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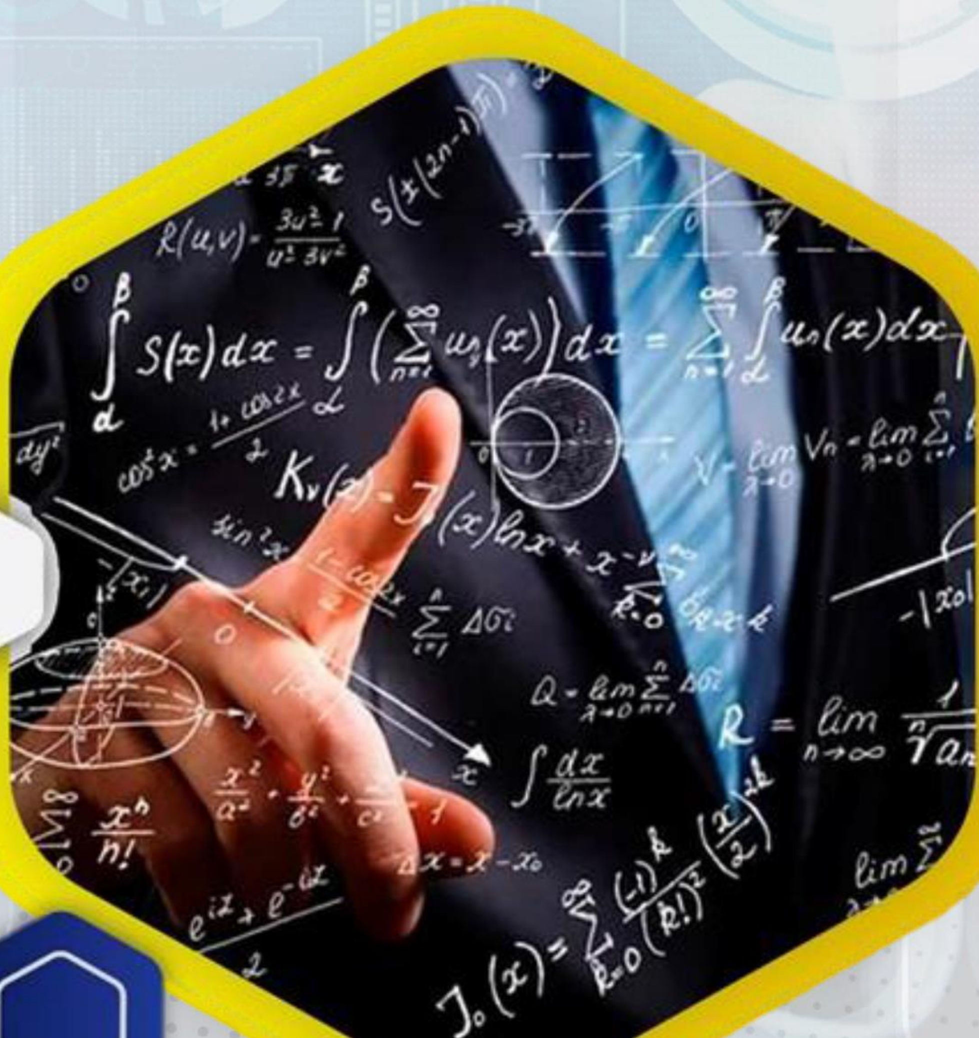

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# CURRENT PROBLEMS OF PHYSICS, MATHEMATICS AND ARTIFICIAL INTELLIGENCE TECHNOLOGIES

## INTERNATIONAL SCIENTIFIC AND THEORETICAL CONFERENCE

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# ON SUPERPOSITION OF LINEAR AND QUADRATIC STOCHASTIC OPERATOR

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**ABSTRACT:** In this paper, we consider the superposition of a single linear operator and quadratic stochastic Volterra operators on a two-dimensional simplex and which depend on parameters. Fixed points are found and their types are studied, as well as the limit behavior of the trajectory.

**Key words:** Linear operator;quadratic stochastic operator;Volterra operator;trajectory;simplex.

Quadratic operators appear in many models of mathematical genetics, particular, in the theory of heredity. (see e.g. [ 1, 2]).

A quadratic stochastic operator may arise in mathematical genetics as follows. Consider a biological (ecological) population, i.e., a community of organisms closed with respect to reproduction.

