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# UZBEK MATHEMATICAL JOURNAL

Journal was founded in 1957. Until 1991 it was named by "Izv. Akad. Nauk UzSSR, Ser. Fiz.-Mat. Nauk". Since 1991 it is known as "Uzbek Mathematical Journal". It has 4 issues annually.

3. 2020\_\_\_\_\_

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Uzbek Mathematical Journal, 2020, №3, pp.117-125 DOI: 10.29229/uzmj.2020-3-12

#### Boundary value problem for a quasilinear elliptic equation with two perpendicular line of degeneration Rasulov X.R.

**Abstract:** The unique solvability of a boundary value problem for a quasilinear equation of elliptic type with two perpendicular lines of degeneration is the main of the present work. For this aim, we equivalently reduce the formulated problem to the integral-differential equation and solve it by the successive iteration method. Certain properties of hypergeometric functions have been used to obtain necessary estimations.

**Keywords:** Elliptic type equation, integral-differential equation, unique solvability, method of successive approximations, hypergeometric functions.

MSC (2010): 35A01; 35A02; 35L02; 35L03; 35R03.

#### 1 Introduction

Intensive studies of quasilinear equations of elliptic and mixed types are due to the emergence of new, theoretically interesting problems. Moreover, they have numerous applications in the study of problems in mechanics, physics, engineering, and biology. Boundary value problems for equations of elliptic and mixed types with one line of degeneration were studied in many works, for instance, see [1, 2]. However, boundary value problems for equations with two perpendicular lines of degeneracy have been studied relatively little. We would like to note the works [3, 4].

## 2 Formulation of the problem

This work is devoted to the study of the boundary value problem for a quasilinear equation of elliptic type with two perpendicular lines of degeneracy.

Consider the equation:

$$y^{m}u_{xx} + x^{m}u_{yy} = f(x, y, u, u_{x}, u_{y}), \ m = const > 0.$$
(2.1)

Let  $\Omega$ - be a finite simply-connected domain bounded by a normal curve  $\sigma_0$ :  $x^{m+2} + y^{m+2} = 1$  with ends at points A(1,0), B(0,1) and by segments of coordinate axes: OA: y = 0 axes and OB: x = 0.

Let us introduce the notation:

$$P = \{ (x, y) : (x, y) \in \Omega, -\infty < u, u_x, u_y < +\infty \},\$$

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