BUXORO DAVLAT UNIVERSITETI

BUXORO DAVLAT UNIVERSITETI
ILMIY AXBOROTI

3/2022

3/2022 Научный вестник Бухарского государственного университета
Scientific reports of Bukhara State University

E-ISSN 2181-1466
9 772181 146004

ISSN 2181-6875
9 772181 687004

@buxdu_uz @buxdu1 @buxdu1 www.buxdu.uz

PUBLISHED SINCE 2000 (Online since 2020)

PUBLISHED SIX TIMES A YEAR

2022/3(91)

SCIENTIFIC REPORTS OF BUKHARA STATE UNIVERSITY

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НАУЧНЫЙ ВЕСТНИК БУХАРСКОГО ГОСУДАРСТВЕННОГО УНИВЕРСИТЕТА

The journal is published in the Bukhara Regional Department of Press and Information of the Press and Information Agency of Uzbekistan on August 24, 2020 With registered certificate № 1103

The journal "Scientific reports of Bukhara state university" is included in the list of scientific publications recommended to publish the main scientific results of doctoral dissertations of the Higher Attestation Commission under the Cabinet of Ministers of the Republic of Uzbekistan on philology and physical and mathematical sciences.

The journal is intended for professors and teachers of higher educational institutions, senior researchers, students, scientific staff of scientific research institutions, teachers of academic lyceums, professional colleges, as well as researchers in general secondary education and various fields.

Founder: BUKHARA STATE UNIVERSITY

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UDC: 517

NUMERICAL STUDY OF THE FIRST AND SECOND BOUNDARY VALUE PROBLEMS POSED FOR A MODEL EQUATION OF MIXED TYPE IN SPACE, AND THE CALCULATION OF THE APPROXIMATION OF A STABLE FINITE-DIFFERENCE SCHEME OF THESE PROBLEMS

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Abstract:

Introduction. *When analyzing difference schemes for partial differential equations, we always carry out research, dividing it into two stages. The first step is to check the approximation. The second stage is to test the so-called sustainability. In this article, stable difference schemes are constructed for the multidimensional case, and the order of approximation of the stable difference scheme in the one-dimensional case is determined.*

Research methods. *To solve a given boundary value problem, we will use an approximate (numerical) method. Among the approximate methods for solving differential ones used now, "finite-difference schemes" are widely used. In this method we discretize the considered area where the problem is solved, i.e. build a difference network; we approximate the initial differential problem by a finite-difference scheme on the constructed network; prove stability; we find a numerical solution to the differential problem using a finite difference scheme.*

Results. *Today, the use of modern electronic computers plays an important role in the development of the theory of differential equations. When studying differential equations, computational experiments to determine one or another property of solutions can then be theoretically justified and serve as the basis for further theoretical research.*

Discussions. *The study of direct and inverse problems posed to a mixed type equation is one of the advanced critical and rapidly emerging areas of world science. Their numerical implementation provides an applied application for the study of these problems. In this paper, we numerically study the boundary value problem posed to a model equation of mixed type. To do this, you need to know the concept of approximation and stability. The stability of the difference scheme has been proven. The order of approximation is calculated in the work. Further, when the stability and approximation are proved, it is possible to show the approximation of the numerical solution to the exact solution.*

Conclusion. *Conclusion. A priori estimates have been obtained for the solutions of equations of mixed type. The analytical solution of nonclassical equations in mathematical physics is a very complex process, so the boundary value problems in these equations are approximated by differential schemes and the stability is checked, which allows to solve a number of boundary value problems for mixed type equations.*

Keywords: *mixed type equations, stability, approximation, difference scheme.*

Introduction. The theory of differential equations is one of the largest branches of modern mathematics. To illustrate its place in modern mathematical science, it is worth noting the peculiarities of two major branches of the theory of differential equations: ordinary differential equations and the theory of partial differential equations.

The first peculiarity is the widespread application of the theory of differential equations in life. If we consider mathematics as a science that reveals the secrets of nature, its main purpose is to build mathematical models of real life. When a researcher studies a physical phenomenon, he constructs its mathematical ideal, in other words, he sees a mathematical model. In most cases, this model takes the form of a differential equation. Models of phenomena in the mechanics of contiguous media, chemical reactions, electrical, magnetic, and other phenomena take the form of differential equations.

By studying the obtained differential equations with additional, i.e., initial and boundary conditions, the mathematician can obtain information about the event that is taking place, and sometimes be aware of the past and future of the event.

Mathematical, the study of a model by mathematical methods allows not only to obtain qualitative characteristics of physical phenomena, but also to know the basis of a physical phenomenon, sometimes to discover new physical phenomena. Mathematically, the criterion for determining the correct choice of the model is the comparison of the results obtained with the help of mathematical research with experimental data.

As A. Poincare said: “Mathematics is the art of calling different things by one name,” that is, mathematics studies various phenomena of real life in one way - a mathematical model.

In order to construct a mathematical model in the form of differential equations, it is not necessary to have complete knowledge of the whole physical phenomenon, but only to know the necessary connections.

The study of the basic equations of mathematical physics led to the classification of equations and systems of partial differential equations. In the first 30s of the last century I.G. Petrovsky classified partial differential equations. Today, hyperbolic, parabolic and elliptic types of equations are widely studied. Mixed-type partial differential equations are considered unstudied by the class of partial differential equations.

Hence, a distinctive feature of the theory of differential equations is its wide application. The second distinctive aspect is the relationship of the theory of differential equations with other branches of mathematics: functional analysis, algebra, and probability theory.

The theory of differential equations, mainly the theory of special derivative differential equations, makes extensive use of basic concepts, ideas, and methods in various fields of mathematics and also influences their problems and the nature of research.

