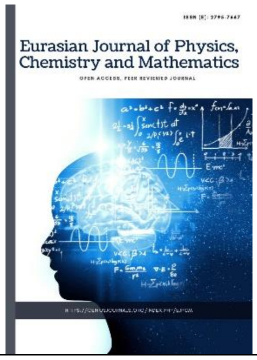

Uniqueness Of Solution To The Problem For One Mixed Integral-Differential Equation

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ABSTRACT

In this article we consider integro-differential equations of mixed type. We prove the uniqueness of the solution.

Keywords: differential equations of mixed type, integro-differential equations, boundary value problem.

1. Introduction.

Integro-differential equations are a class of equations in which the unknown function is contained both under the integral sign and under the differential or derivative sign. In this article we consider an integro-differential equation of mixed type.

Various questions of physics and technology lead to the study of integro-differential equations in ordinary and partial derivatives and to the formulation of certain tasks. The exact solution of which either cannot be expressed in a closed form using modern methods, or these solutions are expressed in a cumbersome form. In this regard, the theory such equations have long attracted the attention of both theoretical physicists and mathematicians, The mixed type equation occurs in the case of application - for example, in problems related to transonic gas dynamics. [1][2].

Direct and inverse problems for an equation of mixed parabolic-hyperbolic type were studied in [3]-[6], [8]-[10]. A criterion for uniqueness and existence has been established solving the inverse problem of determining the unknown right-hand side. Also in [11], other formulations of the inverse problem were considered. Generally speaking, direct and inverse problems for mixed type equations are not as well known as similar problems for classical equations. However, these tasks are also relevant to practice.

This article considers the problem for the integro-differential equation mixed type and proves the uniqueness of the solution to the problem posed.

2. Statement of the boundary value problem.

Let us present the formulation of the inverse problem for an integro-differential equation of mixed type.

Consider in the domain $G = \{(x,t): 0 < x < 1, -\alpha < t < \beta\}$ }, an integro-differential equation of mixed type:

$$\begin{cases} u_t - u_{xx} = f(x), \quad t > 0 \\ u_{tt} - u_{xx} = f(x) + \int_0^t K(\tau) u(x,\tau) d\tau \,.\, t < 0 \end{cases} \tag{1}$$

where α, β are given positive numbers.

Find in the domain G the function $u(x,t)$ to equation (1) and the following conditions:

boundary conditions:

$$u|_{x=0} = 0 = 0, u|_{x=1} = 0, \quad -\alpha \le t \le \beta, \tag{2}$$

and we assume that the local condition holds:

$$u(x,-\alpha) = \varphi(x), \quad x \in [0,1], \tag{3}$$

as well as gluing conditions at $t = 0$:

$$\lim_{t\to -0} u(x,t) = \lim_{t\to +0} u(x,t), \ \lim_{t\to -0} \frac{\partial u(x,t)}{\partial t} = \lim_{t\to +0} \frac{\partial u(x,t)}{\partial t}, \ x \in [0,1], \tag{4}$$

here $\varphi(\cdot)$ is a given sufficiently smooth function.

Let's denote

$$G_+ = G \cap \{t > 0\}, G_- = G \cap \{t < 0\}.$$

Relations (1)-(4) are a direct problem, i.e., if the functions φ, f are known, then the solution $u(x,t)$ can be found from relations (1)-(4).

Definition 1. The solution to problem (1)-(4) is the function $u(x,t)$ from the class $C(\bar{G}) \cap C^1(G) \cap C^{2,1}_{x,t}(G_+ \cup \{t = \beta\}) \cap C^2(G_- \cup \{t = -\alpha\})$ and satisfying relations (1)-(4).

Now let's pose the inverse problem in a given area:

Inverse problem: It is necessary to determine the function K(t) if the following additional information is known about the solution of the direct problem (1)-(4):

$$u(x,\beta) = \psi(x), \quad x \in [0,1]. \tag{5}$$

here $\psi(\cdot)$is a given sufficiently smooth function

Definition 2. The solution to problem (1)-(5) is the functions $u(x,t)$ and $K(t)$ from the class $C(\bar{G}) \cap C^1(G) \cap C^{2,1}_{x,t}(G_+ \cup \{t = \beta\}) \cap C^2(G_- \cup \{t = -\alpha\})$ and $C[0,1]$ respectively, satisfying relations (1)-(5).