Let $S^{m-1}=\left\{\mathbf{x}=(x_1,x_2,\dots,x_m)\in R^m : x_i\geq 0,\ \sum_{i=1}^{m}x_i=1\right\}$

be the $(m-1)$-dimensional simplex. A map $V$ of $S^{m-1}$ into itself is called a *quadratic stochastic operator* (QSO) if

$$(V\mathbf{x})_k=\sum_{i,j=1}^{m}p_{ij,k}x_ix_j \tag{1}$$

for any $\mathbf{x}\in S^{m-1}$ and for all $k=1,\dots,m$, where

$$p_{ij,k}\geq 0,\quad p_{ij,k}=p_{ji,k},\quad \sum_{k=1}^{m}p_{ij,k}=1,\ \forall i,j,k=1,\dots,m. \tag{2}$$

Assume $\{\mathbf{x}^{(n)}\in S^{m-1}: n=0,1,2,\ \dots\}$ is the trajectory of the initial point $\mathbf{x}\in S^{m-1}$, where $\mathbf{x}^{(n+1)}=V(\mathbf{x}^{(n)})$ for all $n=0,1,2,\dots$, with $\mathbf{x}^{(0)}=\mathbf{x}$.

**Definition 1.** A point $\mathbf{x}\in S^{m-1}$ is called a fixed point of a QSO $V$ if $V(\mathbf{x})=\mathbf{x}$.

**Definition 2.**[1] A fixed point $\mathbf{x}^*$ is called hyperbolic if its Jacobian $D_{\mathbf{x}}V(\mathbf{x}^*)$ has no eigenvalues on the unit circle.

**Definition 3.**[1] A hyperbolic fixed point $\mathbf{x}^*$ is called:

i) attracting if all the eigenvalues of the Jacobian $D_{\mathbf{x}}V(\mathbf{x}^*)$ are less than 1 in absolute value;

ii) repelling if all the eigenvalues of the Jacobian $D_{\mathbf{x}}V(\mathbf{x}^*)$ are greater than 1 in absolute value;

iii) a saddle otherwise.

In this work we will consider the superposition of A linear operator and a quadratic stochastic operator V, $B=A(V(x))$, where

$$V: S^2\to S^2$$

$$V: x_1=x_1^2\qquad x_2=x_2^2+2x_1x_2\qquad x_3=x_3^2+2x_1x_2+2x_2x_3$$

and $A=\begin{pmatrix}\alpha & 0 & 0\\ \beta & \gamma & 0\\ 1-\alpha-\beta & 1-\gamma & 1\end{pmatrix}, 0<\alpha,\beta,\gamma\leq 1.$

In this case, operator B will take the form:

$$B=A(V(x))=\begin{pmatrix} \alpha & 0 & 0 \\ \beta & \gamma & 0 \\ 1-\alpha-\beta & 1-\gamma & 1 \end{pmatrix}\begin{pmatrix} x_1^2 \\ x_2^2+2x_1x_2 \\ x_3^2+2x_1x_2+2x_2x_3 \end{pmatrix}$$
$$=\begin{pmatrix} \alpha x_1^2 \\ \beta x_1^2+\gamma x_2^2+2\gamma x_1x_2 \\ (1-\alpha-\beta)x_1^2+(1-\gamma)x_2^2+2(1-\gamma)x_1x_2+x_3^2+2x_1x_3+2x_2x_3 \end{pmatrix} \quad (3)$$

Let us find the fixed points of the operator B (B(x)=x):

$$\begin{cases} x_1=\alpha x_1^2 \\ x_2=\beta x_1^2+\gamma x_2^2+2\gamma x_1x_2 \\ x_3=(1-\alpha-\beta)x_1^2+(1-\gamma)x_2^2+2(1-\gamma)x_1x_2+x_3^2+2x_1x_3+2x_2x_3 \end{cases} \quad (4)$$

Theorem. The following statements are true for operator B:

a) for α=1,γ<1 it has two fixed points (0,0,1) and (1,0,0), which are attracting and saddle points, respectively;

saddle points, respectively;

c) for α<1 ,γ<1 it has a single fixed point (0,0,1) and it is attracting;

d) in all cases there are no periodic trajectories.

References

[1] Devaney R. L. *An introduction to chaotic dynamical systems*, (New York: Westview Press) 2003.

[2] Lyubich Yu. I. 1992 *Mathematical Structures in Population Genetics* (Berlin: Sprenger) 1992.