Many sections of the theory of differential equations have developed in such a way that as a result they have been formed as separate directions. One such direction is mixed type equations. The classical equations of mathematical physics and the problems posed to them are studied.

Today's time is the use of computers, i.e. computers in the study of differential equations is considered relevant. When studying differential equations, and when finding certain properties of a solution, computational experiments are conducted, and this will become the basis for further theoretical research.

Computational experiments are also used in physics and in theoretical research. This is done using a mathematical model of a physical process. The purpose of a computational experiment using a computer in the right accuracy, spending little computer time to express the physical process. On the basis of such experiments lies in many cases partial differential equations. This leads to the connection between theories of differential equations and computational mathematics, mainly with the sections of the finite difference method, the finite element method, etc. It means that their widespread use is characteristic of differential equations.

Physics and other natural sciences provide the theory of differential equations with unsolved problems, but it can also happen that mathematical studies over time find their exact life applications. Tricomi's task is an example of this. After a quarter century, the solution to this problem has found its application in gas dynamics.

The sections of differential equations have developed so much that they are formed as separate directions. One such direction of the partial differential equation.

The relevance of the work. The classical equations of mathematical physics and the problems posed by him have been studied sufficiently. There are enough experimental and theoretical works on the analytical and numerical solution of classical equations. The study of mixed equations, called non-classical problems of mathematical physics, began at the beginning of the last century.

The study of equations of mixed and composite type from the mathematical side is very important because they are used in different branches of mechanics and physics. For example, the problem of motion in gas dynamics, where there are subsonic and post-sound areas. Usually such free waves are called mixed, subsonic or transonic waves.

In today's time of existence and uniqueness of the solution of a mixed type equation, it has been proved in a functional way, and much has been done. One can say that a theory of mixed type equations has been created. There are enough works done to prove in a funkitsonal way the existence and uniqueness of a solution to a mixed type equation, we can say how the theory of mixed type equations was formulated. The next task is to create a theory of numerical methods for a mixed type equation. Enough work has been done in this area today. Now the task is to solve these types of equations by numerical methods and create a theory of numerical calculations of mixed types. Currently, a lot of work has been done in this direction. Basically, replacing mixed type equations with finite division schemes can prove that these schemes are elastic, which proves the validity of the boundary value problem of the mixed type equation, as well as the existence and uniqueness of the solution.

A priori estimates were obtained for solving mixed equations, and many scientists devoted and devoted their research to this field. The study of a discrete model of boundary value problems for mixed equations today has not lost its relevance. In our opinion, if we study the discrete model of boundary value problems for mixed equations, then we first obtain numerical analogues of the a priori estimates obtained for solving these equations and prove that these problems can be solved numerically. The main task here is to demonstrate the adequacy of the considered new discrete models or the original differential problem.

The scope of the studied problem. Mixed flows are usually determined by equations of a mixed-type elliptic-hyperbolic type, therefore, the study of the dynamics of a transition gas is closely connected with the development of mixed-type equations. First of all, the importance of studying mixed type equations was emphasized by S. A. Chaplygin in his 1902 work, Gas Flows. The study of systematically mixed types of equations began in the 1920s. The Italian scientist F. Tricomi began to study equations of various kinds and generalized them to S. Gellerstend. For the first time, they introduced boundary conditions for mixed equations and began to study them. These equations are now called by their names.

M. V. Keldysh, M. A. Lavrentiev, A. V. Bitsadze, K. I. Babenko, F. I. Frankel, A new stage in the development of differential equations with differential equations, equations of a mixed type and the theory of nonlinear classical equations. L.V. Ossyannikov, L.D. Kudryavtsev and a number of their students.
Further development of the theory of equations of mixed type A.V. Bitsadze, O.A. Alynik, E.V. Radkevich, M.M. Smirnov, G. Fiker, T. Dzhuraev, D.G. Karatoprakliev, M.S. Salakhitdinov, V.P. Glushko, T.S. Kalmenov, A.I. Kiprianova, V.K. The names of N. Vragov, B. A. Bubnov and other authors are connected.

Scientific and practical significance of the work.

The analytical solution of nonlinear equations of mathematical physics is a very complex process, therefore, stable boundary schemes for boundary value problems for these equations are constructed, which allow solving a finite number of boundary value problems for mixed equations.

Formulation of the problem: Let be $\Omega \subset R^n$ - limited area $\gamma \in C^2$ the boundary of a given region, a simply connected region.

Let be $Q = \Omega \times (-T,T)$, $S = \gamma \times (-T,T)$, where Γ - Q border of a given area. $Q^+ = Q \cap \{t > 0\}$, $Q^- = Q / Q^+$.

In the field, consider the following differential equation:

$$Lu \equiv K(x,t)\cdot u_{tt} - \sum_{i,j=1}^{n} a_{ij}(x,t)\cdot u_{x_i x_j} + a(x,t)\cdot u_t + \sum_{i=1}^{n} a_i(x,t)\cdot u_{x_i} + c(x,t)\cdot u = f(x,t) \qquad (1)$$

where, $K(x,t), a_{i,j}(x,t) \in C^2(\overline{Q})$, $i,j = \overline{1,n}$; $a(x,t), a_i(x,t) \in C^1(\overline{Q})$, $c(x,t) \in C(Q)$,

$a_{ij}(x,t) = a_{ji}(x,t)$, $i,j = \overline{1,n}$; $t \cdot K(x,t) > 0$, $t \neq 0$; $K(x,0) = 0$, $x \in \overline{\Omega}$; $\sum_{i,j=1}^{n} a_{ij}\cdot \xi_i \cdot \xi_j > 0$, $x \in \Omega$,

$\forall \xi \in R^n,\ |\xi| \neq 0;\ \sum_{i,j=1}^{n} a_{ij}(x,t) \cdot n_i \cdot n_j = 0,\ (x,t) \in S;\ n = (n_1, n_2, ..., n_n, n_t)$ - vector inner normals to. $\beta(x) \equiv a(x,0) - K_t(x,0) > 0,\ x \in \overline{\Omega}$.

