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THE CONVEX COMBINATIONS OF QUADRATIC OPERATORS ON S^2

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Quadratic operators attract the attention of specialists in various fields of mathematics and its applications (see, for example, [2], [3]). We will use the definition and notations of the reference [1]. In [1] on the two-dimensional simplex the following quadratic stochastic operator was studied: $V_0 : S^2 \to S^2$, $V_0(x_1, x_2, x_3) = (x_1', x_2', x_3')$, where

$$x_1' = x_1^2 + 2x_2x_3; \quad x_2' = x_2^2 + 2x_1x_3; \quad x_3' = x_3^2 + 2x_1x_2.$$

It is proved that $M_1(1,0,0)$, $M_2(0,1,0)$, $M_3(0,0,1)$, $C(1/3,1/3,1/3)$ are fixed points of the operator V_0.

Note that in [4] by author studied the following quadratic stochastic operator V_1:

$$V_1 : \begin{cases} x_1' = 1/3x_1^2 + 1/3x_2^2 + 1/3x_3^2 + 2x_2x_3, \\ x_2' = 1/3x_1^2 + 1/3x_2^2 + 1/3x_3^2 + 2x_1x_3, \\ x_3' = 1/3x_1^2 + 1/3x_2^2 + 1/3x_3^2 + 2x_1x_2. \end{cases}$$

It is proved that the operator V_1 has a unique fixed point C and it is a regular operator.

In present paper, we shall consider a convex combination of the operators V_0 and V_1

$$V_\lambda : S^2 \to S^2; \ V_\lambda = (1-\lambda)V_0 + \lambda V_1, (0 \le \lambda \le 1).$$

It is easy to see that the operator V_λ has the form:

$$V_\lambda : \begin{cases} x_1' = (1 - 2\lambda/3)x_1^2 + \lambda/3x_2^2 + \lambda/3x_3^2 + 2x_2x_3, \\ x_2' = \lambda/3x_1^2 + (1 - 2\lambda/3)x_2^2 + \lambda/3x_3^2 + 2x_1x_3, \\ x_3' = \lambda/3x_1^2 + \lambda/3x_2^2 + (1 - 2\lambda/3)x_3^2 + 2x_1x_2. \end{cases}$$

Obviously, the operator V_λ is also a quadratic stochastic operator.

Theorem. For the operator V_λ the following statements are true:

a) The operator V_λ has a unique fixed point $C(1/3, 1/3, 1/3)$;

b) if $\lambda = (3 - \sqrt{6})/2$ then the fixed point C is a non-hyperbolic point;

c) if $0 < \lambda < (3 - \sqrt{6})/2$ then the fixed point C is a repelling point;

d) if $(3 - \sqrt{6})/2 < \lambda < 1$ then C is an attracting point.

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