagi masofa 20 bo‘lgan giperbolaning ekssentrisitetini toping.

A) $\varepsilon = \frac{5}{4}$ C) $\varepsilon = \frac{4}{5}$

B) $\varepsilon = \frac{5}{3}$ D) $\varepsilon = \frac{3}{5}$

29. Quyidagi nuqtalardan qaysi biri ushbu giperbolani qanoatlantiradi:

$\frac{x^2}{20} - \frac{y^2}{14} = 1$

A) $(2\sqrt{10}\,;\ \sqrt{14})$ C) $(\sqrt{10}\,;\ \sqrt{14})$

B) $(2\sqrt{10}\,;\ 2\sqrt{14})$ D) $(\sqrt{10}\,;\ 2\sqrt{14})$

30. Teng tomonli giperbola $x^2 - y^2 = 18$ berilgan. Unga fokusdosh bo‘lib, $M(10; 8)$ nuqtadan o‘tuvchi giperbolaning tenglamasi topilsin.

A) $\frac{x^2}{16} - \frac{y^2}{20} = 1$ C) $\frac{x^2}{20} - \frac{y^2}{16} = 1$

B) $\frac{x^2}{30} - \frac{y^2}{6} = 1$ D) $\frac{x^2}{6} - \frac{y^2}{30} = 1$

31.$A(4; \frac{2\pi}{3})$ nuqtaga qutb o‘qiga nisbatan simmetrik bo‘lgan B nuqtani toping.

A) $B(-4; \frac{2\pi}{3})$ B) $B(4; \frac{5\pi}{3})$ C) $B(4; \frac{\pi}{3})$ D) $B(-4; \frac{4\pi}{3})$

32. $B(2; \frac{2\pi}{3})$ nuqtaga qutb o‘qiga nisbatan simmetrik bo‘lgan C nuqtani toping.

A) $C(2; \frac{\pi}{3})$ B) $C(-2; \frac{2\pi}{3})$ C) $C(2; \frac{5\pi}{3})$ D) $C(2; \frac{4\pi}{3})$

33. $A(3; \frac{\pi}{6})$ nuqtani qutb o‘qi atrofida $\frac{3\pi}{4}$ burchakka musbat yo‘nalishda burilsa bu nuqtaning koordinatalarini aniqlang.

A) $(3; \frac{17\pi}{12})$ B) $(3; \frac{2\pi}{3})$ C) $(-3; \frac{2\pi}{5})$ D) $(3; \frac{5\pi}{6})$

34. Qutb koordinatalar sistemasida $A(8; -\frac{2\pi}{3})$ va $B(6; \frac{\pi}{3})$ nuqtalar berilgan. AB kesma o‘rtasining koordinatalarini toping.

A) $(3; -\frac{2\pi}{3})$ B) $(2; \frac{\pi}{3})$ C) $(1; -\frac{2\pi}{3})$ D) $(1; \frac{2\pi}{3})$

35. Dekart koordinatalar sistemasida $M(\sqrt{3}; 1)$ nuqta berilgan.Uni qutb koordinatalarini toping.

A) $(2;\frac{\pi}{6})$ B) $(1;\frac{2\pi}{3})$ C) $(2;\frac{\pi}{3})$ D) $(3;\frac{\pi}{3})$

36.Qutb koordinatalarida a radiusli, markazi koordinatalar boshida bo‘lgan aylana tenglamasini toping.

A) $r = 2a$ B) $r = a$ C) $r = a^2$ D) $r = 3a$

37.Qutb koordinatalar sistemasida $M(3;\frac{5\pi}{6})$ va $N(2;\frac{\pi}{6})$ nuqtalar orasidagi masofani toping.

A) $\sqrt{19}$ B) 2 C) 3 D)$\sqrt{5}$

38.Qutb koordinatalar sistemasida $r = \frac{2}{1-\cos\varphi}$ tenglama bilan berilgan chiziqni dekart koordinatalar sistemasida tenglamasini toping.

A) $y^2 = 4(x+1)$ B) $y^2 + x^2 = 1$

C) $x = y^2$ D) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

39. Qutb koordinatalar sistemasida $M(3;\frac{5\pi}{6})$ va $N(4;\frac{\pi}{3})$ nuqtalar orasidagi masofani toping.

A) 6 B) 3 C) 7 D) 5

40. $\rho = \frac{4cos\varphi}{sin^2\varphi}$ parabolaning direktrisa tenglamasini toping.

A) $x = 5$ B) $x = -3$ C) $x = -1$ D) $x = -2$

41. Quyidagi $3x^2 - 2xy + 3y^2 + 2x - 4y + 1 = 0$ egri chiziqni markazi topilsin.

A) $(-1;1)$ B) $(-\frac{1}{8};\frac{5}{8})$ C) $(\frac{1}{2};-4)$ D) $(2;-3)$

42. $4xy + 3y^2 + 16x + 12y - 36 = 0$ berilgan ikkinchi tartibli chiziqning turini aniqlang.

A)giperbola B)parabola C)parallel to‘g‘ri chiziqlar D)ellips

43. Ushbu $9x^2 - 16y^2 - 54x - 64y - 127 = 0$ ikkinchi tartibli chiziqning eksentrisitetini aniqlang.

A) $\frac{3}{4}$ B) $\frac{4}{5}$ C) $\frac{5}{4}$ D) $\frac{7}{5}$

44. Quyidagi $32x^2 + 52xy - 7y^2 + 180 = 0$ egri chiziqning asimptotalarini toping.

A) $\pm\frac{2}{3}x$ B) $\pm\frac{5}{2}x$ C) $\pm\frac{4}{3}x$ D) $\pm\frac{1}{3}x$

45. $14x^2 + 24xy + 21y^2 - 4x + 18y - 139 = 0$ ellipsning fokuslari orasidagi masofani aniqlang.

A) 5 B) 6 C) 10 D) 8

46. Ushbu $7x^2 + 60xy + 32y^2 - 14x - 60y + 7 = 0$ ikkinchi tartibli chiziqning tipini aniqlang.

A) giperbola B) parallel to‘g‘ri chiziqlar
C) ellips D) parabola

47. Quyidagi $9x^2 + 24xy + 16y^2 - 230x + 110y - 475 = 0$ tenglama bilan berilgan ikkikinchi tartibli chiziqning direktrisasini aniqlang.

A) $x = -\frac{5}{3}$ B) $x = -\frac{7}{2}$ C) $x = -\frac{4}{3}$ D) $x = -\frac{5}{2}$

48. Ushbu $5x^2 + 12xy - 12x - 22y - 19 = 0$ egri chiziqning haqiqiy o‘qining burchak koeffitsiyentini aniqlang.

A) $k = \frac{1}{3}$ B) $k = \frac{3}{4}$ C) $k = \frac{2}{3}$ D) $k = \frac{1}{2}$

49. Quyidagi $5x^2 + 8xy + 5y^2 - 18x - 18y + 9 = 0$ ikkinchi tartibli chiziqning markazi qaysi nuqtada joylashgan?

A) $(1; 1)$ B) $(-2; 3)$ C) $(-3; 1)$ D) $(-1; 1)$

50. $6xy - 8y^2 + 12x - 26y - 11 = 0$ tenglama bilan berilgan ikkinchi tartibli chiziqning turini aniqlang.

A) parabola B) ellips C) parallel to‘g‘ri chiziqlar
D) giperbola.