Note that in the region Q equation (1) is a mixed type equation. Namely, when $t > 0$ - hyperbolic-parabolic type, with $t < 0$ - elliptic-parabolic type.

The first boundary value problem. Find solutions to the equation:

$$Lu = f(x,t) \text{ в } Q \tag{1}$$

such that

$$u(x,-T) = 0,\ \ x \in \overline{\Omega} \tag{2}$$

Suppose in equation (1) given in a cylindrical region Q:

$$\sum_{i,j=1}^{n} a_{ij}(x,t) \cdot u_{x_i x_j} = h(x,t) \cdot \Delta_x u\ ,$$

when

$$\Delta_x u = \sum_{i,j=1}^{n} u_{x_i x_j},\ \ h(x,t) \in C^\infty(\overline{Q});\ \ a(x,-T) = 0;\ \ h(x,t) > 0,\ \ (x,t) \in Q;\ \ h(x,t) = 0,\ \ (x,t) \in S\ ;$$

$$\beta(x) \equiv a(x,0) - K_t(x,0) < 0,\ x \in \overline{\Omega}.$$

And then we get the following equation:

$$Lu \equiv K(x,t) \cdot u_{tt} - h(x,t) \cdot \Delta_x u + a(x,t) \cdot u_t + \sum_{i=1}^{n} a_i(x,t) \cdot u_{x_i} + c(x,t) \cdot u = f(x,t) \tag{1'}$$

We assume that the coefficients of an equation from the class $C^\infty(Q)$.

Second boundary value problem: Find in the field of solution of the equation:

$$Lu = f(x,t) \tag{1'}$$

satisfying condition

$$u(x,-T) = u(x,T) = u_t(x,T) = 0\ x \in \overline{\Omega}\ . \tag{3}$$

Numerical solution of assigned tasks. The numerical solution of the boundary value problems (1) - (2) and - (3) is not an easy task because a stable difference scheme is not constructed for them.

In this paper, we propose stable difference schemes and numerically solve (1) - (2) and - (3) the first and second boundary value problems for an equation of mixed type.

Using the functional approach in [1], the following inequalities were proved for the first boundary-value problem:

$$\int_Q e^{-\lambda t} u_t LudQ + \mu_0 \int_Q uLudQ \geq m\|u\|_{H_1}^2 \qquad (4.a)$$

$$\|Lu\|_0 \geq m\|u\|_{H_1}, \qquad (4.b)$$

and for the second boundary-value problem, the following estimates:

$$-\int_{Q^+} e^{\lambda t} u_t LudQ^+ \geq m \cdot \int_{Q^+} (u_t^2 + h(x,t) \cdot u_x^2 + u^2) dQ^+ \qquad (5.a)$$

$$\|Lu\|_{o,Q^+} \geq m \cdot \|u\|_{H_1(Q^+)}, \quad \forall u \in C_Q \qquad (5.b)$$

$$\int_{Q^-} uLudQ^- \geq m_1 \cdot \int_{Q^-} \left(| K(x,t) | \cdot u_t^2 + h(x,t) \cdot u_x^2 + u^2 \right) dQ^- \qquad (5.c)$$

where are the constants m, m_1 independent of function $u(x,t)$ and the uniqueness theorem for solving problems is proved (1)-(2) и (1')-(3).

<u>Difference schemes.</u> For the numerical solution of the mixed problem (1) - (2) we offer the following scheme:

$$\begin{cases} L^- u \equiv K\left(\dfrac{\tau \bar{\tau} u}{\Delta^2}\right) - \sum\limits_{i,j=1}^{n} \dfrac{1}{\Delta \chi_i^{\,2}} \cdot a_{ij} \xi_i \overline{\xi_j} u + \dfrac{a \tau u}{\Delta} + \sum\limits_{i=1}^{n} \dfrac{1}{\Delta \chi_i} \cdot a_i \overline{\xi_i} u + cu = f \\ \qquad\qquad k = \overline{-m+1,0}; \quad l_i = \overline{0, N_i}, i = \overline{1,n} \qquad (6) \\ L^+ u \equiv K\left(\dfrac{\overline{\tau \tau} u}{\Delta^2}\right) - \sum\limits_{i,j=1}^{n} \dfrac{1}{\Delta \chi_i^{\,2}} \cdot a_{ij} \xi_i \overline{\xi_j} u + \dfrac{a \tau u}{\Delta} + \sum\limits_{i=1}^{n} \dfrac{1}{\Delta \chi_i} \cdot a_i \overline{\xi_i} u + cu = f \\ \qquad\qquad k = \overline{1,m}; \quad l_i = \overline{0, N_i}, i = \overline{1,n} \qquad (6') \\ u^{-m}_{l_1, l_2, \ldots, l_n} = 0, \qquad l_i = \overline{0, N_i}, i = \overline{1,n} \qquad (7) \end{cases}$$

and for the numerical solution of the problem - (3) we offer the following difference scheme:

$$\begin{cases} L^- u \equiv K\left(\dfrac{\tau \bar{\tau} u}{\Delta^2}\right) - h \cdot \sum\limits_{i,j=1}^{n} \dfrac{1}{\Delta \chi_i^{\,2}} \cdot \xi_i \overline{\xi_j} u + \dfrac{a \tau u}{\Delta} + \sum\limits_{i=1}^{n} \dfrac{1}{\Delta \chi_i} \cdot a_i \overline{\xi_i} u + cu = f \\ \qquad\qquad k = \overline{-m+1,0}; \quad l_i = \overline{0, N_i}, i = \overline{1,n} \qquad (8) \\ L^+ u \equiv K\left(\dfrac{\overline{\tau \tau} u}{\Delta^2}\right) - h \cdot \sum\limits_{i,j=1}^{n} \dfrac{1}{\Delta \chi_i^{\,2}} \cdot \xi_i \overline{\xi_j} u + \dfrac{a \tau u}{\Delta} + \sum\limits_{i=1}^{n} \dfrac{1}{\Delta \chi_i} \cdot a_i \overline{\xi_i} u + cu = f \\ \qquad\qquad k = \overline{1,m}; \quad l_i = \overline{0, N_i}, i = \overline{1,n} \qquad (8') \\ u^{-m} = u^m = \dfrac{1}{\Delta} \tau u^m = 0, \qquad l_i = \overline{0, N_i}, i = \overline{1,n} \qquad (9) \end{cases}$$

