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BUXORO DAVLAT UNIVERSITETI

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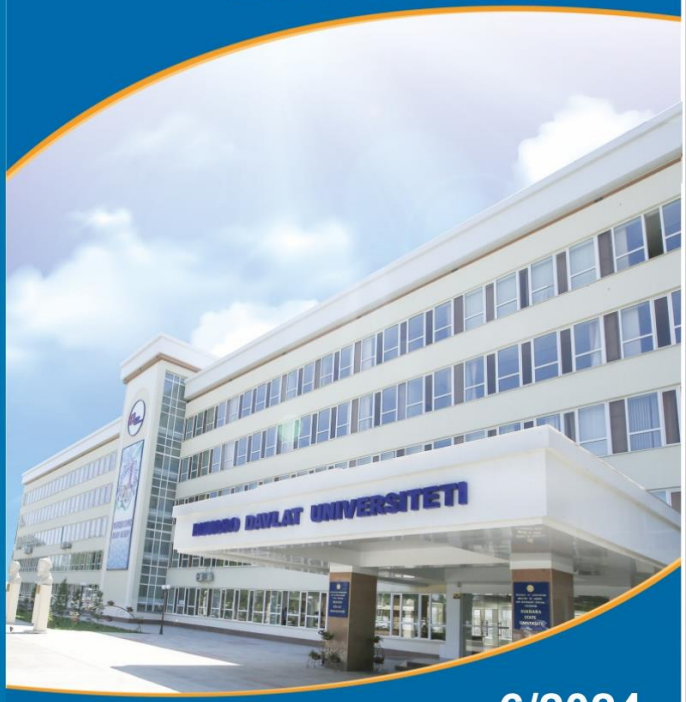

6/2024

BUXORO DAVLAT UNIVERSITETI ILMIY AXBOROTI
SCIENTIFIC REPORTS OF BUKHARA STATE UNIVERSITY
НАУЧНЫЙ ВЕСТНИК БУХАРСКОГО ГОСУДАРСТВЕННОГО УНИВЕРСИТЕТА

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KASR TARTIBLI HOSILA VA UNING BA’ZI BIR TATBIQLARI

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Annotatsiya. *Ushbu maqolada ilm-fan sohasida keng o‘rganilayotgan tushunchalardan biri kasr tartibli hosila va uning ba’zi tadbiqlari haqida ma’lumotlar keltirilgan. Maqolada keltirilgan ma’lumotlardan matematika yo‘nalishida tahsil olayotgan bakalavrlar, magistrlar foydalanishlari mumkin.*

Kalit so‘zlar: *hosila, maxsusu funksiylar, kasr tartibli hosila, differensial tenglamalar.*

ПРОИЗВОДНАЯ ДРОБНОГО ПОРЯДКА И НЕКОТОРЫЕ ЕЕ ПРИМЕНЕНИЯ

Аннотация. *В данной статье представлена информация об одном из понятий, широко изучаемом в области науки, дробной производной и некоторых ее применениях. Информация, представленная в статье, может быть использована бакалаврами и магистрами, обучающимся в направление математика.*

Ключевые слова: *производная, специальные функции, дробная производная, дифференциальные уравнения.*

FRACTIONAL ORDERED DERIVATIVE AND SOME OF ITS IMPLEMENTATIONS

Abstract. *This article provides information about one of the concepts widely studied in the field of science, fractional derivative and some of its applications. The information presented in the article can be used by bachelors and masters students studying mathematics.*

Keywords: *derivative, special functions, fractional derivative, differential equations.*

Kirish. Kasr tartibli hosila tushunchasi hozirgi kunda fanga kirib keldi va chuqur amaliy tatbiqqa ega. Butun tartibli hosilalar qatnashgan tenglamalardan tashqari kasr tartibli hosila qatnashgan tenglamalarga qo‘yilgan masalalar chuqur o‘rganilyapti. Kasr tartibli hosila meteorologiya, kimyoda juda ko'p amaliy masalalarda yuzaga keladi va suyuqlik oqimlarining harakatini modellashtirishda, murakkab akustik tebranishlarning tarqalishida va, albatta, fanning eng birinchi pog'onasida - kvant fizikasida qo'llaniladi. Shu sababli ushbu maqolada biz qisqacha kasr tartibli hosila haqida ma’lumotlar keltirib, ba’zi bir funksiyalarning kasr tartibli hosilalarini hisoblash usullarini keltiramiz.