TEST SINOVI JAVOBLARI

№	0	1	2	3	4	5	6	7	8	9
0		A	C	A	D	B	C	D	B	A
1	B	C	D	B	C	A	A	B	D	C
2	A	D	B	B	C	A	D	C	B	A
3	C	B	D	A	C	A	B	A	A	D
4	C	B	A	C	A	C	B	D	C	A
5	D									

Javoblar

1.1.2. 1) $S(3;0), r=3$; 2) $S(-3;4), r=5$; 3) $S(5;-12), r=15$; 4) $S(-1;\frac{2}{3}),\ r=\frac{4}{3}$; 5) $(x-1)^2+(y+2)^2-5=0$; 6) $(x-\frac{1}{3})^2+$ $+(y+\frac{7}{6})^2-\frac{41}{36}=0$. **1.1.3.** A,C,D nuqtalar aylana tashqarisida, B nuqta aylanada yotadi. **1.1.4.** 1) Izlangan nuqtalar markazi $S(1;3)$ nuqtada va radiusi 5 ga teng aylanada, yoki uning tashqarisida yotadi; 2) nuqtalar markazi $(1;-3)$ nuqtada va radiuslari 4 va 5 ga teng konsentrik aylanalarda, yoki bu aylanalar orasida yotadi; 3) markazlari $S(1;2), S(4;6)$ nuqtalarda bo‘lgan va radiuslari mos ravishda 5 va 3 ga teng bo‘lgan doiralarning umumiy qismiga va chegaralariga tegishli; 4) nuqtalar markazi $(3;0)$ nuqtada, radiusi 3 ga teng bo‘lgan yarim aylananing OX o‘qidan yuqorida joylashgan bo‘lagini to‘ldiradi; 5) nuqtalar $x^2+y^2-4y=0$ aylanadan $x=\pm1$ to‘g‘ri chiziqlar ajratgan sigmentlarni to‘ldiradi. **1.1.5.** $(x-6)^2+(y-5)^2=25$. **1.1.6.** $(x-2)^2+(y-3)^2-\frac{9}{5}=0$. **1.1.8.** $(x-3)^2+(y-2)^2-$ $-26=0$, $(x+3)^2+(y-6)^2-26=0$. **1.1.9.** 1)$x^2+y^2=9$; 2)$(x-2)^2+(y+3)^2=49$; 3)$(x-6)^2+(y+8)^2=100$; 4) $(x+1)^2+(y-2)^2=25$; 5) $(x-1)^2+(y-4)^2=8$; 6) x^2+ $+y^2=16$; 7) $(x-1)^2+(y+1)^2=4$; 8)$(x-2)^2+(y-4)^2=10$; 9) $(x-1)^2+y^2=1$; 10) $(x-2)^2+(y-1)^2=25$. **1.1.10.** $(x+2)^2+(y+1)^2=20$. **1.1.11.** 1) $x+5y-3=0$; 2) $x+2=0$; 3)$3x-y-9=0$; 4)$y+1=0$. **1.1.12.**$x^2+y^2+6x-9y-17=0$. **1.1.13.** $7x-4y=0$. **1.1.14.** 1)$\frac{x^2}{25}+\frac{y^2}{16}=1$; 2)$\frac{x^2}{25}+\frac{y^2}{9}=1$; 3)$\frac{x^2}{169}+\frac{y^2}{25}=1$. **1.1.15.** $(\pm3;0)$. **1.1.16.** $(0;\pm12)$. **1.1.17.** 1) ichki; 2) ichki; 3) tashqi; 4) tashqi; 5)elepsga tegishli. **1.1.18.** $3x^2+5y^2=32$. **1.1.19.** $3x^2+2xy+$ $+3y^2-4x-4y=0$. **1.1.20.** $\frac{x^2}{16}+\frac{y^2}{7}=1$. **1.1.21.** $x=\pm9$. **1.1.22.** $\frac{x^2}{32}+\frac{y^2}{16}=1$. **1.1.23.** $\left(-\frac{15}{2};\pm\frac{3\sqrt{7}}{2}\right)$.**1.1.24.** 1) $\frac{x^2}{25}+\frac{y^2}{4}=1$; 2) $\frac{x^2}{25}+\frac{y^2}{9}=1$; 3) $\frac{x^2}{169}+\frac{y^2}{144}=1$; 4) $\frac{x^2}{25}+\frac{y^2}{16}=1$; 5) $\frac{x^2}{100}+\frac{y^2}{64}=1$; 6) $\frac{x^2}{169}+\frac{y^2}{25}=1$; 7) $\frac{x^2}{5}+5=1$; 8) $\frac{x^2}{16}+\frac{y^2}{12}=1$; 9) $\frac{x^2}{13}+\frac{y^2}{9}=1$ yoki $\frac{x^2}{\frac{117}{4}}+\frac{y^2}{9}=1$; 10)$\frac{x^2}{64}+\frac{y^2}{48}=1$. **1.1.25.**1) $\frac{x^2}{4}+\frac{y^2}{9}=1$; 2) $\frac{x^2}{9}+\frac{y^2}{25}=1$; 3) $\frac{x^2}{25}+\frac{y^2}{169}=1$;

4) $\frac{x^2}{64}+\frac{y^2}{100}=1$; 5) $\frac{x^2}{16}+\frac{y^2}{25}=1$; 6) $\frac{x^2}{7}+\frac{y^2}{16}=1$. **1.1.26.** 1) 4 va 3; 2) 2 va 1; 3) 5 va 1; 4) $\sqrt{15}$ va $\sqrt{3}$; 5) $\frac{5}{2}$ va $\frac{5}{3}$; 6) $\frac{1}{3}$ va $\frac{1}{5}$; 7) 1 va $\frac{1}{2}$; 8) 1 va 4; 9) $\frac{1}{5}$ va $\frac{1}{3}$; 10) $\frac{1}{3}$ va 1. **1.1.27.** 1) 5 va 3; 2) $F_1(-4;0)$, $F_2(4;0)$; 3) $\varepsilon=\frac{4}{5}$; 4) $x=\pm\frac{25}{4}$. **1.1.28.** 1) $\sqrt{5}$ va 3; 2)$F_1(0;-2)$, $F_2(0;-2)$; 3)$\varepsilon=\frac{2}{3}$; 4)$y=\pm\frac{9}{2}$. **1.1.30.** 1) $\frac{x^2}{36}+\frac{y^2}{9}=1$; 2) $\frac{x^2}{16}+\frac{y^2}{\frac{16}{3}}=1$; 3) $\frac{x^2}{20}+\frac{y^2}{15}=1$; 4) $\frac{x^2}{20}+\frac{y^2}{4}=1$; 5) $\frac{x^2}{9}+\frac{y^2}{5}=1$; 6) $\frac{x^2}{256}+\frac{y^2}{192}=1$; 7) $\frac{x^2}{15}+\frac{y^2}{6}=1$. **1.2.1.** A – ichki, B- tashqi, C- giperbola nuqtasi. **1.2.2.** 1)$\frac{x^2}{25}-\frac{y^2}{9}=1$; 2)$\frac{x^2}{16}-\frac{y^2}{9}=1$. **1.2.3.** 1)$\frac{x^2}{576}-\frac{y^2}{100}=1$; 2) $\frac{x^2}{64}-\frac{y^2}{36}=1$. **1.2.4.** $\sqrt{2}$. **1.2.5.** $\frac{x^2}{432}-\frac{y^2}{75}=1$. **1.2.6.** $F_1(-13;0)$, $F_2(13;0)$. **1.2.7.** $F_1(0;17)$, $F_2(0;-17)$. **1.2.8.** 1)$\frac{x^2}{16}-\frac{y^2}{9}=1$; 2)$\frac{x^2}{9}-\frac{y^2}{3}=1$. **1.2.9.** $\frac{x^2}{16}-\frac{y^2}{9}=1$. **1.2.10.** 1) $F_1(5;0)$, $F_2(-5;0)$; 2)$e=\frac{5}{3}$; 3)$y=\pm\frac{4}{3}x$, $x=\pm\frac{9}{5}$; 4) $\frac{y^2}{16}-\frac{x^2}{9}=1, e=\frac{5}{4}$.
1.2.11. 1) $a=2\sqrt{3}$, $b=2$; 2) $a=b=6$; 3) $a=\sqrt{5}$, $b=2\sqrt{5}$; 4) $a=\frac{3\sqrt{19}}{5}$, $b=\sqrt{19}$. **1.2.12.** $\frac{x^2}{10}-\frac{y^2}{6}=1$. **1.2.13.** 1)$\frac{x^2}{25}-\frac{y^2}{16}=1$; 2) $\frac{x^2}{9}-\frac{y^2}{16}=1$; 3) $\frac{x^2}{4}-\frac{y^2}{5}=1$; 4) $\frac{x^2}{64}-\frac{y^2}{36}=1$; 5) $\frac{x^2}{36}-\frac{y^2}{64}=1$; 6) $\frac{x^2}{144}-\frac{y^2}{25}=1$; 7) $\frac{x^2}{16}-\frac{y^2}{9}=1$; 8) $\frac{x^2}{4}-\frac{y^2}{5}=1$; 9) $\frac{x^2}{64}-\frac{y^2}{36}=1$. **1.2.14.** 1)$\frac{x^2}{36}-\frac{y^2}{324}=-1$; 2) $\frac{x^2}{16}-\frac{y^2}{9}=-1$; 3) $\frac{x^2}{100}-\frac{y^2}{576}=-1$; 4) $\frac{x^2}{24}-\frac{y^2}{25}=-1$; 5) $\frac{x^2}{9}-\frac{y^2}{16}=-1$.
1.2.15. 1) $a=3$, $b=2$; 2) $a=4$, $b=1$; 3) $a=4$, $b=2$; 4) $a=1$, $b=1$; 5) $a=\frac{5}{2}, b=\frac{5}{3}$; 6) $a=\frac{1}{5}, b=\frac{1}{4}$; 7) $a=\frac{1}{3}, b=\frac{1}{8}$. **1.2.16.** 1) $a=3, b=4$; 2) $F_1(-5;0)$, $F_2(5;0)$; 3) $\varepsilon=\frac{5}{3}$; 4) $y=\pm\frac{4}{3}x$; 5)$x=\pm\frac{9}{5}$. **1.2.17.** 1) $a=3, b=4$; 2) $F_1(0;-5)$, $F_2(0;5)$; 3) $\varepsilon=\frac{5}{4}$; 4) $y=\pm\frac{4}{3}x$; ; 5) $y=\pm\frac{16}{5}$. **1.2.18.** 12. **1.2.20.** $x-4\sqrt{5}y+10=0$ yoki $x-10=0$. **1.2.21.** $r_1=2\frac{1}{4}$; $r_2=10\frac{1}{4}$. **1.2.22.** $\left(10;\frac{9}{2}\right)$ va $\left(10;-\frac{9}{2}\right)$.