Here $u = u\left(t^k, x_{1,l_1}, x_{2,l_2}, \ldots, x_{n,l_n}\right) = u^k_{l_1, l_2, \ldots, l_n}$, $i = \overline{1,n}$,

$\varphi, \varphi^{-1}, \psi_i, \psi_i^{-1}$ - shift operators. $\varphi u = u_{l_i}^{k+1} = u^{k+1}$, $\varphi \; u = u_{l_i}^{k+1} = u^{k+1}$,

$\psi_i u = u_{l_i+1}^{k} = u_{l_i+1}$, $\psi_i^{-1} u = u_{l_i-1}^{k} = u_{l_i-1}$,

а также $\tau, \bar{\tau}, \xi_i, \bar{\xi}_i$ - difference operators:

$\tau = \varphi - 1$, $\bar{\tau} = 1 - \varphi^{-1}$, $\xi_i = \psi_i - 1$, $\bar{\xi}_i = 1 - \psi_i^{-1}$,

Δ - step by t, a $\Delta\chi_i$- step by x_i, $i = \overline{1,n}$.

System of linear algebraic equations (6)-(6')-(7) and (8)-(8')-(9) relatively unknown $\left\{u_{l_1,l_2,\dots,l_n}^{k}\right\}_{l_i=\overline{0,N_i},i=\overline{1,n}}^{k=\overline{-m,m}}$, forms a complete system.

Here, using some examples, we will show the unique solvability of the difference scheme and present the results of a numerical calculation.

When $n=1$ в work [25], that is, for the equation given on the plane R^2 The solvability of the first boundary value problem was proved numerically.

Now let us show how to calculate the order of approximation of these schemes for the one-dimensional case.

When analyzing difference schemes for partial differential equations, we always conduct research, dividing it into two stages.

First stage consists in checking that the solution u of the differential equation

$$Lu = f \qquad (10)$$

of interest to us, after substitution in the approximating difference equation $L_h u = f$, almost exactly satisfies this equation. As a rule, the validity of equalities of the type

$$L_h u - Lu = O(h),$$

$$L_h u - Lu = O(h^2 + \tau^2) \text{ etc.}$$

(h, τ are the steps of the difference scheme). Testing the validity of this kind of statement is called an approximation test.

At the beginning, we give the exact meaning of the approximation of the differential boundary value problem.

Let problem (1) be given. A difference scheme was constructed for a numerical solution:

$$L_h u^{(h)} = f^{(h)}, \qquad (11)$$

which approximates problem (10) on the solution u for some of order h^k. This means the residual $\delta f^{(h)}$

$$L_h[u]_h = f^{(h)} + \delta f^{(h)}, \qquad (12)$$

arising from the substitution of the grid function $[u]_h$-table of the desired solution u - in equation (11), satisfies an estimate of the form:

$$\left\|\delta f^{(h)}\right\|_{F_h} \le C_1 h^k. \qquad (13)$$

where $C_1$is some constant independent of h.

The tendency of the residual value $\delta f^{(h)}$to zero as $h \to 0$ is taken as the definition of an approximation [2]. The approximation is insufficient for convergence. We also need stability.

Second stage consists in checking the so-called stability. Stability is understood as the fulfillment of the inequality for solutions of the difference equations $L_h u_h = f_h$.

$$\|u_h\| \leq M\|f_h\|.$$

Here $\|u_h\|$, $\|f_h\|$ are any norms in which the "value" of the difference solution u_h and the right-hand side of $f_h$is measured; M is a constant independent of the steps of the difference scheme[1].

Definition 1 [2]. Difference scheme (2) will be called stable if there exist numbers $h_0 > 0$ and δ> 0 such that for any $h < h_0$and any $\varepsilon^h \in F_h$, $\left\|\varepsilon^{(h)}\right\|_{F_h} < \delta$, difference problem (12) obtained from problem (11) by adding to the right-hand side of the perturbation $\varepsilon^{(h)}$, has one and only one solution $z^{(h)}$, and this solution deviates from the solution $u^{(h)}$unperturbed problem (11) on the grid function $z^{(h)} - u^{(h)}$satisfying the estimate

$$\left\|z^{(h)} - u^{(h)}\right\|_{U_h} \leq C\left\|\varepsilon^{(h)}\right\|_{F_h} \tag{14}$$

where C is some constant independent of h, $F_h$and $U_h$are linear, normed spaces consisting, respectively, of the elements $f^{(h)}$or $\delta f^{(h)}$initially given and $u^{(h)}$of the sought elements.

Let the operator $L_h$mapping $U_h$to $F_h$be linear. Then the above definition of stability is equivalent to the following:

Definition 2 [2]. We call difference scheme (11) with linear operator $L_h$stable if, for any $f^{(h)} \in F_h$, equation (11) has a unique solution $u^{(h)} \in U_h$, and

$$\left\|u^{(h)}\right\|_{U_h} \leq C\left\|f^{(h)}\right\|_{F_h} \tag{15}$$

where C is some constant independent of h.