We expand the unknown function and the initially given functions into Fourier series (where $\lambda_n = \pi n$)

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin \lambda_n x,$$

$$\varphi(x) = \sum_{n=1}^{\infty} \varphi_n \sin \lambda_n x,$$

$$f(x) = \sum_{n=1}^{\infty} f_n \sin \lambda_n x.$$

Substituting the series for the functions $u(x,t)$ and $f(x)$ in (1) we get the following:

$$\begin{cases} u'_n(t) + \lambda_n^2 u_n(t) = f_n, & t > 0, \\ u''_n(t) + \lambda_n^2 u_n(t) = f_n + \int_{-\alpha}^{t} K(\tau) u_n(\tau) d\tau, & t < 0. \end{cases} \tag{6}$$

Let us introduce the following notation:

$$\mathrm{F}_n(t;u) = \int_{-\alpha}^{t} K(\tau) u_n(\tau) d\tau. \tag{7}$$

Substitute (7) into (6) we get the following:

$$\begin{cases} u'_n(t) + \lambda_n^2 u_n(t) = f_n, \ t > 0 \\ u''_n(t) + \lambda_n^2 u_n(t) = f_n + \mathrm{F}_n(t;u), \ t < 0 \end{cases} \tag{8}$$

We solve inhomogeneous ordinary differential equations in (9) by the method of variation of constants and obtain a general solution in the following form:

$$\begin{cases} u_n(t) = \mathrm{C}e^{-\lambda_n^2 t} + \dfrac{f_n}{\lambda_n^2}, \qquad t > 0, \\ u_n(t) = C_1 cos(\lambda_n t) + C_2 sin(\lambda_n t) + \dfrac{f_n}{\lambda_n^2} + \dfrac{1}{\lambda_n}\displaystyle\int_{-\alpha}^{t} \mathrm{F}_n(\tau;u)\cdot \sin\,(\lambda_n(t-\tau))d\tau\,, t < 0. \end{cases} \tag{9}$$

Using gluing conditions (4) and initial conditions (3) for unknown coefficients, we obtain the following system of linear equations:

$$\begin{cases} \mathrm{C} + \dfrac{f_n}{\lambda_n^2} = C_1 + \dfrac{f_n}{\lambda_n^2} - \dfrac{1}{\lambda_n}\displaystyle\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \sin\,(\lambda_n \tau)d\tau\,, \\ -\lambda_n^2 \mathrm{C} = \lambda_n C_2 + \displaystyle\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \cos\,(\lambda_n \tau)d\tau\,, \\ C_1 cos(\lambda_n \alpha) - C_2 sin(\lambda_n \alpha) + \dfrac{f_n}{\lambda_n^2} = \varphi_n. \end{cases}$$

From here we find the coefficients:

$$\begin{cases} C_1 = \mathrm{C} + \dfrac{1}{\lambda_n}\displaystyle\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \sin\,(\lambda_n \tau)d\tau\,, \\ C_2 = -\lambda_n \mathrm{C} - \dfrac{1}{\lambda_n}\displaystyle\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \cos\,(\lambda_n \tau)d\tau\,, \\ C\big(cos(\lambda_n \alpha) + \lambda_n sin(\lambda_n \alpha)\big) - \dfrac{1}{\lambda_n}\displaystyle\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \sin\big(\lambda_n(\alpha-\tau)\big)\,d\tau = \varphi_n - \dfrac{f_n}{\lambda_n^2}. \end{cases}$$

Which implies

$$C = \frac{\varphi_n - \frac{f_n}{\lambda_n^2} + \frac{1}{\lambda_n}\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \sin\big(\lambda_n(\alpha-\tau)\big)\,d\tau}{cos(\lambda_n \alpha) + \lambda_n sin(\lambda_n \alpha)}.$$

