UZBEKISTAN ACADEMY OF SCIENCES
V.I.ROMANOVSKIY INSTITUTE OF MATHEMATICS

UZBEK MATHEMATICAL JOURNAL

Journal was founded in 1957. Until 1991 it was named by "Izv. Akad. Nauk UzSSR, Ser. Fiz.-Mat. Nauk". Since 1991 it is known as "Uzbek Mathematical Journal". It has 4 issues annually.

3. 2020

Uzbek Mathematical Journal is abstracting and indexing by

MathSciNet **Zentralblatt** **Math VINITI**

Starting from 2018 all papers published in Uzbek Mathematical Journal you can find in **EBSCO** and **CrossRef**.

TASHKENT - 2020

Uzbek Mathematical
Journal, 2020, №3, pp.117-125
DOI: 10.29229/uzmj.2020-3-12

Boundary value problem for a quasilinear elliptic equation with two perpendicular line of degeneration

Rasulov X.R.

Abstract: The unique solvability of a boundary value problem for a quasilinear equation of elliptic type with two perpendicular lines of degeneration is the main of the present work. For this aim, we equivalently reduce the formulated problem to the integral-differential equation and solve it by the successive iteration method. Certain properties of hypergeometric functions have been used to obtain necessary estimations.

Keywords: Elliptic type equation, integral-differential equation, unique solvability, method of successive approximations, hypergeometric functions.

MSC (2010): 35A01; 35A02; 35L02; 35L03; 35R03.

1 Introduction

Intensive studies of quasilinear equations of elliptic and mixed types are due to the emergence of new, theoretically interesting problems. Moreover, they have numerous applications in the study of problems in mechanics, physics, engineering, and biology. Boundary value problems for equations of elliptic and mixed types with one line of degeneration were studied in many works, for instance, see [1, 2]. However, boundary value problems for equations with two perpendicular lines of degeneracy have been studied relatively little. We would like to note the works [3, 4].

2 Formulation of the problem

This work is devoted to the study of the boundary value problem for a quasilinear equation of elliptic type with two perpendicular lines of degeneracy.

Consider the equation:

$$y^m u_{xx} + x^m u_{yy} = f(x, y, u, u_x, u_y),\ m = const > 0. \tag{2.1}$$

Let $\Omega-$ be a finite simply-connected domain bounded by a normal curve σ_0 : $x^{m+2} + y^{m+2} = 1$ with ends at points $A(1, 0), B(0, 1)$ and by segments of coordinate axes: $OA : y = 0$ axes and $OB : x = 0$.

Let us introduce the notation:

$$P = \{(x, y) : (x, y) \in \Omega, -\infty < u, u_x, u_y < +\infty\},$$

References

1. Aziz K.F. and Schneider M. The Existence of Generaliezed Solutions for a Class of Quasi-linear Equation of Mixed Type // Journal of Math.anal. and applications, 1985. -107.-p.425-445.
2. Bateman G., Erdelyi A. Higher transcendental functions. - M .: Science, 1973. - T. 1. - 294 p.
3. Islomov B.I. On local and nolocal boundary value problems for a mixed-type equation with two internal lines of degeneration // Uzbek mathematical journal. 1993. N.2. pp.36-42.
4. Mengziyayev B. The Neumann-Tricomi problem for one equation of mixed type with two lines of degeneration // Proceedings of the All-Union conference on partial differential equations dedicated to the 75th anniversary of the birth of Academician I.G. Petrovsky, January 27-31 1976. -M.: 1978. pp. 381-382.
5. Rasulov X.R. Dirichlet problem for a quasilinear equation of elliptic type with two lines of degeneration // Reports of UzAS, 1996. - N.12. - c.12-16.
6. Rasulov X.R. On one boundary value problem for a quasilinear equation of elliptic type (Actual problems of differential equations and their applications, Republican scientific conference with the participation of foreign scientists, December 15-17, 2017, Tashkent), p.63-64.
7. Salokhitdinov M.S., Islomov B.I. Mixed type equations with two lines of degeneration. Tashkent, Mumtoz Suz , 2009. 262 p.
8. Shimkovich E.V. On a boundary value problem for a nonlinear equation of mixed type // Lithuanian Mathematical Collection, 1986.-T.26. - N.3. pp.582-591.
9. Smirnov M.M. Mixed type equations. - M .: Science, 1970.- 294 p.

Rasulov Xaydar,
Bukhara State University, Institute of Mathematics, Bukhara branch, Uzbekistan Academy of Sciences, Bukhara State University, M.Ikbol 11, Bukhara 705018, Uzbekistan.
e-mail: xrasulov71@mail.ru.

Contents