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BOUNDARY VALUE PROBLEM IN A DOMAIN WITH DEVIATION FROM THE CHARACTERISTICS FOR ONE NONLINEAR EQUATION OF A MIXED TYPE

Rasulov Kh.

Bukhara Branch of the V.I.Romanovskiy Institute of Mathematics, Bukhara State University, Bukhara, Uzbekistan,
xrasulov71@mail.ru

Consider the equation

$$T(U) \equiv K(y)U_{xx} + N(x)U_{yy} + C(x,y)U = f(x,y,U), \tag{1}$$

where $K(y), N(x), C(x,y), f(x,y,U)$ are given functions, when $K(t) \gtreqless 0$ and $N(t) \gtreqless 0$ for $t \gtreqless 0$.

Let Ω be a finite simply connected convex domain on the plane of variables (x,y), bounded for $x>0, y>0$ by a smooth curve σ with endpoints at the points $A(1,0)$ and $B(0,1)$, for $x>0, y<0$ and with characteristics $x<0, y>0$.

$$OD_1 : \int_0^x \sqrt{N(t)}dt + \int_0^y \sqrt{-K(t)}dt = 0 \quad \left(OC_1 : \int_0^x \sqrt{-N(t)}dt + \int_0^y \sqrt{K(t)}dt = 0\right)$$

equation and smooth curves $D_1A(BC_1)$, lying inside the characteristic triangle $ODA(OBC)$, respectively.

Suppose that curves D_1A, σ and BC_1 satisfy the condition:

$$(n_1 + n_2)|_{D_1A\cup\sigma\cup BC_1} < 0, \tag{2}$$

where $(n_1, n_2)-$ internal normal vector to $\Gamma = D_1A \cup \sigma \cup BC_1$.

Problem T. Find a solution $U(x,y)$ of equation (1) in the domain Ω such that

$$U(x,y)\big|_\Gamma = 0. \tag{3}$$

Note that the classical solvability of boundary value problems for linear equations of mixed types with one and two lines of degeneration has been studied rather deeply. However, the generalized solvability of boundary value problems for quasilinear equations of mixed type has not been fully studied, since there is no general theory that can be applied to study such equations. Works in this direction one can see in [1].

In this article we study the existence of a generalized solution of problem **T** for equation (1) in the weighted space Sobolev S.L.. In addition, concrete functions $K(y), N(x), C(x,y), f(x,y,U)$ and examples of the considered domain Ω that satisfy the conditions of the lemmas and theorems on the solvability of the problem **T** have been constructed.

Moreover, the existence of a generalized solution of the boundary value of problem **T** for the equation

$$yU_{xx} + xU_{yy} + c(x,y)U = f(x,y,U),$$

in the weightless space Sobolev S.L. under weaker restricted on the given functions with $c(x,y)$ and $f(x,y,U)$.

Constructed an example which shows that under conditions of the theorem, the existence of a generalized solution of problem **T** for equation (1) the solution can be non unique.

References

1. Aziz K. F. and Schneider M. The Existence of Generalized Solutions for a Class of quasi-linear Equation of Mixed Type // Journal of Math. anal. and applications. 1985. Vol. 107, Page. 425-445.