Then, substitute the found coefficients into (9) and obtain the formal solution:

$$\begin{cases} u_n(t) = \dfrac{\varphi_n - \frac{f_n}{\lambda_n^2} + \frac{1}{\lambda_n}\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \sin\big(\lambda_n(\alpha-\tau)\big)\,d\tau}{cos(\lambda_n \alpha) + \lambda_n sin(\lambda_n \alpha)} e^{-\lambda_n^2 t} + \dfrac{f_n}{\lambda_n^2}, \qquad t > 0, \\ u_n(t) = \dfrac{\varphi_n - \frac{f_n}{\lambda_n^2} + \frac{1}{\lambda_n}\int_{-\alpha}^{0} \mathrm{F}_n(\tau;u)\cdot \sin\big(\lambda_n(\alpha-\tau)\big)\,d\tau}{cos(\lambda_n \alpha) + \lambda_n sin(\lambda_n \alpha)}\big(cos(\lambda_n t) - \lambda_n sin(\lambda_n t)\big) + \\ \quad + \dfrac{f_n}{\lambda_n^2} + \dfrac{1}{\lambda_n}\displaystyle\int_{-\alpha}^{t} \mathrm{F}_n(\tau;u)\cdot \sin\,(\lambda_n(t-\tau))d\tau\,, t < 0. \end{cases} \tag{10}$$

Let's perform the following calculations:

$$\int_{-\alpha}^{t} \mathrm{F}_n(\tau;u)\cdot \sin\big(\lambda_n(t-\tau)\big)\,d\tau = \int_{-\alpha}^{t}\left[\int_{-\alpha}^{\tau} K(s)u_n(s)ds\right]\cdot \sin\big(\lambda_n(t-\tau)\big)\,d\tau =$$

$$= \left| \begin{matrix} U = \int_{-\alpha}^{\tau} K(s)u_n(s)ds\,, \rightarrow dU = K(\tau)u_n(\tau)d\tau \\ dV = \sin\big(\lambda_n(t-\tau)\big)\,d\tau, \rightarrow \quad V = \frac{1}{\lambda_n}\cos\big(\lambda_n(t-\tau)\big) \end{matrix} \right| =$$

$$= \int_{-\alpha}^{\tau} K(s)u_n(s)ds \cdot \frac{1}{\lambda_n}\cos\big(\lambda_n(t-\tau)\big)\Big|_{\tau=-\alpha}^{t} -$$

$$-\frac{1}{\lambda_n}\int_{-\alpha}^{t} K(\tau)cos\big(\lambda_n(t-\tau)\big)u_n(\tau)d\tau = \frac{1}{\lambda_n}\int_{-\alpha}^{t} K(\tau)u_n(\tau)d\tau -$$

$$-\frac{1}{\lambda_n}\int_{-\alpha}^{t} K(\tau)cos\big(\lambda_n(t-\tau)\big)u_n(\tau)d\tau =$$

$$=\frac{1}{\lambda_n}\int_{-\alpha}^{t}K(\tau)\left(1-cos\left(\lambda_n(t-\tau)\right)\right)u_n(\tau)d\tau,$$

that is

$$\int_{-\alpha}^{t}\mathrm{F}_n(\tau;u)\cdot\sin\left(\lambda_n(t-\tau)\right)d\tau=\frac{1}{\lambda_n}\int_{-\alpha}^{t}K(\tau)\left(1-cos\left(\lambda_n(t-\tau)\right)\right)u_n(\tau)d\tau. \quad (12)$$