Dastlab Riman-Liuvill, Kaputo ma’nosidagi kasr tartibli hosila va integrallarining ta’riflarini keltiramiz hamda haqiqiy sonlar o‘qining chegaralarangan oralig’ida berilgan uzluksiz funksiyalar uchun ayrim xossalarini umumlashtiramiz.

Asosiy qism. Kasr tartibli hosila matematik tushunchalaridan biri bo‘lgan oddiy hosila tushunchasining umumlashtirilganidir. Ushbu tushunchaning bir necha xil berilih usullari mavjud, ammo ularning barchasi natural tartibda oddiy hosila tushunchasi bilan ustma-ust tushadi. Hosilaning faqat kasr emas, balki manfiy tartiblari ham ko‘rib chiqish mumkin. Kasr tartibli hosila haqidagi dastlabki tushunchalar I.Nyuton, G.V. Leybnits ishlarida uchraydi [1]. Hozirgi kunda amaliy tadbig‘ini topgan bu tushuncha matematiklar tomonidan keng tadqiq qilinyapti.

R.R.Ashurov, D.Q.Durdiyev, A.V.Psxu, A.A.Rahmonov, J.Jumayev [2-7] va boshqalarning ilmiy maqolalarida kasr tartibli tenglama uchun qo‘yilgan to‘g‘ri va teskari masalalar tadbiq qilingan.

1. Riman-Liuvill kasr tartibli hosilalari.

$\mathbb{R}$ haqiqiy sonlar to‘plamida chegaralangan $\Omega = [a,b]$ $(-\infty < a < b < \infty)$ to‘plam berilgan bo‘lsin. $[a,b]$ segmentda aniqlangan $f(x)$ funktsiya uchun $\alpha \in \mathbb{C}$ $(\mathcal{R}(\alpha) > 0)$ tartibli *Riman-Liuvill kasr tartibli integrallari* $I_{a+}^{\alpha} f$ va $I_{b-}^{\alpha} f$ mos ravishda quyidagi tengliklar bilan aniqlanadi[1(Kilbas)]:

$$(I_{a+}^{\alpha} f)(x) := \frac{1}{\Gamma(\alpha)} \int_a^x \frac{f(t)dt}{(x-t)^{1-\alpha}} \quad (x > a;\ \mathcal{R}(\alpha) > 0) \tag{1.1}$$

$$(I_{b-}^{\alpha} f)(x) := \frac{1}{\Gamma(\alpha)} \int_x^b \frac{f(t)dt}{(t-x)^{1-\alpha}} \quad (x < b;\ \mathcal{R}(\alpha) > 0) \tag{1.2}$$

Bu yerda $\Gamma(\alpha)$ Gamma funksiya. Bu integrallar *chap tomonlama va o‘ng tomonlama kasr tartibli integrallar* deyiladi.

$\alpha = n \in \mathbb{N}$ bo‘lganda (1.1) va (1.2) formulalar quyidagi n karrali integrallar bilan mos tushadi:

$$(I_{a+}^{n} f)(x) = \int_a^x dt_1 \int_a^{t_1} dt_2 \dots \int_a^{t_{n-1}} f(t_n)\, dt_n =$$

$$= \frac{1}{(n-1)!} \int_a^x (x-t)^{n-1} f(t)\, dt \quad (n \in \mathbb{N}) \tag{1.3}$$

$$(I_{b-}^{n} f)(x) = \int_x^b dt_1 \int_{t_1}^b dt_2 \dots \int_{t_{n-1}}^b f(t_n)\, dt_n =$$

$$= \frac{1}{(n-1)!} \int_x^b (t-x)^{n-1} f(t)\, dt \quad (n \in \mathbb{N}) \tag{1.4}$$