If the difference equations approximate the differential equations and if the difference equations are stable, then it is easy to prove the closeness of the exact and approximate solutions [1,2]. One of the important problems in the theory of difference schemes is the construction of stable and exact schemes for second-order differential equations containing terms with first derivatives. The problem becomes much more complicated if the equations are of mixed type.

A number of methods for constructing difference schemes for partial differential equations were indicated by Godunov, Ryabenky, Samarsky and other authors (see, for example, [1-4]).

Much work has been done to prove the existence and uniqueness of solutions of equations of mixed type, and it can be said that the theory of equations of mixed type has been developed. The task now is to solve this type of equation by numerical methods and to create a theory of numerical calculation of mixed type equations. Much work is currently being done in this regard. Basically, by substituting mixed-type equations with finite-difference schemes, it is possible to prove the stability of these schemes, and on this basis to prove the correctness of the boundary value problem and the solution of the problem that exists and is unique.

A priori estimates have been obtained for the solutions of equations of mixed type, and many scientists have devoted and continue to dedicate their scientific work in this direction. The study of the differential model of boundary value problems for mixed-type equations has not lost its relevance to this day. In our view, if a differential model of boundary value problems for mixed-type equations is studied, first, a numerical analogue of the a priori estimates obtained for the solutions of these equations is obtained, and it is proved that these problems can be solved numerically. The main task is to show the adequacy of the new discrete models under consideration or construction to the initial differential problem.

Mixed flows are usually defined by equations of the elliptic-hyperbolic type of the mixed type, so the study of transistor gas dynamics is closely related to the development of the theory of mixed-type equations. The importance of studying equations of mixed type was first emphasized by S.A. Chaplygin in his 1902 work On Gas Flows. The study of systemically mixed type equations dates back to the 1920s. The Italian scientist F. Trikomi began to study equations of mixed type, which were generalized by S. Gellerstend [5-8]. They were the first to set boundary conditions for mixed-type equations and to study them. These equations are now called by their name.

A new stage in the development of the theory of differential equations of variable type, equations of mixed type and equations of nonclassical type is M.V Keldysh, M.A. Lavrentev, AV Bitsadze, K.I. Babenko, F.I. There were a number of works by LV Ovsyannikov, LD Kudryavtsev and their students.

Further development of the theory of equations of mixed type was carried out by A.V. Bitsadze, O.A. Oleynik, E.V. Radkevich, M.M.Smirnov, G.Fiker, T.D.Djuraev, D.G.Karatoprakliev, M.S.Salaxitdinov, V.P.Glushko, T.Sh.Kalmenov, A.I.Kipriyanova, V. Associated with the names of N. Vragov, B.A. Bubnov and other authors [7-11],[14-16].

Consider the following boundary value problem for equations of mixed type in the one-dimensional case.

In the $D=\{(x,t):\ 0<x<l, -T<t<T\}$domain, we consider the following equation:

$$Lu \equiv K(t)\cdot u_{tt} - h(x)\cdot u_{xx} + a(x,t)\cdot u_t + b(x,t)\cdot u_x + c(x,t)\cdot u = f(x,t), \qquad (16)$$

where L through we have defined a linear partial differential operator of the second order.

$K(t)$, $h(x)$, $a(x,t)$, $b(x,t)$, $c(x,t)$ are given functions that satisfy the following conditions:

1) $K(t) \in C^2([-T,T])$, at $t \neq 0$ $t \cdot K(t) > 0$ and $K(0) = 0$.

2) $h(x) \in C^2([0,l])$, $h(x) > 0$, if $x \in (0,l)$ and $h(0) = h(l) = 0$.

3) $a(x,t)$, $b(x,t) \in C^1(\overline{D})$, $c(x,t) \in C(\overline{D})$.

4) $\beta(x) = a(x,0) - K_t(0) > 0$, $x \in [0,1]$.

C - space of continuous functions, $\overline{D}$- closure D. We divide the area D into three areas: $D = D^+ \cup D^- \cup \{t = 0\}$, where

$$D^+ = D \cap \{t > 0\} = \{(x,t) : 0 < x < l, 0 < t < T\} \quad ,$$

$$D^- = D \cap \{t < 0\} = \{(x,t) : 0 < x < l, -T < t < 0\},$$

$\Gamma = \partial D$- the border of the area D. $\vec{n} = (n_x, n_t)$ is the internal normal drawn to the boundary.

According to the classification of second-order partial differential equations, equation (16) in the domain is a mixed-type equation. The study of the basic equations of mathematical physics made it possible to conduct a classification of differential equations and systems of equations with special derivatives. In the 1930s, a class of elliptic, parabolic, and hyperbolic equations was first studied by I.G. Petrovsky. And now these classes are a class of equations that have been studied enough. A class of mixed-type equations can be considered as an unstudied class.

For equation (16) we consider the following boundary value problem:

Boundary value problem: Find a function $u(x,t)$ that satisfies equation (16) in the domain D, and under the condition

$t = -T$

$$u(x,-T) = 0, \; x \in [0,l] \tag{17}$$

Let the space C_2 – of functions lying in the class $C^2(\overline{D})$ and satisfying condition (17).

Boundary value problems for an equation of mixed type were studied in the works of Tricomi [5], Fiker [6], Vragov [14-16], Nakhushev [10-11], Dzhuraev [8], Salakhitdinov [7] and many other researchers (see, for example, [18-20]). Mixed type equations are of an applied nature. Today, the use of modern electronic computers plays an important role in the development of differential equations. In the study of differential equations, it is possible to show computational experiments in the determination of solutions or these properties, to prove them further theoretically, and to lay the groundwork for the next attractions.

The analytical solution of such problems is difficult, and one has to solve them by numerical methods.

To solve the boundary value problem (16) - (17), we use an approximate (numerical) method.