Let us calculate the following integral in exactly the same way

$$\int_{-\alpha}^{0}\mathrm{F}_n(\tau;u)\cdot\sin\left(\lambda_n(\alpha-\tau)\right)d\tau=\int_{-\alpha}^{0}\left[\int_{-\alpha}^{\tau}K(s)u_n(s)ds\right]\cdot\sin\left(\lambda_n(\alpha-\tau)\right)d\tau=$$

$$=\left|\begin{matrix} U=\int_{-\alpha}^{\tau}K(s)u_n(s)ds,\rightarrow dU=K(\tau)u_n(\tau)d\tau \\ dV=\sin\left(\lambda_n(\alpha-\tau)\right)d\tau,\rightarrow \quad V=\frac{1}{\lambda_n}\cos\left(\lambda_n(\alpha-\tau)\right)\end{matrix}\right|=$$

$$=\int_{-\alpha}^{\tau}K(s)u_n(s)ds\cdot\frac{1}{\lambda_n}\cos\left(\lambda_n(\alpha-\tau)\right)\Big|_{\tau=-\alpha}^{0}-$$

$$-\frac{1}{\lambda_n}\int_{-\alpha}^{0}K(\tau)cos\left(\lambda_n(\alpha-\tau)\right)u_n(\tau)d\tau=$$

$$=\frac{\cos(\lambda_n\alpha)}{\lambda_n}\int_{-\alpha}^{0}K(\tau)u_n(\tau)d\tau-\frac{1}{\lambda_n}\int_{-\alpha}^{0}K(\tau)cos\left(\lambda_n(\alpha-\tau)\right)u_n(\tau)d\tau=$$

$$=\frac{1}{\lambda_n}\int_{-\alpha}^{0}K(\tau)\left(\cos(\lambda_n\alpha)-cos\left(\lambda_n(\alpha-\tau)\right)\right)u_n(\tau)d\tau,$$

that is

$$\int_{-\alpha}^{0}\mathrm{F}_n(\tau;u)\cdot\sin\left(\lambda_n(\alpha-\tau)\right)d\tau=\frac{1}{\lambda_n}\int_{-\alpha}^{0}K(\tau)\left(\cos(\lambda_n\alpha)-cos\left(\lambda_n(\alpha-\tau)\right)\right)u_n(\tau)d\tau. \quad (13)$$

Substituting (12) and (13), into (11) we obtain the following integral equations:

$$u_n(t)=\frac{\varphi_n-\frac{f_n}{\lambda_n^2}+\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\left(\cos(\lambda_n\alpha)-cos\left(\lambda_n(\alpha-\tau)\right)\right)u_n(\tau)d\tau}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}e^{-\lambda_n^2 t}+\frac{f_n}{\lambda_n^2},\quad t>0,\ (14)$$

$$u_n(t)=\frac{cos(\lambda_n t)-\lambda_n sin(\lambda_n t)}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}\left[\varphi_n-\frac{f_n}{\lambda_n^2}+\right.$$

$$\left.+\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\left(\cos(\lambda_n\alpha)-cos\left(\lambda_n(\alpha-\tau)\right)\right)u_n(\tau)d\tau\right]+$$

$$+\frac{f_n}{\lambda_n^2}+\frac{1}{\lambda_n}\int_{0}^{t}K(\tau)\left(1-cos\left(\lambda_n(t-\tau)\right)\right)u_n(\tau)d\tau,\qquad t<0. \qquad (15)$$

If the free term and the kernel are continuous functions, then the unique solvability of equations (13) can be proven by the method of successive approximations.

We substitute the solution of equations (14) and (15) into the next series

$$u(x,t)=\sum_{n=1}^{\infty}u_n(t)\sin\lambda_n x \qquad (16)$$

and find the solution (1) of the equation that satisfies the given conditions.

$$u(x,t)=\sum_{n=1}^{\infty}\left[\frac{\varphi_n-\frac{f_n}{\lambda_n^2}+\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\left(\cos(\lambda_n\alpha)-cos\left(\lambda_n(\alpha-\tau)\right)\right)u_n(\tau)d\tau}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}e^{-\lambda_n^2 t}+\right.$$

$$\left.+\frac{f_n}{\lambda_n^2}\right]\sin(\lambda_n x),\qquad t>0, \qquad (17)$$