$\alpha \in \mathbb{C}$ $(\mathcal{R}(\alpha) \geq 0)$ tartibli *Riman-Liuvill kasr tartibli hosilalari* $D_{a+}^{\alpha} y$ va $D_{b-}^{\alpha} y$ mos ravishda quyidagi tengliklar bilan aniqlanadi:

$$(D_{a+}^{\alpha} y)(x) := \left(\frac{d}{dx}\right)^n (I_{a+}^{n-\alpha} y)(x) =$$

$$= \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dx}\right)^n \int_a^x \frac{y(t)dt}{(x-t)^{\alpha-n+1}}, \quad (n = [\mathcal{R}(\alpha)] + 1;\ x > a) \tag{1.5}$$

$$(D_{b-}^{\alpha} y)(x) := \left(-\frac{d}{dx}\right)^n (I_{b-}^{n-\alpha} y)(x) =$$

$$= \frac{1}{\Gamma(n-\alpha)} \left(-\frac{d}{dx}\right)^n \int_x^b \frac{y(t)dt}{(t-x)^{\alpha-n+1}}, \quad (n = [\mathcal{R}(\alpha)] + 1;\ x < b) \tag{1.6}$$

bu yerda $[\mathcal{R}(\alpha)]$ **ifoda** $\mathcal{R}(\alpha)$ **ning butun qismini** bildiradi. Xususiy holda, $\alpha = n \in \mathbb{N}_0$ bo‘lganda

$$(D_{a+}^{0} y)(x) = (D_{b-}^{0} y)(x) = y(x)$$

$$(D_{a+}^{n} y)(x) = y^{(n)}(x)$$

$$(D_{b-}^{n} y)(x) = (-1)^n y^{(n)}(x) \quad (n \in \mathbb{N}) \tag{1.7}$$

bu yerda $y^{(n)}(x)$ ifoda $y(x)$ funksiyaning n-tartibli oddiy hosilasi.

Agar $0 < \mathcal{R}(\alpha) < 1$ bo‘lsa,

$$(D_{a+}^{\alpha} y)(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_a^x \frac{y(t)dt}{(x-t)^{\alpha-[\mathcal{R}(\alpha)]}} \quad (0 < \mathcal{R}(\alpha) < 1; x > a) \tag{1.8}$$

$$(D_{b-}^{\alpha} y)(x) = -\frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_x^b \frac{y(t)dt}{(t-x)^{\alpha-[\mathcal{R}(\alpha)]}} \quad (0 < \mathcal{R}(\alpha) < 1; x > b) \tag{1.9}$$

tengliklar o‘rinli.

Endi esa Kaputo kasr tartibli hosilalari va ularning xossalarini keltiramiz.

2. Kaputo kasr tartibli hosilalari.

$[a,b]$ haqiqiy sonlar o‘qi $\mathbb{R}$ da chegaralangan oraliq bo‘lsin hamda

$$D_{a+}^{\alpha}[y(t)](x) \equiv (D_{a+}^{\alpha}y)(x) \text{ va } D_{b-}^{\alpha}[y(t)](x) \equiv (D_{b-}^{\alpha}y)(x)$$

lar mos ravishda (1.8) va (1.9) tengliklar bilan aniqlangan $\alpha \in \mathbb{C}$ $(\Re(\alpha) \geq 0)$ tartibli Riman-Liuvill kasr tartibli hosilalari bo‘lsin. Ushbu $\alpha \in \mathbb{C}$ $(\Re(\alpha) \geq 0)$ tartibli Riman-Liuvill kasr tartibli hosilalari $(cD_{a+}^{\alpha}y)(x)$ va $(cD_{b-}^{\alpha}y)(x)$ lar Riman-Liuvill kasr tartibli hosilalari orqali mos ravishda quyidagi

$$({}^{C}D_{a+}^{\alpha}y)(x) := \left(D_{a+}^{\alpha}\left[y(t) - \sum_{k=0}^{n-1}\frac{y^{(k)}(a)}{k!}(t-a)^{k}\right]\right)(x), \tag{2.1}$$

$$({}^{C}D_{b-}^{\alpha}y)(x) := \left(D_{b-}^{\alpha}\left[y(t) - \sum_{k=0}^{n-1}\frac{y^{(k)}(b)}{k!}(b-t)^{k}\right]\right)(x) \tag{2.2}$$

tengliklar bilan aniqlangan, bu yerda

$$n = [\Re(\alpha)] + 1, \text{ agar } \alpha \notin \mathbb{N}_0 \text{ va } n = \alpha, \text{ agar } \alpha \in \mathbb{N}_0\,. \tag{2.3}$$

Bu hosilalar *chap tomonlama va o‘ng tomonlama α tartibli Kaputo kasr tartibli hosilalari* deyiladi.