To solve boundary value problems posed to differential equations, he uses different approximate methods (see, for example, [2, 4, 17, 18, 22]).

Among the approximate methods for solving differential equations, "finite-difference schemes" are now widely used.

This method can be divided into the following steps:

1) Discretization of the considered area where the problem is solved, i.e. building a difference network;

2) Approximation of the original differential problem by a finite-difference scheme on the constructed network;

3) Substantiate the constructed finite-difference scheme, i.e. show its correctness, i.e. check the existence, uniqueness and stability of the solution;

4) Find a numerical solution to the differential problem using a finite-difference scheme.

Now let us check the approximation of the difference scheme for the following problem.

Let us apply the method of finite-difference schemes to the boundary value problem (16) - (17).

In the area $\overline{D}=\{(x,t):\ 0\le x\le l, -T\le t\le T\}$, construct a difference grid with steps $\Delta t=\Delta,\ \Delta x=\Delta_x$, ($T=m\cdot\Delta,\ l=n\cdot\Delta_x$).

We denote by u_i^k the approximate solution of the boundary value problem at a (t^k,x_i) point.

Here (t^k,x_i) is the anchor point formed by the intersection of straight lines $t=t^k=k\cdot\Delta,\ x=x_i=i\cdot\Delta_x$. We introduce $\varphi,\psi,\tau,\overline{\tau},\xi,\overline{\xi}$ shift and difference operators as follows:

$\varphi u_i^k=u_i^{k+1}=u^{k+1}=\hat{u},\ \varphi^{-1}u_i^k=u_i^{k-1}=u^{k-1}=\breve{u}$, $\psi^{\pm1}u_i^k=u_{i\pm1}^k=u_{i\pm1}$

$\tau=\varphi-1$ - right difference operator on a variable t,

$\overline{\tau}=1-\varphi^{-1}$ - left difference operator on a variable t,

$\xi=\psi-1$ - right difference operator on a variable x,

$\overline{\xi}=1-\psi^{-1}$ - left difference operator on a variable x,

$r=\dfrac{\Delta}{\Delta_x}$ - step ratio.

In this case, we approximate the boundary value problem (16) - (17) by the following finite-difference scheme [23-25], the stability of which was proved in [25]:

$$\begin{cases} L^{-}u \equiv \left[K^{k}\dfrac{\tau\bar{\tau}}{\Delta^{2}} - h_{i}\dfrac{\xi\bar{\xi}}{\Delta_{x}^{2}} + a_{i}^{k}\dfrac{\bar{\tau}}{\Delta} + b_{i}^{k}\dfrac{\bar{\xi}}{\Delta_{x}} + c_{i}^{k}\right]u = f_{i}^{k} \\ \qquad\qquad k = \overline{-m+1,0}; \quad i = \overline{0,n} \\ L^{+}u \equiv \left[K^{k}\dfrac{\bar{\tau}\bar{\tau}}{\Delta^{2}} - h_{i}\dfrac{\xi\bar{\xi}}{\Delta_{x}^{2}} + a_{i}^{k}\dfrac{\bar{\tau}}{\Delta} + b_{i}^{k}\dfrac{\bar{\xi}}{\Delta_{x}} + c_{i}^{k}\right]u = f_{i}^{k} \\ \qquad\qquad k = \overline{1,m}; \quad i = \overline{0,n} \\ u_{i}^{-m} = 0, \qquad i = \overline{0,n} \end{cases} \tag{18}$$

This scheme has a first order approximation with respect to Δ, Δ_x.

Results and discussions. Let us check the order of approximation of this scheme with respect to Δ, Δ_x. To do this, we first analyze the difference operators, i.e. let's open them:

$$\bar{\tau}u = \left(1-\varphi^{-1}\right)u = u - \varphi^{-1}u = u_i^k - u_i^{k-1},$$

$$\tau\bar{\tau}u = \tau\left(1-\varphi^{-1}\right)u = u_i^{k+1} - 2u_i^k + u_i^{k-1},$$

$$\bar{\tau}\bar{\tau}u = \bar{\tau}\left(1-\varphi^{-1}\right)u = u_i^k - 2u_i^{k-1} + u_i^{k-2},$$

$$\bar{\xi}u = \left(1-\psi^{-1}\right)u = u - \psi^{-1}u = u_i^k - u_{i-1}^k,$$

$$\xi\bar{\xi}u = \xi\left(1-\psi^{-1}\right)u = u_{i+1}^k - 2u_i^k + u_{i-1}^k,$$

$$\bar{\xi}\bar{\xi}u = \bar{\xi}\left(1-\psi^{-1}\right)u = u_i^k - 2u_{i-1}^k + u_{i-2}^k.$$

Here,

$$u_i^k = u(x_i, t_k),\ u_i^{k+1} = u(x_i, t_{k+1}) = u(x_i, t_k + \Delta),\ u_i^{k-1} = u(x_i, t_{k-1}) = u(x_i, t_k - \Delta),$$

$$u_{i+1}^k = u(x_{i+1}, t_k) = u(x_i + \Delta_x, t_k),\ u_{i-1}^k = u(x_{i-1}, t_k) = u(x_i - \Delta_x, t_k).$$

Now, to study the approximation, we will use, as usual, the Taylor formula. Let us first recall the definition of a Taylor series. The Taylor series is an expansion of a function into an infinite sum of power functions.

The Taylor series was known long before Taylor's publications [13] - it was used in the 14th century in India, as well as in the 17th century by Gregory and Newton.

Taylor series are used to approximate a function by polynomials. In particular, the equations are linearized by expanding into a Taylor series and cutting off all terms above the first order.