1.2.23. $(-6; 4\sqrt{3})$ va $(-6; -4\sqrt{3})$. **1.2.24.** 1) $\frac{x^2}{32}-\frac{y^2}{8}=1$; 2) $x^2-y^2=16$ 3) $\frac{x^2}{18}-\frac{y^2}{8}=1$; 4) $\frac{x^2}{4}-\frac{y^2}{5}=1$ yoki $\frac{x^2}{\frac{61}{9}}-\frac{y^2}{\frac{305}{16}}=1$; 5) $\frac{x^2}{16}-\frac{y^2}{9}=1$. **1.2.25.** $\varepsilon=\sqrt{2}$. **1.2.26.** $\frac{x^2}{4}-\frac{y^2}{12}=1$. **1.2.27.** $\frac{x^2}{60}-\frac{y^2}{40}=1$. **1.2.30.** 1) $\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1$; 2) $\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=-1$. **1.3.4.** $(1;0)$. **1.3.5.** $(0;1)$. **1.3.6.** $(-2;0)$. **1.3.7.** $x=-\frac{3}{2}$. **1.3.8.** $y^2=12x$. **1.3.9.** $y^2=4x$. **1.3.10.** $y^2=8x-8$. **1.3.11.** $y^2=\pm 12x$. **1.3.12.** $y^2=10x-25$. **1.3.13.** $y^2=16x$. **1.3.14.** $x^2=8y$. **1.3.15.** $x^2=-18y$. **1.3.16.** $(18;12)$ va $(18;-12)$. **1.3.17.** $y^2=4x$. **1.3.18.** $y^2=-9x$. **1.3.19.** $x^2=y$. **1.3.20.** $x^2=-2y$. **1.3.21.** $x^2=-12y$. **1.3.22.** $F(6;0)$, $x+6=0$. **1.3.25.** 12. **1.3.26.** 6. **1.3.27.** $(9;12)$, $(9;-12)$. **1.3.30.** $y^2=-28x$.

2.1.2. 1), 2), 3) markazlari qutbda va radiuslari mos ravishda 1,5 va a ga teng bo‘lgan aylanalarda. 4), 5), 6), 7) qutbdan chiquvchi va qutb o‘qi bilan 30^0, 60^0, 90^0 va φ burchaklar tashkil etuvchi nurlarda joylashgan. **2.1.3.** a) $\left(1;\frac{5\pi}{4}\right)$, $\left(3;\frac{5\pi}{3}\right)$, $\left(\frac{2}{3};\frac{5\pi}{6}\right)$, 4) $(\rho;\ \varphi+\pi)$; b) $\left(1;\frac{7\pi}{4}\right)$, $\left(3;\frac{4\pi}{3}\right)$, $\left(\frac{2}{3};\frac{\pi}{6}\right)$, 4) $(\rho;\ 2\pi-\varphi)$. **2.1.4.** $(a;0)$, $\left(a\sqrt{3};\frac{\pi}{6}\right)$, $\left(2a;\frac{\pi}{3}\right)$, $\left(a\sqrt{3};\frac{\pi}{2}\right)$, $\left(a;\frac{2\pi}{3}\right)$. $\left(0;\frac{0}{0}\right)$. Izoh. Qutb nolga teng radius – vektorga va noma’lum amplitudaga ega. **2.1.5.** Jadvalga qaralsin.

φ	0^0	15^0	30^0	45^0	60^0	75^0	90^0
2φ	0^0	30^0	60^0	90^0	120^0	150^0	180^0
$\rho=a\cdot sin2\varphi$	0	$\frac{a}{2}$	$\frac{a\sqrt{3}}{2}$	a	$\frac{a\sqrt{3}}{2}$	$\frac{a}{2}$	0

2.1.6. $AB=\sqrt{3}, CD=10, EF=5$. **2.1.7.** $AB=BC=CA=7$. **2.1.8.** $M_1(1;0)$ va $M_2(7;0)$. **2.1.9.** $S=\frac{1}{2}\rho_1\rho_2\cdot\sin(\varphi_2-\varphi_1)$. Ko‘rsatma. Uchburchakning yuzi uchun trigonometrik $S=\frac{a\cdot b\cdot sinC}{2}$ formuladan foydalanamiz. **2.1.10.** $S=1$ kv birlik. **2.1.11.** $S=6(5\sqrt{3}-3)$ kv birlik. Ko‘rsatma. Shaklni yasang va izlanayotgan yuzni, bir uchi qutbda bo‘lgan OAB, OBC va OAC uchburchakning yuzlari orqali hisoblang. **2.1.12.** 1) $\rho=a$; 2) $\rho=2a\cdot cos2\varphi$; 3) $\rho^2-2\rho_1\rho\cos(\varphi-$