Xususiy holda, $0 < \Re(\alpha) < 1$ bo‘lganda (2.1) va (2.2) tengliklar quyidagi ko‘rinishlarni oladi:

$$({}^{C}D_{a+}^{\alpha}y)(x) = (D_{a+}^{\alpha}[y(t) - y(a)])(x), \tag{2.4}$$

$$({}^{C}D_{b-}^{\alpha}y)(x) = (D_{b-}^{\alpha}[y(t) - y(b)])(x). \tag{2.5}$$

Agar $\alpha \notin \mathbb{N}_0$ hamda $y(x)$ funksiya $\alpha \in \mathbb{C}$ $(\Re(\alpha) \geq 0)$ tartibli Kaputo kasr tartibli hosilalari $({}^{C}D_{a+}^{\alpha}y)(x)$ va $({}^{C}D_{b-}^{\alpha}y)(x)$ bilan birgalikda Riman-Liuvill kasr tartibli hosilalari $(D_{a+}^{\alpha}y)(x)$ va $(D_{b-}^{\alpha}y)(x)$ lar bir vaqtda mavjud bo‘ladigan funksiya bo‘lsa, (1.1) va (1.9) ga ko‘ra, ular quyidagi tengliklar bilan bog’langan:

$$({}^{C}D_{a+}^{\alpha}y)(x) = (D_{a+}^{\alpha}y)(x) - \sum_{k=0}^{n-1}\frac{y^{(k)}(a)}{\Gamma(k-\alpha+1)}(x-a)^{k-\alpha}, \tag{2.6}$$

$$({}^{C}D_{b-}^{\alpha}y)(x) = (D_{b-}^{\alpha}y)(x) - \sum_{k=0}^{n-1}\frac{y^{(k)}(b)}{\Gamma(k-\alpha+1)}(b-x)^{k-\alpha}, \tag{2.7}$$

bu yerda $n = [\Re(\alpha)] + 1$.

Xususiy holda, $0 < \Re(\alpha) < 1$ bo‘lganda biz

$$({}^{C}D_{a+}^{\alpha}y)(x) = (D_{a+}^{\alpha}y)(x) - \frac{y(a)}{\Gamma(1-\alpha)}(x-a)^{-\alpha}, \tag{2.8}$$

$$({}^{C}D_{b-}^{\alpha}y)(x) = (D_{b-}^{\alpha}y)(x) - \frac{y(b)}{\Gamma(1-\alpha)}(b-x)^{-\alpha} \tag{2.9}$$

tengliklarga egamiz.

Agar $\alpha \notin \mathbb{N}_0$ bo‘lsa, u holda Kaputo kasr tartibli hosilalari (2.1) va (2.2) lar Riman-Liuvill kasr tartibli hosilalari (2.1.5) va (2.1.6) lar bilan quyidagi hollarda o‘zaro mos keladi:

$$({}^{C}D_{a+}^{\alpha}y)(x) = (D_{a+}^{\alpha}y)(x), \tag{2.10}$$

agar $y(a) = y'(a) = \cdots = y^{(n-1)}(a) = 0$ $(n = [\Re(\alpha)] + 1)$ bo‘lsa;

$$({}^{C}D_{b-}^{\alpha}y)(x) = (D_{b-}^{\alpha}y)(x), \tag{2.11}$$

agar $y(b) = y'(b) = \cdots = y^{(n-1)}(b) = 0$ $(n = [\Re(\alpha)] + 1)$ bo‘lsa.

Xususiy holda, $0 < \Re(\alpha) < 1$ bo‘lganda, biz

$$({}^{C}D_{a+}^{\alpha}y)(x) = (D_{a+}^{\alpha}y)(x), \quad y(a) = 0 \tag{2.12}$$

$$({}^{C}D_{b-}^{\alpha}y)(x) = (D_{b-}^{\alpha}y)(x), \quad y(b) = 0 \tag{2.13}$$

tengliklarga egamiz.

