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## ON SUPERPOSITION OF LINEAR AND QUADRATIC STOCHASTIC OPERATOR

B.M.MAMUROV

Bukhara State University, Bukhara, Uzbekistan,

[b.j.mamurov@buxdu.uz](mailto:b.j.mamurov@buxdu.uz)

**ABSTRACT:** In this paper, we consider the superposition of a single linear operator and quadratic stochastic Volterra operators on a two-dimensional simplex and which depend on parameters. Fixed points are found and their types are studied, as well as the limit behavior of the trajectory.

**Key words:** Linear operator; quadratic stochastic operator; Volterra operator; trajectory; simplex.

Quadratic operators appear in many models of mathematical genetics, particular, in the theory of heredity. (see e.g. [1, 2]).

A quadratic stochastic operator may arise in mathematical genetics as follows. Consider a biological (ecological) population, i.e., a community of organisms closed with respect to reproduction.

$$\text{Let } S^{m-1} = \left\{ \mathbf{x} = (x_1, x_2, \dots, x_m) \in R^m : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}$$

be the  $(m-1)$ -dimensional simplex. A map  $V$  of  $S^{m-1}$  into itself is called a *quadratic stochastic operator* (QSO) if

$$(V\mathbf{x})_k = \sum_{i,j=1}^m p_{ij,k} x_i x_j \quad (1)$$

for any  $\mathbf{x} \in S^{m-1}$  and for all  $k = 1, \dots, m$ , where

$$p_{ij,k} \geq 0, \quad p_{ij,k} = p_{ji,k}, \quad \sum_{k=1}^m p_{ij,k} = 1, \quad \forall i, j, k = 1, \dots, m. \quad (2)$$

Assume  $\{\mathbf{x}^{(n)} \in S^{m-1} : n = 0, 1, 2, \dots\}$  is the trajectory of the initial point  $\mathbf{x} \in S^{m-1}$ , where  $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$  for all  $n = 0, 1, 2, \dots$ , with  $\mathbf{x}^{(0)} = \mathbf{x}$ .

**Definition 1.** A point  $\mathbf{x} \in S^{m-1}$  is called a fixed point of a QSO  $V$  if  $V(\mathbf{x}) = \mathbf{x}$ .

**Definition 2.**[1] A fixed point  $\mathbf{x}^*$  is called hyperbolic if its Jacobian  $D_{\mathbf{x}}V(\mathbf{x}^*)$  has no eigenvalues on the unit circle.

**Definition 3.**[1] A hyperbolic fixed point  $\mathbf{x}^*$  is called:

- i) attracting if all the eigenvalues of the Jacobian  $D_{\mathbf{x}}V(\mathbf{x}^*)$  are less than 1 in absolute value;
- ii) repelling if all the eigenvalues of the Jacobian  $D_{\mathbf{x}}V(\mathbf{x}^*)$  are greater than 1 in absolute value;
- iii) a saddle otherwise.

In this work we will consider the superposition of A linear operator and a quadratic stochastic operator  $V$ ,  $B = A(V(\mathbf{x}))$ , where

$$V: S^2 \rightarrow S^2$$

$$V: \begin{cases} x_1 = x_1^2 & x_2 = x_2^2 + 2x_1x_2 & x_3 = x_3^2 + 2x_1x_2 + 2x_2x_3 \\ \alpha & 0 & 0 \\ \beta & \gamma & 0 \\ 1 - \alpha - \beta & 1 - \gamma & 1 \end{cases}, 0 < \alpha, \beta, \gamma \leq 1.$$

In this case, operator  $B$  will take the form:



$$B=A(V(x))=\begin{pmatrix} \alpha & 0 & 0 \\ \beta & \gamma & 0 \\ 1-\alpha-\beta & 1-\gamma & 1 \end{pmatrix} \begin{pmatrix} x_1^2 \\ x_2^2 + 2x_1x_2 \\ x_3^2 + 2x_1x_2 + 2x_2x_3 \end{pmatrix}$$

$$=\begin{pmatrix} \alpha x_1^2 \\ \beta x_1^2 + \gamma x_2^2 + 2\gamma x_1x_2 \\ (1-\alpha-\beta)x_1^2 + (1-\gamma)x_2^2 + 2(1-\gamma)x_1x_2 + x_3^2 + 2x_1x_3 + 2x_2x_3 \end{pmatrix} \quad (3)$$

Let us find the fixed points of the operator B ( $B(x)=x$ ):

$$\begin{cases} x_1 = \alpha x_1^2 \\ x_2 = \beta x_1^2 + \gamma x_2^2 + 2\gamma x_1x_2 \\ x_3 = (1-\alpha-\beta)x_1^2 + (1-\gamma)x_2^2 + 2(1-\gamma)x_1x_2 + x_3^2 + 2x_1x_3 + 2x_2x_3 \end{cases} \quad (4)$$

Theorem. The following statements are true for operator B:

- a) for  $\alpha=1, \gamma<1$  it has two fixed points (0,0,1) and (1,0,0), which are attracting and saddle points, respectively;
- b) saddle points, respectively;
- c) for  $\alpha<1, \gamma<1$  it has a single fixed point (0,0,1) and it is attracting;
- d) in all cases there are no periodic trajectories.

#### References

- [1] Devaney R. L. *An introduction to chaotic dynamical systems*, (New York: Westview Press) 2003.
- [2] Lyubich Yu. I. 1992 *Mathematical Structures in Population Genetics* (Berlin: Springer) 1992.

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