



ABSTRACTS

of the international conference

**MATHEMATICAL ANALYSIS AND ITS
APPLICATIONS IN MODERN
MATHEMATICAL PHYSICS**

PART I

Samarkand
September 23-24, 2022

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Regularity of a non-Volterra quadratic stochastic operator

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The evolution of a population can be studied by a dynamical system of a quadratic stochastic operator [2].

Let $E = \{1, \dots, m\}$ be a finite set and the set of all probability distributions on E

$$S^{m-1} = \{\mathbf{x} = (x_1, x_2, \dots, x_m) \in \mathbf{R}^m : x_i \geq 0, \text{ for any } i \text{ and } \sum_{i=1}^m x_i = 1\}$$

the $(m - 1)$ -dimensional simplex.

A *quadratic stochastic operator* (QSO) is a mapping $V : S^{m-1} \rightarrow S^{m-1}$ of the simplex into itself, of the form $V(x) = x' \in S^{m-1}$, where

$$V : x'_k = \sum_{i,j=1}^m p_{ij,k} x_i x_j, \quad k = 1, \dots, m \quad (1)$$

and the coefficients $p_{ij,k}$ satisfy

$$p_{ij,k} = p_{ji,k} \geq 0, \quad \sum_{k=1}^m p_{ij,k} = 1, \quad i, j \in E \quad (2)$$

The trajectory $\mathbf{x}^{(n)}$, $n = 0, 1, 2, \dots$, of V for an initial point $\mathbf{x}^{(0)} \in S^{m-1}$ is defined by

$$\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)}) = V^{n+1}(\mathbf{x}^{(0)}), \quad n = 0, 1, 2, \dots$$

One of the main problems in mathematical biology consists of the study of the asymptotical behaviour of the trajectories. Note that the main problem is open even in two-dimensional case.

The main problem deeply studied for Volterra quadratic stochastic operators. In [1] developed the theory of Volterra QSOs. A *Volterra* QSO is defined by (1), (2) and with the additional assumption

$$p_{ij,k} = 0 \quad \text{if} \quad k \notin \{i, j\}, \quad \forall i, j, k \in E.$$

Consider the following non-Volterra quadratic stochastic operator which has the form

$$V : \begin{cases} x'_1 = (\frac{1}{3} + \alpha)x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + \frac{1}{3}x_1x_2, \\ x'_2 = (\frac{1}{3} - \alpha)x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2, \\ x'_3 = \frac{1}{3}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + \frac{2}{3}x_1x_2, \end{cases} \quad (3)$$

where $\alpha \in [0, 1/3]$.

Theorem. a) The operator (3) has a unique fixed point $\mathbf{x}^* = (x_1^*, x_2^*, x_3^*) \in S^2$, where $x_1^* = 1 - x_2^* - x_3^*$,

$$x_2^* = \frac{3\alpha\sqrt{17} + \sqrt{216\alpha - 78 - 72\alpha\sqrt{17} + 34\sqrt{17}} - \sqrt{17} - 3\alpha - 5}{4(3\alpha - 2)}, \quad x_3^* = \frac{5 - \sqrt{17}}{4};$$

- b) The operator (3) has no-periodic points except the fixed point;
- c) Any trajectory of the operator converges to the unique fixed;
- d) The operator (3) is a regular transformation.

References

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