Agar $\alpha = n \in \mathbb{N}_0$ bo‘lsa hamda oddiy n –tartibli hosila $y^{(n)}(x)$ mavjud bo‘lsa, unda $({}^{C}D_{a+}^{\alpha}y)(x)$ hosila $y^{(n)}(x)$ bilan ustma-ust tushadi, shuningek $({}^{C}D_{b-}^{\alpha}y)(x)$ hosila $y^{(n)}(x)$ ning $(-1)^n$ ga ko‘paytmasi bilan ustma-ust tushadi:

$$({}^{C}D_{a+}^{\alpha}y)(x) = y^{(n)}(x) \text{ va } ({}^{C}D_{b-}^{\alpha}y)(x) = (-1)^n y^{(n)}(x) \quad (n \in \mathbb{N}). \tag{2.14}$$

Kaputo kasr tartibli hosilalari $({}^{C}D_{a+}^{\alpha}y)(x)$ va $({}^{C}D_{b-}^{\alpha}y)(x)$ lar (2.1) va (2.2) ning o‘ng tomonlama Riman-Liuvill kasr tartibli hosilalari mavjud bo‘lgan $y(x)$ funksiyalar uchun aniqlangan. Xususiy holda, ular $AC^{n}[a,b]$ absolyut uzluksiz funksiyalar fazosiga tegishli $y(x)$ lar uchun aniqlangan. Quyidagi teorema o‘rinli.

2.1-teorema. $\Re(\alpha) \geq 0$ o‘lsin va n (2.3) shartni qanoatlantirsin. Agar $y(x) \in AC^{n}[a,b]$ bo‘lsa, Kaputo kasr tartibli hosilalari $({}^{C}D_{a+}^{\alpha}y)(x)$ va $({}^{C}D_{b-}^{\alpha}y)(x)$ lar $[a,b]$ ning deyarli hamma yerda aniqlangan.

a) Agar $\alpha \notin \mathbb{N}_0$ bo‘lsa, $({}^C D_{a+}^{\alpha} y)(x)$ va $({}^C D_{b-}^{\alpha} y)(x)$ lar mos ravishda quyidagi

$$({}^C D_{a+}^{\alpha} y)(x) = \frac{1}{\Gamma(n-\alpha)} \int_a^x \frac{y^{(n)}(t)dt}{(x-t)^{\alpha-n+1}} =: (I_{a+}^{n-\alpha} D^n y)(x), \qquad (2.15)$$

$$({}^C D_{b-}^{\alpha} y)(x) = \frac{(-1)^n}{\Gamma(n-\alpha)} \int_x^b \frac{y^{(n)}(t)dt}{(t-x)^{\alpha-n+1}} =: (-1)^n (I_{b-}^{n-\alpha} D^n y)(x) \qquad (2.16)$$

tengliklar orqali aniqlanadi, bu yerda $D = \frac{d}{dx}$ va $n = [\Re(\alpha)] + 1$.

Xususiy holda, $0 < \Re(\alpha) < 1$ va $y(x) \in AC[a,b]$ bo‘lsa,

$$({}^C D_{a+}^{\alpha} y)(x) = \frac{1}{\Gamma(1-\alpha)} \int_a^x \frac{y'(t)dt}{(x-t)^{\alpha}} =: (I_{a+}^{1-\alpha} Dy)(x), \qquad (2.17)$$

$$({}^C D_{b-}^{\alpha} y)(x) = -\frac{1}{\Gamma(1-\alpha)} \int_a^x \frac{y'(t)dt}{(t-x)^{\alpha}} =: -\left(I_{b-}^{1-\alpha} D^n y\right)(x) \qquad (2.18)$$

tengliklar o‘rinli.

b) Agar $\alpha = n \in \mathbb{N}_0$ bo‘lsa $({}^C D_{a+}^{\alpha} y)(x)$ va $({}^C D_{b-}^{\alpha} y)(x)$ lar (2.14) tenglik bilan aniqlangan. Xususiy holda,

$$\left({}^C D_{b-}^{0} y\right)(x) = \left({}^C D_{b-}^{0} y\right)(x) = y(x) \qquad (2.19)$$

bo‘ladi.

3. Misollar.

Misol 1. $y = x$ funksiyaning $a = \frac{1}{2}$ nuqta atrofida o‘ng tomonlama $\frac{1}{2}$-tartibli hosilasini hisoblang.