$-\varphi_1) = a^2 - \rho_1^2$. **2.1.13.** $\rho^2 = \frac{b^2}{1-e^2cos^2\varphi}$. **2.1.14.** $\varphi = \arccos\left(\pm\frac{4}{5}\right)$. **2.1.15.** 1) $\rho = \frac{P}{1-ecos\varphi}$; 2) $\rho = \frac{P}{1+ecos\varphi}$, bunda 1) $P = \frac{b^2}{a}$ miqdor ellipsning parametri deyiladi. **2.1.16.** $a = 2\sqrt{2}$; $b = \sqrt{6}$; $2c = 2\sqrt{2}$. **2.1.17.** $\rho^2 = \frac{-b^2}{1-e^2cos^2\varphi}$. **2.1.18.** Ichiga giperbola joylashgan burchaklardan biri θ bilan belgilansa, $\theta = \frac{2\pi}{3}$. **2.1.19.** $\rho = \frac{P}{1-ecos\varphi}$, bunda $P = \frac{b^2}{a}$. **2.1.20.** Asimptotalarning tenglamalari: $\rho = -\frac{2}{\sin\left(\varphi-\frac{\pi}{4}\right)}$ va $\rho = -\frac{2}{\sin\left(\varphi-\frac{3\pi}{4}\right)}$; direktrisalar tenglamalari: $\rho = -\frac{\sqrt{2}}{\cos\varphi}$ va $\rho = -\frac{3\sqrt{2}}{\cos\varphi}$. **2.1.21.** $\rho = \frac{2Pcos\varphi}{sin^2\varphi}$. **2.1.22.** $M(3; arccos\frac{1}{3})$ – qutb o‘qiga nisbatan simmetrik bo‘lgan ikkita nuqta. **2.1.23.** $\rho = \frac{P}{1-\cos\varphi}$. **2.1.24.** 1) $(\frac{P}{2}; \pi)$ – parabolaning uchi; 2) ikkita nuqta: $(P; \frac{\pi}{2})$ va $\left(P; \frac{3\pi}{2}\right)$. **2.1.25.** $|\delta_1\delta_2| = P^2$. **2.1.26.** 1) $\frac{x^2}{169} + \frac{y^2}{25} = 1$; 2) $y^2 = \frac{2}{3}x$; 3) $\frac{x^2}{16} - \frac{y^2}{9} = 1$; 4) $\frac{x^2}{5} + \frac{y^2}{4} = 1$. **2.1.27.** $x + 3y + 2 = 0$. **2.1.28.** 1) $x + 3y + 2 = 0$; 2) $x - 4 = 0$. **2.1.29.** 1) $x - 6y + 1 = 0$; 2) $2x + 3y - 3 = 0$. **2.1.30.** $(5; 1)$. **3.1.1.** $5x^2 + 16xy + 5y^2 - 5x - 5y = 0$. **3.1.2.** 1) $O'(1; 1)$; 2) $O'(-1; 2)$; 3) $4x + 2y - 5 = 0$ markazlar to‘g‘ri chizig‘i; 4) bu chiziqning markazi yo‘q. **3.1.3.** 1) Qarama- qarshi tomonlarining o‘rtalarining birlashtiruvchi to‘g‘ri chiziqlar egri chiziq diametrlari; 2) Qarama- qarshi tomonlarining urinish nuqtalarini tutashtiruvchi egri chiziqlar to‘g‘ri chiziq diametri.

3.1.6. $3x^2 + 2xy + 2y^2 + 3x - 4 = 0$. Ikkinchi tartibli egri chiziqning umumiy tenglamasini olamiz: $a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + +2a_{23}y + a_{33} = 0$. Agar bu tenglama izlangan egri chiziqni tasvirlasa, berilgan nuqtalarning koordinatalari tenglamani qanoatlantirishi kerak. Berilgan har bir nuqtaning koordinatalarini umumiy tenglamaga qo‘yib, a_{ik} koeffitsiyentlarni bog‘lovchi 5 ta shart hosil qilamiz, bu munosabatlardan beshta koeffitsiyentning 6 - koeffitsiyentga nisbatlarini aniqlaymiz va ularni,oldin oltinchi koeffitsiyentga bo‘lingan umumiy tenglamaga qo‘yamiz. **3.1.7.** $x^2 + 4xy + 4y^2 - -6x - 12y = 0$. **3.1.8.** Masalaning shartini ikkita parabola tipidagi egri chiziq qanoatlantiradi: $x^2-8x-y+15=0$ va $9x^2+6xy+$ $+y^2-72x-24y+135=0$. **3.1.9.** $x^2-4xy+3y^2-4y+3=0$. **3.1.10.** $xy+15=0$. **3.1.11.** $x^2-8y=0$. **3.1.12.** 1) $(7;5)$; 2) $(-1;-1)$; 3) $(0;1)$; 4) markazi bo‘lmagan, parabola tipidagi egri chiziq; 5) egri chiziq $x+y+1=0$ markazlar chizig‘iga ega; 6) $\left(\frac{10}{3};\frac{4}{3}\right)$. **3.1.13.** Agar $a\neq 9$ bo‘lsa, tenglama markaziy chiziqni ifodalaydi. Agar $a=9$ va $b\neq 9$ bo‘lsa, tenglama parabola tipidagi egri chiziqni ifodalaydi.Agar $a=9$; $b\neq 9$ bo‘lsa, egri chiziq $2x+6y+$ $+3=0$ markazlar chizig‘iga ega bo‘ladi. *Ko‘rsatma.* Masala markazining koordinatalari aniqlanadigan ikkita tenglama $\begin{cases}2x+6y+3=0\\6x+2ax+b=0\end{cases}$ sistemasini tekshirishga olib keladi. Javob esa bu sistemaning aniq, birgalikda bo‘la olmaydi, yoki aniqmas bo‘lishligiga bog‘liqdir. **3.1.14.** a), b), c) egri chiziqlar koordinatalar boshida markazga ega; d) egri chiziq $3x-2y=0$ markazlar chizig‘iga ega; agarda $\delta\neq 0$ bo‘lsa. e) egri chiziqning markazi koordinatalar boshida bo‘ladi, agarda $\delta=0$ bo‘lsa, egri chiziq $a_{11}x+a_{12}y=0$ ko‘rinishdagi markazlar chizig‘iga ega. **3.1.15.** $2x^2-6xy+5y^2-$ $-11=0$. Ko‘rsatma. Egri chizqning markazi koordinatalar boshi deb olinganda uning tenglamasi $a_{11}x^2++2a_{12}xy+a_{22}y^2+\frac{\Delta}{\delta}=0$ bo‘ladi. Berilgan $(2x^2-6xy+5y^2)$ tenglamaning yuqori hadlarini o‘zgartirmasdan birinchi darajali hadlarini tanlab, yangi tenglamaning ozod hadini topish uchun ikkita $\Delta=-11$ va $\delta=1$ diskriminantni hisoblashimiz kerak. **3.1.16.** a) $7x^2+4xy+4y^2-83=0$; b) x^2- $-2xy+4=0$; c) $6x^2-4xy+9y^2-40=0$. **3.1.17.** $a_{11}(x--x_0)^2+2a_{12}(x-x_0)(y-y_0)+a_{22}(y-y_0)^2+a_{33}=0$. *Ko‘rsatma.* Egri chiziqning markaziga nisbatan yozilgan tenglamasini olib, qaytadan oldingi koordinatalar sistemasiga o‘tish kerak. **3.1.18.** $5x^2-5xy+2y^2-5x-2y=0$. **3.1.19.** $3x+y=0$ to‘g‘ri chiziq. *Ko‘rsatma.* Markazning koordinatalarini topish uchun tenglamalar tuzib, ularda a parametrni yo‘qotib, izlanayotgan geometrik o‘rinning tenglamasini hosil qilamiz. **3.1.20.** $4x^2-8xy-$ $-2y^2+9y-4=0$. Berilgan to‘rtta nuqta orqali cheksiz ko‘p to‘g‘ri chiziqlar o‘tadi; ularning hammasi $2x^2-4\lambda xy+(4\lambda+1)y^2-4x-$ $-(4\lambda+1)y=0$ tenglama bilan ifodalanadi, bunda $\lambda-$ o‘zgaruvchi parametr=.**4.1.1.** $17x-4y-4=0$. **4.1.2.** $2x+y+6=0$. **4.1.3.**