$$u(x,t)=\sum_{n=1}^{\infty}\left[\frac{cos(\lambda_n t)-\lambda_n sin(\lambda_n t)}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}\left[\varphi_n-\frac{f_n}{\lambda_n^2}+\right.\right.$$

$$\left.+\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\Big(\cos(\lambda_n\alpha)-cos\big(\lambda_n(\alpha-\tau)\big)\Big)u_n(\tau)d\tau\right]+$$

$$\left.+\frac{f_n}{\lambda_n^2}+\frac{1}{\lambda_n}\int_0^t K(\tau)\Big(1-cos\big(\lambda_n(t-\tau)\big)\Big)u_n(\tau)d\tau\right]\sin(\lambda_n x),t<0. \quad (18)$$

Now we will prove the uniqueness of the solution.

We solve problem (1)-(4) for the case $f(x)=0$ and $\varphi(x)=0$, then we obtain solutions in the following form:

$$u(x,t)=\sum_{n=1}^{\infty}\frac{\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\Big(\cos(\lambda_n\alpha)-cos\big(\lambda_n(\alpha-\tau)\big)\Big)u_n(\tau)d\tau}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}e^{-\lambda_n^2 t}\sin(\lambda_n x),$$

$t>0,$ (19)

$$u(x,t)=\sum_{n=1}^{\infty}\left[\frac{cos(\lambda_n t)-\lambda_n sin(\lambda_n t)}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\Big(\cos(\lambda_n\alpha)-cos\big(\lambda_n(\alpha-\tau)\big)\Big)u_n(\tau)d\tau+\right.$$

$$\left.+\frac{1}{\lambda_n}\int_0^t K(\tau)\Big(1-cos\big(\lambda_n(t-\tau)\big)\Big)u_n(\tau)d\tau\right]\sin(\lambda_n x),t<0. \quad (20)$$

Common members of these series

$$u_n(t)=\frac{\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\Big(\cos(\lambda_n\alpha)-cos\big(\lambda_n(\alpha-\tau)\big)\Big)u_n(\tau)d\tau}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}e^{-\lambda_n^2 t},\quad t>0,\ (21)$$

$$u_n(t)=\frac{cos(\lambda_n t)-\lambda_n sin(\lambda_n t)}{cos(\lambda_n\alpha)+\lambda_n sin(\lambda_n\alpha)}\frac{1}{\lambda_n^2}\int_{-\alpha}^{0}K(\tau)\Big(\cos(\lambda_n\alpha)-cos\big(\lambda_n(\alpha-\tau)\big)\Big)u_n(\tau)d\tau$$

$$+\frac{1}{\lambda_n}\int_{-\alpha}^{t}K(\tau)\Big(1-cos\big(\lambda_n(t-\tau)\big)\Big)u_n(\tau)d\tau\ ,t<0. \quad (22)$$

For $t>0$ we have a homogeneous Fredholm equation of the 2nd kind of the form (21). In this case $\{\lambda_n,\sin(\lambda_n x)\}\in L^2(0,1)$ and $0=u_n=(u,\sin(\lambda_n x))=0$. Then $u(x,t)\equiv 0$.

Now consider the case $t<0$. We have obtained an integral equation of the Volterra-Fredholm type. The book [12] shows the fulfillment of the following inequality

Theorem 1[12]. Let $u(t),a(t),b(t),f(t),g(t)\in C(I,R_+)$.

(a_1) Let $a(t)$ be continuously differentiable on I, $a'(t)\ \geq\ 0$ and

$$u\ (t)\ \leq a(t)+\int_{\alpha}^{t}b(s)u(s)ds\ +\int_{\alpha}^{\beta}c(s)u(s)ds,$$

for $t\in I$. If

$$p_1=\int_{\alpha}^{\beta}c(s)\exp\left(\int_{\alpha}^{s}b(\sigma)\,d\sigma\right)ds<1,$$

then

$$u\ (t)\leq M_1\exp\left(\int_{\alpha}^{t}b(s)\,ds\right)+\int_{\alpha}^{\beta}a'(s)\exp\left(\int_{\alpha}^{t}b(\sigma)\,d\sigma\right)ds,$$

for $t\in I$, where

$$M_1=\frac{1}{1-p_1}\left[a(\alpha)+\int_{\alpha}^{\beta}c(s)\left(\int_{\alpha}^{s}a'(\tau)\exp\left(\int_{\tau}^{t}b(\sigma)\,d\sigma\right)d\tau\right)ds\right]$$