Yechish: Buning uchun quyidagi lemmadan foydalanamiz:

Lemma. $\mathcal{R}(\alpha) \geq 0$ va $n = [\mathcal{R}(\alpha)] + 1$ berilgan bo‘lsin. Agar $y(x) \in AC^n[a,b]$ bo‘lsa, $D_{a+}^{\alpha} y$ va $D_{b-}^{\alpha} y$ kasr tartibli hosilalar $[a,b]$ oraliqning deyarli hamma yerida mavjud va mos ravishda quyidagi tengliklar bilan ifodalanishi mumkin:

$$(D_{a+}^{\alpha} y)(x) = \sum_{k=0}^{n-1} \frac{y^{(k)}(a)}{\Gamma(1+k-\alpha)} (x-a)^{k-\alpha} + \frac{1}{\Gamma(n-\alpha)} \int_a^x \frac{y^{(n)}(t)dt}{(x-t)^{\alpha-n+1}},$$

$$(D_{b-}^{\alpha} y)(x) = \sum_{k=0}^{n-1} \frac{(-1)^k y^{(k)}(b)}{\Gamma(1+k-\alpha)} (b-x)^{k-\alpha} + \frac{(-1)^n}{\Gamma(n-\alpha)} \int_x^b \frac{y^{(n)}(t)dt}{(t-x)^{\alpha-n+1}}.$$

Demak bizning misolda:

$$\alpha = \frac{1}{2}, \ \ a = 1, \ \ \Gamma\left(1-\frac{1}{2}\right) = \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \ \ n = [\mathcal{R}(\alpha)] + 1 = 1.$$

$$\left(D_{1+}^{\frac{1}{2}} y\right)(x) = \frac{y(1)}{\Gamma\left(\frac{1}{2}\right)} (x-1)^{-\frac{1}{2}} + \frac{1}{\Gamma\left(\frac{1}{2}\right)} \int_1^x \frac{y'(t)dt}{(x-t)^{\frac{1}{2}-1+1}} =$$

$$= \frac{1}{\sqrt{\pi}} (x-1)^{-\frac{1}{2}} + \frac{1}{\sqrt{\pi}} \int_1^x \frac{dt}{(x-t)^{\frac{1}{2}}} = \frac{1}{\sqrt{\pi}} \left[(x-1)^{-\frac{1}{2}} - 2(x-t)^{\frac{1}{2}} \Big|_1^x \right] =$$

$$= \frac{1}{\sqrt{\pi}} \left[(x-1)^{-\frac{1}{2}} - 2(x-x)^{\frac{1}{2}} + 2(x-1)^{\frac{1}{2}} \right] = \frac{1}{\sqrt{\pi}} \left[(x-1)^{-\frac{1}{2}} + 2(x-1)^{\frac{1}{2}} \right].$$

Javob: $\left(D_{1+}^{\frac{1}{2}} y\right)(x) = \frac{1}{\sqrt{\pi}} \left[(x-1)^{-\frac{1}{2}} + 2(x-1)^{\frac{1}{2}} \right]$.

Biz bilamizki, $y = x$ funksiyaning birinchi tartibli natural tartibli hosilasi 1 ga teng. Bu natijani hosila ta’rifidan foydalanib aniqlash qiyin emas:

$$y'(x) = \lim_{\Delta x \to 0} \frac{y(x+\Delta x) - y(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x} = 1.$$

Misol 2. $y = x^m, m > -1$ funksiyaning a nuqta atrofida o‘ng tomonlama α-tartibli hosilasini hisoblang.

Yechish: Buning uchun **lemma1** dan foydalanamiz va Kaputo ma'nosidagi kasr tartibli hosilani hisoblaymiz.:

$$({}^{C}D_{a+}^{\alpha}y)(x) = \frac{1}{\Gamma(n-\alpha)}\int_{a}^{x}\frac{y^{(n)}(t)dt}{(x-t)^{\alpha-n+1}} = \frac{1}{\Gamma(n-\alpha)}\int_{a}^{x}\frac{\frac{d^n}{dt^n}(t^m)dt}{(x-t)^{\alpha-n+1}}$$

$$= \frac{1}{\Gamma(n-\alpha)}\int_{a}^{x}(x-t)^{n-\alpha-1}\left[\frac{m!}{(m-n)!}t^{m-n}\right]dt$$

$$= \frac{\Gamma(1+m)}{\Gamma(n-\alpha)\Gamma(1+m+n)}\int_{a}^{x}x^{n-\alpha-1}\left(1-\frac{t}{x}\right)^{n-\alpha-1}t^{m-n}dt = \frac{\Gamma(1+m)}{\Gamma(1+m-\alpha)}x^{m-\alpha}$$

Xulosa. Ushbu maqolada keltrilgan ma'lumotlar o'z ilmiy ishlarida kasr tartibli hosila tushunchasidan foydalanadigan tadqiqotchilarga dastlabki ma'lumotlarni olish imkonini beradi.

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