$4x-6y+1=0$. **4.1.4.** $y=x-1$. **4.1.5.** $7x-y-3=0$. **4.1.6.** $20x-9y-91=0$. **4.1.8.** $\left(\pm\frac{ab}{\sqrt{b^2-a^2}};\pm\frac{ab}{\sqrt{b^2-a^2}}\right)$; masala $b>a$ holdagina o‘rinli. **4.1.9.** $\frac{2b^2}{a}$. **4.1.10.** $\frac{2ab}{\sqrt{a^2+b^2}}$. **4.1.11.** $8x+25y=0$. **4.1.12.** $32x+25y-89=0$. **4.1.13.** $2p$. **4.1.14.** $y=2x-5$. **4.1.15.** $(2;1),(-6;9)$. **4.1.16.** $(-4;6)$. **4.1.17.** Kesishmaydi. **4.1.18.** $(6;12)$ va $(6;-12)$. **4.1.19.** $(10;\sqrt{30})$, $(10;-\sqrt{30})$, $(2;\sqrt{6})$, $(2;-\sqrt{6})$. **4.1.20.** $(2;1)$, $(-1;4)$, $\left(\frac{3+\sqrt{13}}{2};\frac{7+\sqrt{13}}{2}\right)$ va $\left(\frac{3-\sqrt{13}}{2};\frac{7-\sqrt{13}}{2}\right)$. **4.1.21.** $\left(4;\frac{3}{2}\right)$; $(3;2)$. **4.1.22.** $\left(3;\frac{8}{5}\right)$. **4.1.23.** To‘g‘ri chiziq ellips bilan kesishmaydi. **4.1.24.** 1) To‘g‘ri chiziq ellips bilan kesishadi; 2)To‘g‘ri chiziq ellips bilan kesishmaydi; 3) To‘g‘ri chiziq ellips bilan urinadi.**4.1.25.** 1)$|m|<5$ bo‘lganda to‘g‘ri chiziq ellipsni kesib o‘tadi; 2) $m=\pm5$ to‘g‘ri chiziq ellipsga urinadi; 3) $|m|>5$ bo‘lganda to‘g‘ri chiziq ellipsni kesib o‘tmaydi. **4.1.26.** $(6;2)$ va $\left(\frac{14}{3};-\frac{2}{3}\right)$. **4.1.27.** $\left(\frac{25}{4};3\right)$ nuqtada urinadi. **4.1.28.** To‘g‘ri chiziqni giperbola bilan kesishmaydi. **4.1.29.** 1) $|m|>4{,}5$ bo‘lganda to‘g‘ri chiziq giperbolani kesib o‘tadi; 2) $m=\pm4{,}5$ bo‘lganda to‘g‘ri chiziq giperbolaga urinadi; 1)$|m|<4{,}5$ bo‘lganda to‘g‘ri chiziq giperbola bilan kesishmaydi. **4.1.30.** k^2a^2- $-b^2=m^2$. **4.2.1.** $(x-2)^2-(x-y)^2=0$ yoki ikkita to‘g‘ri chiziq: $2x-y-2=0$; $y-2=0$. **4.2.2.** 1) $3x-y+3=0$; $2y+3=0$; 2)$3x-y=0$; $4y-9=0$. **4.2.3.**$2x+3y-5=0$; $5x+3y-8=0$. **4.2.4.** 1) $7x-35y+22=0$; $7x+14y+20=0$; 2) $6x-2y+$ $+19=0$; $2x+2y-1=0$; 3) $3x+4y+14=0$; $x+y-3=0$; 4) $25x-5y+13=0$; $5y+3=0$. **4.2.5.** 1) $x-4y-2=0$, 2) $x+$ $+4y-3=0$. **4.2.6**. $x+y-1=0$; $3x+3y+13=0$. **4.2.7.** $7x+$ $+1=0$. **4.2.8.** $7x-2y-13=0$; $x-3=0$. **4.2.9.** $\begin{cases} x'=\alpha x+\beta y+\gamma \\ y'=Ax+By+C \end{cases}$ almashtirishda chiziq tenglamasi $x'^2+2y'=0$. ko‘rinishga ega. **4.2.10.** Diametr tenglamasi $(a_{11}x+a_{12}y)-(a_{11}x_0+a_{12}y_0)=0$, urunma tenglamasi $(a_1+a_{11}x_0+a_{12}y_0)x+(a_2+a_{12}x_0+$ $+a_{22}y_0)y+(a_1x_0+a_2y_0+a)=0$; bu yerda $a_{11}=\alpha^2$, $a_{12}=\alpha\beta$, $a_{22}=\beta^2$. **4.2.12.** $x+y-1=0$. **4.2.13.** $3x+y-8=0$. **4.2.14.** 1) $x=1$; 2) $5x-2y+3=0$. **4.2.15.** 1) $3x-y\pm3\sqrt{5}=0$; 2) $5x-$ $-2y\pm9=0$. **4.2.16.** $x^2-\frac{y^2}{4}=1$. **4.2.17.** $3x-4y-10=0$;

$3x-4y+10=0$. **4.2.18.** $10x-3y-32=0$; $10x-3y+32=0$. **4.2.19.** $x+2y-4=0$; $x+2y+4=0$; $d=\frac{8\sqrt{5}}{5}$. **4.2.20.** $M_1(-6;3)$; $d=\frac{11\sqrt{13}}{11}$. **4.2.21.** $5x-3y-16=0$, $13x+5y+48=0$. **4.2.22.** $2x+5y-16=0$. **4.2.23.** $24x+25y=0$. **4.2.24.** $3x+4y-24=0$. **4.2.25.** 1) $y=4$; 2) $16x-15y-100=0$. **4.2.26.** $x+y\pm5=0$. **4.2.27.** $\pm3x\pm4y+15=0$. **4.2.28.** $x\pm y\pm3=0$. **4.2.29.** $arcctg\frac{c^2}{2ab}\le\varphi\le\frac{\pi}{2}$. **4.2.30.** $\frac{\sqrt{a^2+b^2}}{\sqrt{2}}$. **4.2.31.** Yig'indi $\frac{2(a^2+b^2)}{a}$ ga teng. **4.2.32.** 1) $C(2;-3)$, $a=3$, $b=4$, $\varepsilon=\frac{5}{3}$, direktisa tenglama $5x-1=$ $=0$; $5x-19=0$, assimptota tenglamasi: $4x-3y-17=0$; $4x+$ $+3y+1=0$; 2) $C(-5;1)$, $a=8$, $b=6$, $\varepsilon=1{,}25$, direktisa tenglama $x=-11{,}4$ va $x=1{,}4$, assimptota tenglamasi: $3x+4y+11=0$; $3x-4y+19=0$; 3) $C(2;-1)$, $a=3$, $b=4$, $\varepsilon=1{,}25$, direktisa tenglama $y=-4{,}2$, $y=2{,}2$, asimptota tenglamasi: $4x+3y-5=0$; $4x-3y-11=0$. **4.2.34.** $x-3y+9=0$. **4.2.35.** $y^2=4x$. **4.2.36.** 1) $k<\frac{1}{2}$; 2) $k=\frac{1}{2}$; 3) $k>\frac{1}{2}$. **4.2.37.** $y_1y=p(x+x_1)$. **4.2.38.** $x+$ $+y+2=0$. **4.2.39.** $2x-y-16=0$. **4.2.40.** $y-p=0$.