(a2) Suppose that

$$u(t) \le a(t) + b(t)\int_{\alpha}^{t} f(s)u(s)ds + c(t)\int_{\alpha}^{\beta} g(s)u(s)ds.$$

for $t \in I$. If

$$p_2 = \int_{\alpha}^{\beta} g_2(s)K_2(s)ds < 1,$$

then

$$u(t) \le K_1(t) + M_2K_2(t)$$

for $t \in I$, where

$$K_1(t) = a(t) + b(t)\int_{\alpha}^{t} f(\tau)a(\tau)\exp\left(\int_{\tau}^{t} f(\sigma)b(\sigma)d\sigma\right)d\tau,$$

$$K_2(t) = c(t) + b(t)\int_{\alpha}^{t} f(\tau)c(\tau)\exp\left(\int_{\tau}^{t} f(\sigma)b(\sigma)d\sigma\right)d\tau,$$

and

$$M_2 = \frac{1}{1-p_2}\int_{\alpha}^{\beta} g(s)K_1 ds.$$

In our case

1) $a(t) \equiv 0,$

2) $b(\tau) = \frac{1}{\lambda_n}K(\tau)\left(1 - cos\left(\lambda_n(t-\tau)\right)\right),\ K \in C[-\alpha, 0],$

3) $c(\tau) = \frac{cos(\lambda_n t) - \lambda_n sin(\lambda_n t)}{cos(\lambda_n \alpha) + \lambda_n sin(\lambda_n \alpha)}\frac{1}{\lambda_n^2}\int_{-\alpha}^{0} K(\tau)\left(\cos(\lambda_n\alpha) - cos\left(\lambda_n(\alpha-\tau)\right)\right)u_n(\tau)d\tau.$

If

$$p_1 = \int_{\alpha}^{\beta} c(s)\exp\left(\int_{\alpha}^{s} b(\sigma)\,d\sigma\right)ds < 1,$$

then $u_n(t) \le M_1\exp\left(\int_{\alpha}^{t} b(s)\,ds\right) + \int_{\alpha}^{\beta} a'(s)\exp\left(\int_{\alpha}^{t} b(\sigma)\,d\sigma\right)ds$

where $\int_{\alpha}^{\beta} a'(s)\exp\left(\int_{\alpha}^{t} b(\sigma)\,d\sigma\right)ds = 0$ and

$$M_1 = \frac{1}{1-p_1}\left[a(\alpha) + \int_{\alpha}^{\beta} c(s)\left(\int_{\alpha}^{s} a'(\tau)\exp\left(\int_{\tau}^{t} b(\sigma)\,d\sigma\right)d\tau\right)ds\right] = 0,$$

from here $u_n(t) = 0$ and

$0 = u_n = (u, \sin(\lambda_n x)) = 0$. Then $u(x,t) \equiv 0$.

Thus we proved the following theorem:

Theorem*. Let the following conditions be satisfied*

$$|cos(\lambda_n\alpha) + \lambda_n sin(\lambda_n\alpha)| \ge \alpha_0 > 0,$$

and $\varphi \in C^2[0,1],\ f \in C^1[0,1],\ K \in C[-\alpha, \beta]$.

Then the solution to problem (1)-(4), presented in the form of series (19)-(20), is unique.

4. Conclusion. The theory of inverse and ill-posed problems is widely used in almost all areas of science, in particular in solving practical problems. Therefore, in this article, at the beginning, direct problems posed for integro-differential equations of parabolic and hyperbolic types are studied and a method for solving them is given.

Currently, the study of inverse problems is relevant, so the formulation of the inverse problem for a mixed interdifferential equation is given.

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