The Taylor polynomial of a function $f(x)$ of a real variable x differentiable k times at a point a is the finite sum

$$f(x) = \sum_{n=0}^{k} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

used in approximate calculations as a generalization of the corollary of Lagrange's theorem on the mean value of a differentiable function:

for $x - a = h \to 0$ it is true

$$f(x) = f(a+h) = f(a) + f'(a)h + O(h^2) \approx f(a) + f'(a)h = f(a) + f'(a)(x-a) \ .$$

When writing the sum, we used the notation $f^{(0)}(x) = f(x)$and the product convention over the empty set: $0! = 1, \ \ (x-a)^0 = 1$.

The Taylor series at a point a of a function $f(x)$ real x, infinitely differentiable in a neighborhood of a point a, is a formal power series

$$f(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

with a common term $\frac{f^{(n)}(a)}{n!}(x-a)^n$, depending on the design a.

In other words, the Taylor series of the function $f(x)$ at the point a is the series of the expansion of the function in positive powers of the binomial $(x-a)$.

The presence of infinite differentiability of the function $f(x)$ in a neighborhood of the point a is not enough for the Taylor series to converge to the function itself anywhere, except for the point a itself.

Suppose that the function $f(x)$ has all derivatives up to order $n+1$ inclusive in some interval, containing the point $x = a$. Find a polynomial $P_n(x)$ of degree at most n, the value of which at the point $x = a$ is equal to the value of the function $f(x)$ at this point, and the values of its derivatives up to the nth order inclusive at the point $x = a$ are equal to the values of the corresponding derivatives of the function $f(x)$ at this point.

It is easy enough to prove that such a polynomial has the form

$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$, that is, this is the nth partial sum of the Taylor series of the function $f(x)$.

Definition 3[26]. The difference between the function $f(x)$ and the polynomial $P_n(x)$ is called the remainder and is denoted by $R_n(x) = f(x) - P_n(x)$.

Definition 4[26].The formula $f(x) = P_n(x) + R_n(x)$ is called the Taylor formula.

The remainder is differentiable $n+1$ times in the considered neighborhood of the point a. Taylor's formula is used to prove a large number of theorems in differential calculus. Loosely speaking, Taylor's formula shows the behavior of a function in a neighborhood of some point.

There are various forms of residual members.

Schloumilch-Roche form: $R_n(x) = \left(\frac{x-a}{x-\xi}\right)^p \frac{(x-\xi)^{n+1}}{n!p} f^{(n+1)}(\xi);$

Lagrange form: $R_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}(a+\theta(x-a)), p = n+1; 0 < \theta < 1;$

Cauchy form: $R_n(x) = \frac{(x-a)^{n+1}(1-\theta)^n}{n!p} f^{(n+1)}(a+\theta(x-a))\,,\; p = 1; 0 < \theta < 1;$

Integral form: $R_n(x) = \frac{1}{n!}\int_a^x (x-a)^n f^{(n+1)}(t)dt.$

We will now use these notions in our research.

$$u_i^{k+1} = u(x_i, t_{k+1}) = u(x_i, t_k + \Delta) = u(x, t+\Delta) = u(x,t) + \frac{\Delta}{1!}u_t + \frac{\Delta^2}{2!}u_{tt} + \frac{\Delta^3}{3!}u_{ttt} + \ldots =$$
$$= u(x,t) + \frac{\Delta}{1!}u_t + \frac{\Delta^2}{2!}u_{tt} + \frac{\Delta^3}{3!}u_{ttt}(\xi,\tau),$$

$$u_i^{k-1} = u(x_i, t_{k-1}) = u(x_i, t_k - \Delta) = u(x, t-\Delta) = u(x,t) - \frac{\Delta}{1!}u_t + \frac{\Delta^2}{2!}u_{tt} - \frac{\Delta^3}{3!}u_{ttt} + \ldots =$$
$$= u(x,t) - \frac{\Delta}{1!}u_t + \frac{\Delta^2}{2!}u_{tt} - \frac{\Delta^3}{3!}u_{ttt}(\xi,\tau),$$

$$u_{i+1}^k = u(x_{i+1}, t_k) = u(x_i + \Delta_x, t_k) = u(x+\Delta_x, t) = u(x,t) + \frac{\Delta_x}{1!}u_x + \frac{\Delta_x^2}{2!}u_{xx} + \frac{\Delta_x^3}{3!}u_{xxx} + \ldots =$$
$$= u(x,t) + \frac{\Delta_x}{1!}u_x + \frac{\Delta_x^2}{2!}u_{xx} + \frac{\Delta_x^3}{3!}u_{xxx}(\xi,\tau)$$

$$u_{i-1}^k = u(x_{i-1}, t_k) = u(x_i - \Delta_x, t_k) = u(x-\Delta_x, t) = u(x,t) - \frac{\Delta_x}{1!}u_x + \frac{\Delta_x^2}{2!}u_{xx} - \frac{\Delta_x^3}{3!}u_{xxx} + \ldots =$$
$$= u(x,t) - \frac{\Delta_x}{1!}u_x + \frac{\Delta_x^2}{2!}u_{xx} - \frac{\Delta_x^3}{3!}u_{xxx}(\xi,\tau)$$

Here, (ξ,τ)some intermediate point.