5.1.1. $\frac{(x-1)^2}{5^2}-\frac{(y+2)^2}{3^2}=1$. **5.1.2.** 1) ellips ($\Delta\neq0$; $\delta>0$); 2) giperbola ($\Delta\neq0$; $\delta<0$); 3)parabola ($\Delta\neq0$; $\delta=0$); 4)haqiqiy $(7;5)$ nuqtada kesishadigan mavhum to'g'ri chiziqlar ($\Delta=0$; $\delta>0$); 5)ikkita kesishuvchi haqiqiy to'g'ri chiziq ($\Delta=0$; $\delta<0$). **5.1.3.** 1) Giperbola ($\Delta=16$; $\delta=-8$); 2) ellips ($\Delta=-64$; $\delta=8$); 3) ikkita haqiqiy kesishuvchi to'g'ri chiziq ($\Delta=0$; $\delta=-1$); 4) ikkita haqiqiy kesishuvchi to'g'ri chiziq $\left(\Delta=0;\ \delta=-\frac{81}{4}\right)$; 5) giperbola $\left(\Delta=-\frac{1}{4};\ \delta=-\frac{5}{4}\right)$. **5.1.4.** 1) Koordinata o'qlariga parallel bo'lgan ikkita to'g'ri chiziq: $x-a=0$ va $y-b=0$; 2) ordinatalar o'qi $x=0$ va $x-2y+5=0$ to'g'ri chiziq; 3) ikki marta olingan $x-2y=0$ to'g'ri chiziq; 4) ikki marta olingan $3x+5y=0$ to'g'ri chiziq; 5) ikkita parallel to'g'ri chiziq $2x-3y+5=0$ va $2x-3y-5=0$. **5.1.5.** $y+5=0$ va $y=x-2$. **5.1.6.** 1) $3x-2y=0$ va $7x+5y=0$; 2) $x+y+1+\sqrt{5}=0$ va $x+y+1-\sqrt{5}=0$; 3) $y-5x=0$ va $x+y-1=0$; 4) $2x-y+3=0$ to'g'ri chiziqlar ustma – ust tushadi. **5.1.7.** 1) $x-y=0$ va $2x+5y=0$ qo'sh to'g'ri chiziq; 2) ikki marta olingan $x + 2y = 0$ to‘g‘ri chiziq; 3) $5x - y = 0$ va $2x - y = 0$ qo‘sh to‘g‘ri chiziq; 4) koordinatalar boshida kesishuvchi qo‘sh mavhum to‘g‘ri chiziq. **5.1.8.** 1) $x^2 - y^2 = 11\sqrt{2}$; $I_1 = 0$; $I_2 = -2$; $I_3 = 44$; 2) $\frac{x^2}{16} - \frac{y^2}{9} = 1$; $I_1 = 7$; $I_2 = -144$; $I_3 = -144^2$; 3)$\frac{x^2}{9} + y^2 = 1$; $I_1 = 10$; $I_2 = 9$; $I_3 = -81$; 4)$\frac{x^2}{4} - \frac{y^2}{9} = 1$; $I_1 = 5$; $I_2 = -36$; $I_3 = 36^2$; 5) $x^2 - -\frac{y^2}{9} = 1$; $I_1 = 8$; $I_2 = -9$; $I_3 = 81$. **5.1.9.** 1) $y^2 = 4\sqrt{2}x$; 2) $y = \frac{6}{\sqrt{5}}x$; 3) $y^2 = \frac{3}{2\sqrt{2}}x$; 4) $y^2 = \frac{1}{5\sqrt{5}}x$; **5.1.10.** 1) $15x^2 - y^2 + 3 = 0$; 2) $\frac{x^2}{3} + +y^2 = 1$; 3)$y^2 = \sqrt{3}x$. **5.1.11.**$xy = \frac{5}{2}$. **5.1.12.** 1)$xy = 1{,}2$; 2) $xy = \frac{29}{25}$; 3) $xy = \frac{3\sqrt{5}}{2}$. **5.1.13.** $9x^2 - 4y^2 \pm 36x = 0$. *Ko‘rsatma.* Oxirgi had oldidagi ishora koordinatalar boshi giperbolaning qaysi bir uchiga ko‘chirilganligiga bog‘liq. **5.1.14.** 1) $C(3; -1)$, yarim o‘qi 3 va $\sqrt{5}$, $\varepsilon = \frac{2}{3}$, direktrisa tenglamalari $2x - 15 = 0$, $2x + 3 = 0$; 2) $C(-1; 2)$, yarim o‘qi 5 va 4, $\varepsilon = \frac{3}{5}$, direktrisa tenglamalari $3x - 22 = 0$, $3x + +28 = 0$; 3) $C(1; -2)$, yarim o‘qi $2\sqrt{3}$ va 4, $\varepsilon = \frac{1}{2}$, direktrisa tenglamalari $y - 6 = 0$, $y + 10 = 0$. **5.1.15.**1) $C(2; -3)$, $a = 3$, $b = 4$, $\varepsilon = \frac{5}{3}$, direktrisa tenglamalari $5x - 1 = 0$, $5x - 19 = 0$, asimptotasi $4x - 3y - 17 = 0$, $4x + 3y + 1 = 0$; 2) $C(-5; 1)$, $a = 8$, $b = 6$, $\varepsilon = = 1{,}25$, direktrisa tenglamalari $x = -11{,}4$ va $x = 1{,}4$; asimptotasi $3x + 4y + 11 = 0$, $3x - 4y + 19 = 0$; 3) $C(2; -1)$, $a = 3$, $b = 4$, $\varepsilon = 1{,}25$, direktrisa tenglamalari $y = -4{,}2$, $y = 2{,}2$; asimptotasi $4x + +3y - 5 = 0$, $4x - 3y - 11 = 0$. **5.1.16.** 1)1, 2, 5 va 8 – yagona markazga; 2) 4 va 6- cheksiz ko‘p markazlarga; 3) 3 va 7 markazga ega emas. **5.1.17.** 1) $(3; -2)$; 2) $(0; -5)$; 3) $(0; 0)$; 4) $(-1; 3)$. **5.1.18.** 1) $x - 3y - 6 = 0$; 2) $2x + y - 2 = 0$; 3) $5x - y + 4 = 0$. **5.1.19.** 1) $9x^2 - 18y + 6y^2 + 2 = 0$; 2) $6x^2 + 4xy + y^2 - 7 = 0$; 3) $4x^2 + +6xy + y^2 - 5 = 0$; 4) $4x^2 + 2xy + 6y^2 + 1 = 0$. **5.1.20.** 1) $m \neq 4$, $n -$ har qanday qiymatida; 2) $m = 4$, $n \neq 6$; 3) $m = 4$, $n = 6$. **5.1.21.** 1) Elliptik tenglama; $\frac{x'^2}{9} + \frac{y'^2}{4} = 1$ ellipsni ifodalaydi; $O'(5; -2) -$ yangi koordinatalar sistemasi; 2) Giperbolik tenglama; $\frac{x'^2}{16} - \frac{y'^2}{9} = 1$ giperbolani ifodalaydi; $O'(3; -2) -$ yangi koordinatalar sistemasi;

3) $\frac{x'^2}{4}+\frac{y'^2}{9}=-1$ elliptik tenglamasi, hech qanday geometrik shaklni ifodalamaydi; $O'(5;-2)-$ yangi koordinatalar sistemasi. **5.1.22.** 1) Giperbolik tenglama; $\frac{x'^2}{9}-\frac{y'^2}{4}=1$giperbolani ifodalaydi; $tg\alpha=-2$, $cos\alpha=\frac{1}{\sqrt{5}}$, $sin\alpha=-\frac{2}{\sqrt{5}}$; 2) Elliptik tenglama; $\frac{x'^2}{16}+\frac{y'^2}{4}=1$ellipsni ifodalaydi; $\alpha=45^0$. **5.1.23.** 1) $\frac{x^2}{30}+\frac{y^2}{30}=1$ ellipsni ifodalaydi; 2) $9x^2-16y^2=5$ giperbolani ifodalaydi. **5.1.24.** 1) 3 va 1; 2) 3 va 2; 3) 1 va $\frac{1}{2}$; 4) 3 va 7. **5.1.25.** 1) $x=2,\ y=3$; 2) $x=3,\ y=-3$; 3) $x=1, y=-1$; 4) $x=-2, y=4$. **5.1.26.** 1) 2 va 1; 2) 5 va 1; 3) 4 va 2; 4) 1 va $\frac{1}{2}$. **5.1.27.** 1) $x+y-1=0,\ \ 3x+y+1=0$; 2) $x-$ $-4y-2=0,\ \ x-2y+2=0$; 3) $x-y=0,\ x-3y=0$; 4) $x+$ $+y-3=0,\ x+3y+3=0$. **5.1.28.** 1) $y^2=6x-$ parabola. **5.1.29.** 1) 3; 2) 3; 3) $\sqrt{2}$; 4) $\frac{1}{2}\sqrt{10}$. **5.1.30.** 1) $2x+y-5=0,\ 2x+y-1=$ $=0$; 2) $2x-3y-1=0,\ \ 2x-3y+11=0$; 3) $5x-y-3=0$, $5x-y+5=0$.

7.1.2. 1) $x-y=0,\ z=0$; 2) $x+y=0,\ z=0$. **7.1.3.** 1) $(6;-2;3)$, $r=7$; 2) $(-4;0;0),\ r=4$; 3) $(1;-2;3),\ r=6$; 4) $(0;0;3),\ r=4$. **7.1.4.** $A-$ ichki nuqta; $B-$ tashqi nuqta; $C-$ sferada yotadi; $D-$ tashqi nuqta. **7.1.5.** $l(x-a)+m(y-b)+n(z-c)=0$. **7.1.6.** $2x+$ $+y+2z-13=0$. **7.1.7.** $x(x-x_0)+y(y-y_0)+z(z-z_0)=0$ (sfera). **7.1.8.** $x^2+y^2+z^2+Rx=0$(sfera). **7.1.9.**$(x-a)(x-x_0)+$ $+(y-b)(y-y_0)+(z-c)(z-z_0)=0$. **7.1.10.** x^2+y^2+ $+z^2 - 10z-9=0$. **7.1.11.** $x^2+y^2+z^2+22x+16y-6z=0$. **7.1.12.** $x^2+y^2+z^2+27x+21y-\frac{33}{2}z+10=0$. **7.1.13.** 1) kesib o‘tadi; 2)$\left(-\frac{7}{3};-\frac{4}{3};\frac{7}{3}\right)$ nuqtada urinadi; 3) kesib o‘tmaydi. **7.1.14.** $6x+$ $+2y+3z-55=0$. **7.1.16.** $(x_0-a)(x-x_0)+(y_0-b)(y-y_0)+$ $+(z_0-c)(z-z_0)=0$. **7.1.17.** $xx_0+yy_0+zz_0-R^2=0$. **7.1.18.** $R^2(A^2+B^2+C^2)-D^2=0$. $\left(-\frac{AR^2}{D};-\frac{BR^2}{D};-\frac{CR^2}{D}\right)$ urinish nuqtasi. **7.1.19.** $x^2+y^2+z^2-2x+4y-4=0$ va $x^2+y^2+z^2-\frac{58}{65}x+$ $+\frac{116}{65}y-\frac{114}{65}z-\frac{188}{65}=0$. **7.1.20.** $x^2+y^2+(z+1)^2=12$ va

$x^2+y^2+(z+4)^2=27$. **7.1.21.** $\left(\frac{31}{12};\frac{31}{12};\frac{31}{12}\right)$. **7.1.22.** $(l^2+m^2+$ $+n^2)R^2=\begin{vmatrix} a-x_0 & b-y_0 \\ l & m \end{vmatrix}^2+\begin{vmatrix} b-y_0 & c-z_0 \\ m & n \end{vmatrix}^2+\begin{vmatrix} c-z_0 & a-x_0 \\ n & l \end{vmatrix}^2$. **7.1.23.**

$$\begin{vmatrix} x-x_1 & y-y_1 \\ l_1 & m_1 \end{vmatrix}^2+\begin{vmatrix} y-y_1 & z-z_1 \\ m_1 & n_1 \end{vmatrix}^2+\begin{vmatrix} z-z_1 & x-x_1 \\ n_1 & l_1 \end{vmatrix}^2=\begin{vmatrix} x-x_2 & y-y_2 \\ l_2 & m_2 \end{vmatrix}^2+$$

$$+\begin{vmatrix} y-y_2 & z-z_2 \\ m_2 & n_2 \end{vmatrix}^2+\begin{vmatrix} z-z_2 & x-x_2 \\ n_2 & l_2 \end{vmatrix}^2.$$

7.1.24. 1) 3; 2) 2; 3) 1; 4) 3; 5) 3; 6) 2; 7) 3; 8) 2; 9) 1. **7.1.25.**

$$\begin{vmatrix} a-x_1 & b-y_1 & c-z_1 \\ x-x_1 & y-y_1 & z-z_1 \\ \boldsymbol{l} & \boldsymbol{m} & \boldsymbol{n} \end{vmatrix}=R^2\left[\begin{vmatrix} y-y_1 & z-z_1 \\ \boldsymbol{m} & \boldsymbol{n} \end{vmatrix}^2+\begin{vmatrix} z-z_1 & x-x_1 \\ \boldsymbol{n} & \boldsymbol{l} \end{vmatrix}^2+\begin{vmatrix} x-x_1 & y-y_1 \\ \boldsymbol{l} & \boldsymbol{m} \end{vmatrix}^2\right]$$

7.1.26. $\begin{vmatrix} b-y_1 & c-z_1 \\ \boldsymbol{m} & \boldsymbol{n} \end{vmatrix}^2+\begin{vmatrix} c-z_1 & a-x_1 \\ \boldsymbol{n} & \boldsymbol{l} \end{vmatrix}^2+\begin{vmatrix} a-x_1 & b-y_1 \\ \boldsymbol{l} & \boldsymbol{m} \end{vmatrix}^2 > R^2(\boldsymbol{l}^2+\boldsymbol{m}^2+\boldsymbol{n}^2)$.

7.1.27. $A(x-a)+B(y-b)+C(z-c)+R\sqrt{A^2+B^2+C^2}=0$.
7.1.28. $10x+15y+6z-90=0$. **7.1.30.** $a^2A^2+b^2B^2+c^2C^2=D^2$.
7.1.31. $a^2A^2+b^2B^2+c^2C^2>D^2$. **7.1.32.** $(x^2+y^2+z^2)^2=$ $=a^2x^2+b^2y^2+c^2z^2$. **7.1.34.** $\left(-\frac{a^2AD}{\Delta};-\frac{b^2BD}{\Delta};-\frac{c^2CD}{\Delta}\right)$. **7.1.36.** Berilgan ellipsoidning $\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}=\frac{1}{d^4}$ ellipsoid bilan kesishish chizig‘i. **7.1.37.** $x^2+y^2=a^2\pm 2az-$ ikkita aylanma paraboloid. **7.1.38.** $\frac{x^2}{25}+\frac{y^2}{9}+\frac{z^2}{16}=1$. **7.1.39.** $\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{36}=1$. **7.1.40.** $\frac{x^2}{12}+\frac{y^2}{9}+$ $\frac{z^2}{7{,}2}=1$.