We insert these calculations into the difference scheme (12) and get:
At $k = \overline{-m+1,0}; \quad i = \overline{0,n}$

$k = \overline{-m+1,0}; \quad i = \overline{0,n}$

$$\left[K^k \frac{\tau\bar{\tau}}{\Delta^2} - h_i \frac{\xi\bar{\xi}}{\Delta_x^2} + a_i^k \frac{\bar{\tau}}{\Delta} + b_i^k \frac{\bar{\xi}}{\Delta_x} + c_i^k\right]u = \frac{K^k}{\Delta^2}\,\tau\bar{\tau}u - \frac{h_i}{\Delta_x^2}\,\xi\bar{\xi}u + \frac{a_i^k}{\Delta}\,\bar{\tau}u + \frac{b_i^k}{\Delta_x}\,\bar{\xi}u + c_i^k u =$$

$$=\frac{K^k}{\varDelta^2}\left(u_i^{k+1}-2u_i^k+u_i^{k-1}\right)-\frac{h_i}{\varDelta_x^2}\left(u_{i+1}^k-2u_i^k+u_{i-1}^k\right)+\frac{a_i^k}{\varDelta}\left(u_i^k-u_i^{k-1}\right)+\frac{b_i^k}{\varDelta_x}\left(u_i^k-u_{i-1}^k\right)+c_i^k u_i^k=$$

$$=\frac{K(t)}{\varDelta^2}\left(u(x,t)+\frac{\Delta}{1!}u_t+\frac{\Delta^2}{2!}u_{tt}+\frac{\Delta^3}{3!}u_{ttt}+...-2u(x,t)+u(x,t)-\frac{\Delta}{1!}u_t+\frac{\Delta^2}{2!}u_{tt}-\frac{\Delta^3}{3!}u_{ttt}+...\right)-$$

$$-\frac{h(x)}{\varDelta_x^2}\left(u(x,t)+\frac{\Delta_x}{1!}u_x+\frac{\Delta_x^2}{2!}u_{xx}+\frac{\Delta_x^3}{3!}u_{xxx}+...-2u(x,t)+u(x,t)-\frac{\Delta_x}{1!}u_x+\frac{\Delta_x^2}{2!}u_{xx}-\frac{\Delta_x^3}{3!}u_{xxx}+...\right)+$$

$$+\frac{a(x,t)}{\varDelta}\left(u(x,t)-\left(u(x,t)-\frac{\Delta}{1!}u_t+\frac{\Delta^2}{2!}u_{tt}-\frac{\Delta^3}{3!}u_{ttt}+...\right)\right)+$$

$$\frac{b(x,t)}{\varDelta_x}\left(u(x,t)-\left(u(x,t)-\frac{\Delta_x}{1!}u_x+\frac{\Delta_x^2}{2!}u_{xx}-\frac{\Delta_x^3}{3!}u_{xxx}+...\right)\right)+c(x,t)u(x,t)=$$

$$=2\frac{K(t)}{\varDelta^2}\left(\frac{\Delta^2}{2!}u_{tt}+\frac{\Delta^4}{4!}u_{tttt}+...\right)-2\frac{h(x)}{\varDelta_x^2}\left(\frac{\Delta_x^2}{2!}u_{xx}+\frac{\Delta_x^4}{4!}u_{xxxx}+...\right)+$$

$$+\frac{a(x,t)}{\varDelta}\left(u(x,t)-\left(u(x,t)-\frac{\Delta}{1!}u_t+\frac{\Delta^2}{2!}u_{tt}-\frac{\Delta^3}{3!}u_{ttt}+...\right)\right)+$$

$$+\frac{b(x,t)}{\varDelta_x}\left(u(x,t)-\left(u(x,t)-\frac{\Delta_x}{1!}u_x+\frac{\Delta_x^2}{2!}u_{xx}-\frac{\Delta_x^3}{3!}u_{xxx}+...\right)\right)+c(x,t)u(x,t)=$$

$$=K(t)u_{tt}-h(x)u_{xx}+a(x,t)u_t+b(x,t)u_x+c(x,t)u+K^k\frac{\Delta^2}{12}\frac{\partial^4 u}{\partial t^4}+...-h_i\frac{\Delta_x^2}{12}\frac{\partial^4 u}{\partial x^4}-...-$$

$$-a_i^k\frac{\Delta}{2!}\frac{\partial u}{\partial t}+...-b_i^k\frac{\Delta}{2!}\frac{\partial u}{\partial x}+...$$

We find the residual for our case. After calculations, we get:

$$\delta f=-a_i^k\frac{\Delta}{2!}\frac{\partial u}{\partial t}-b_i^k\frac{\Delta}{2!}\frac{\partial u}{\partial x}+O\left(\Delta^2+\Delta_x^2\right).$$

We calculate the norm of the residual:

$$\|\delta f\|\le\left\|-a_i^k\frac{\Delta}{2!}\frac{\partial u}{\partial t}-b_i^k\frac{\Delta_x}{2!}\frac{\partial u}{\partial x}\right\|\le\Delta\left\|a_i^k\frac{1}{2}\frac{\partial u}{\partial t}\right\|+\Delta_x\left\|b_i^k\frac{1}{2}\frac{\partial u}{\partial x}\right\|\le C_1\Delta+C_2\Delta_x=O\left(\Delta+\Delta_x\right).$$

The first derivatives of the function are considered to be bounded functions, therefore, the first order of approximation in time and space takes place. As a result, we got an estimate of the type (4).

In the same way, it can be shown for $k=\overline{1,m};\quad i=\overline{0,n}$, from this it is clear that the first order of approximation of this scheme is relative Δ, Δ_x.

Conclusion. A priori estimates have been obtained for the solutions of equations of mixed type, and many scientists have devoted and continue to dedicate their scientific work in this direction. The study of the differential model of boundary value problems for mixed-type equations has not lost its relevance to this day. In our view, if a differential model of boundary value problems for mixed-type equations is studied, first, a numerical analogue of the a priori estimates obtained for the solutions of these equations is obtained, and it is proved that these problems can be solved numerically. The main task is to show the adequacy of the new discrete models under consideration or construction to the initial differential problem. In conclusion, the analytical solution of nonclassical equations in mathematical physics is a very complex process, so the boundary value problems in these equations are approximated by differential schemes and the stability is checked, which allows to solve a number of boundary value problems for mixed type equations.

And so we formulate the following theorem:

Theorem. (18) the finite-difference scheme approximates (16) - (17) the problem in the first order with respect to Δ, Δ_x.

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