8.1.1. 1) Ikkita tekislik: $2x+y=0$; $y+2z-2=0$; 2) ikkita tekislik: $x-2y+3z+2=0$; $x-2y+3z-3=0$. 3) ikkita tekislik: $x+2y+3z+4=0$; $3x-2y+z-6=0$. **8.1.2**. 1) ikkita tekislik: $x+y+z+1=0$; $5x+4y+3z+2=0$; 2) ikkita tekislik: $2x-7y+z+1=0$; $2x-7y+z+3=0$; 3) ustma – ust tushgan qo‘sh tekislik: $(4x+3y+10z+7)^2=0$. **8.1.3.** 1) Ellipsoid; 2) bir pallali giperboloid; 3) ikki pallali giperboloid. **8.1.4**. 1)konus; 2) elliptik paraboloid; 3) giperbolik paraboloid. **8.1.5**. 1) elliptik silindr; 2) giperbolik silindr; 3) parabolik silindr. **8.1.6**. 1) giperbolik paraboloid; 2) bir pallali giperboloid. **8.1.7.** 1) Uchi $\left(\frac{3}{2};1;\frac{1}{2}\right)$ da bo‘lgan $Z=2X^2-$ $-4Y^2$ giperbolik paraboloid ; 2) uchi $(0;1;-2)$ nuqtada bo‘lgan elliptik paraboloid $Z=X^2+3Y^2$; 3) uchi $(-1;-1;-1)$ nuqtada bo‘lgan konus $X^2 + 2Y^2 - 3Z^2 = 0$. **8.1.8**. 1) bir juft tekislik: $x +$ $+y \pm z = 0$. 2) Parabolik silindr: $Z = 5X^2$; 3) $Z = 2X^2$parabolik silindr. **8.1.9**. 1) $z^2 - 2x^2 = 1$ giperbolik silindr; 2) $(3;-1;1)$ markazli $\frac{X^2}{36}+\frac{Y^2}{9}+\frac{Z^2}{4}=1$ ellipsoid; 3) $x^2 - y^2 + z^2 = 0$ konus. **8.1.10**. 1) ikkita tekislik: $x - y \pm (z-1)^2 = 0$; 2) markazi $(5;2;3)$ nuqtada bo‘lgan bir pallali giperboloid $\frac{X^2}{16}+\frac{Y^2}{4}-\frac{Z^2}{16}=1$; 3) giperbolik paraboloid: $x^2 - y^2 = -2z$. **8.1.11**. 1) parabolik silindr: $x^2 - 10y^2 = 0$. 2) $x^2 +$ $+y^2 = 1$ doiraviy silindr; 3) $(x-1)^2+\left(y+\frac{2}{3}\right)^2+z^2=\frac{16}{9}$ sfera. **8.1.12**. 1) $(x-1)^2+\left(y+\frac{2}{3}\right)^2=\frac{16}{9}$ doiraviy silindr; 2) $X^2 + Y^2 - Z^2 = 0$ doiraviy konus. 3) $(2x-1) \pm (y-2) = 0$ ikkita tekislik. **8.1.13.** Markazi $\left(-\frac{1}{3},-\frac{2}{3};\frac{2}{3}\right)$nuqtadagi bir pallali giperboloid $\frac{X^2}{\left(\frac{1}{\sqrt{3}}\right)^2}+\frac{Y^2}{\left(\frac{1}{\sqrt{6}}\right)^2}-\frac{Z^2}{\left(\frac{1}{\sqrt{2}}\right)^2}=1$ yangi sistema birlik vektorlarining koordinatalari: $e_1'=\left\{\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right\}$, $e_2'=\left\{\frac{1}{\sqrt{6}};\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}}\right\}$, $e_3'=\left\{\frac{1}{\sqrt{2}};0;\frac{1}{\sqrt{2}}\right\}$. **8.1.14.** $\frac{X^2}{\left(\sqrt{\frac{2}{3}}\right)^2}+\frac{Y^2}{1}=1$ elliptik silindr, simmetriya o‘qi tenglamalari $x = t,\ y = 2 + 2t,\ z = -1 - t,\ O'X$ o‘qining vektori $e_1'=\left\{-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right\}$, $O'Y$ o‘qining vektori $e_2'=\left\{\frac{1}{\sqrt{2}};0;\frac{1}{\sqrt{2}}\right\}$. **8.1.15.** Parabolik silindr: $6x^2 - 2\sqrt{3}y^2 = 0$. **8.1.16.** Ikkita parallel tekislik: $2x - 3y + z = -1 \pm \sqrt{6}$. **8.1.17.** Markazi $(1;2;-1)$ nuqtada bo‘lgan ellipsoid: $\frac{X^2}{2}+\frac{Y^2}{1}+\frac{Z^2}{\frac{2}{3}}=1$, $e_1'=\left\{\frac{1}{3};\frac{2}{3};\frac{2}{3}\right\}$, $e_2'=\left\{\frac{2}{3};\frac{1}{3};-\frac{2}{3}\right\}$, $e_3'=\left\{\frac{2}{3};-\frac{2}{3};\frac{1}{3}\right\}$. **8.1.18.** Markazi $\left(0,1,-\frac{2}{5}\right)$ nuqtada bo‘lgan ikki pallali giperboloid: $\frac{X^2}{\frac{4}{5}}+\frac{Y^2}{\frac{4}{15}}-\frac{Z^2}{\frac{4}{25}}=-1$, $e_1'=\left\{\frac{1}{\sqrt{2}};-\frac{1}{\sqrt{2}};0\right\}$, $e_2'=\left\{\frac{1}{\sqrt{2}};\frac{1}{\sqrt{2}};0\right\}$, $e_3'=\{0;0;1\}$. **8.1.19.** Uchi $(1;1;-1)$ nuqta bo‘lgan $x^2 + y^2 - 2z^2 = 0$ aylanma konus, $(2;1;-2)$ konus o‘qiga parallel vektor. **8.1.20.**

$\frac{X^2}{\frac{5}{2\sqrt{2}}}+\frac{Y^2}{\frac{1}{\sqrt{2}}}=2Z$ elliptik paraboloid. Paraboloid tomoniga yo‘nalgan o‘qining birlik vektori $\left\{\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}},0\right\}$, $(1;1;-2)$, $(1;1;1)$ paraboloid o‘qlariga perpendikulyar kesimlarning bosh o‘qlariga parallel vektorlar, paraboloid uchi $\left(-\frac{1}{40},-\frac{19}{40},\frac{1}{2}\right)$. **8.1.21.** $\frac{X^2}{2}+\frac{Y^2}{1}=1$ elliptik silindr, $(0;1;0)$ silindr o‘qidagi nuqta: $\{1;0;1\}$, silindr o‘qidagi parallel vektor: $\{1;1;-1\}$ va $\{-1;2;1\}$ silindr o‘qiga perpendikulyar kesimlarining bosh o‘qlariga parallel vektorlar. **8.1.22.** Markazi $0'\left(-\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right)$ nuqtada bo‘lgan $\frac{X^2}{\frac{1}{3}}+\frac{Y^2}{\frac{1}{6}}-\frac{Z^2}{\frac{1}{2}}=1$ bir pallali giperboloid, o‘qlarning birlik vektorlari $e_1'=\left\{-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}}\right\}$, $e_2'=\left\{\frac{1}{\sqrt{6}};\frac{2}{\sqrt{6}};\frac{1}{\sqrt{6}}\right\}$, $e_3'=\left\{\frac{1}{\sqrt{2}};0;-\frac{1}{\sqrt{2}}\right\}$. **8.1.23**. $x^2+y^2=\frac{1}{6}$ doiraviy silindr, o‘qining tenglamalari: $5x-2y-z+5=0$; $x-y+z+1=0$. **8.1.24**. $x^2-y^2=\frac{1}{3}$ giperbolik silindr, markazlar o‘qining tenglamalari: $x+2y-5z+1=0$; $x-y+z+1=0$. Bosh kesim haqiqiy o‘qining yo‘nalishi, $e_1'=\left\{\frac{1}{\sqrt{2}};0;-\frac{1}{\sqrt{2}}\right\}$, mavhum o‘qining yo‘nalishi: $e_2'=\left\{\frac{1}{\sqrt{3}};-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right\}$. **8.1.25.** $\frac{X^2}{\frac{4}{7\sqrt{14}}}-\frac{Y^2}{\frac{2}{\sqrt{14}}}=2Z$ giperbolik paraboloid. $O'XZ$ tekisligi bilan sirt kesishidan hosil bo‘lgan parabola o‘qining musbat yo‘nalishini $\{1;2;-3\}$ vektor aniqlaydi. $O'X$ o‘qining musbat yo‘nalishi $\{4;1;2\}$ vektor bilan, $O'Y$ o‘qining musbat yo‘nalishi $\{-1;2;1\}$ vektor bilan aniqlanadi. Uchi $O'\left(-\frac{617}{392},-\frac{113}{196},\frac{1011}{392}\right)$nuqtada. **8.1.26.** Uchi $\left(-\frac{183}{784},-\frac{499}{784},\frac{509}{392}\right)$ nuqtada bo‘lgan giperbolik paraboloid: $7X^2-2Y^2-\frac{8Z}{\sqrt{14}}=0$. $O'X$ o‘qining yo‘nalishi $\{2;4;1\}$, $O'X$ o‘qining yo‘nalishini $\{1;-1;2\}$ vektor aniqlaydi. $\{-3;1;2\}$ vektor paraboloid o‘qi bo‘ylab kichik parametrli bosh kesim o‘qi tomonga yo‘nalgan ($O'XZ$ tekislik). **8.1.27**. 1) Simmetriya o‘qlari aniqlanadi, hosil bo‘lgan silindrlar avvalgilarga gomotetik bo‘ladi; 2) simmetriya o‘qi o‘ziga parallel siljiydi, hosil bo‘lgan silindr avvalgiga o‘xshash bo‘ladi. **8.1.28**.1) Parametri, botiqlik yo‘nalishi va yasovchilar yo‘nalishi o‘zgarmagan holda silindrning ko‘chishi ro‘y beradi; 2) yasovchilar yo‘nalishi o‘zgaradi, parameter o‘zgaradi. **8.1.29**. $\lambda = \pm 1$; $\mu = \pm\sqrt{2}$ parametrlar $I_3 = 0$, $K_4 = 0$, ${I_1}^2 = 4I_2$ shartlardan aniqlanadi. **8.1.30**. $ab + bc + ca = 0$..

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