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Remark on the bounded non-self-adjoint friedrichs model

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Annotation

In the present paper we consider the bounded, but not-self-adjoint Friedrichs model A with rank one perturbation. We show the for any positive integer number m there exist the parameter functions of the operator A such that this operator has at least m eigenvalues.

Key words: Friedrichs model, perturbation operator, eigenvalue, eigenfunction.

Spectral theory is one of the most important areas of the theory of linear operators. Current trends in the theory of operators dictate the need for appropriate knowledge about the number of eigenvalues. It is known that the solution of differential equations after the application of the corresponding Fourier transform in many cases is reduced to the analysis of the non-self-adjoint Friedrichs model. Information about the finite number of all eigenvalues of this operator is relevant and valuable because it happens that an important object is considered, but it has an infinite number of eigenvalues, which significantly inhibits further research. Thus, the Friedrichs model in an arbitrary interval can have a finite set of eigenvalues under certain restrictions.

Let $L_2[0, 2\pi]$ be the complex Hilbert space of square integrable functions on $[0, 2\pi]$. In this paper we consider the operator A acting in the Hilbert space $L_2[0, 2\pi]$ according to the formula

$$A = A_0 + K \tag{1}$$

where the operators A_0 and K are defined by

$$(A_0 f)(x) = u(x) f(x), \quad (Kf)(x) = \varphi(x) \int_0^{2\pi} \psi(y) f(y) dy.$$

Here $u(\cdot)$, $\varphi(\cdot)$ and $\psi(\cdot)$ are complex valued analytic functions on $[0, 2\pi]$.

Under these assumptions the operator A is bounded, but not-self-adjoint in $L_2[0, 2\pi]$.

Friedrichs [1, 2] was the first to consider an operator of the form (1) in $L_2(-1,1)$ with $u(x) = x$ as a simple model of perturbation theory for the continuous spectrum. The Friedrichs model was further developed in [3, 4, 5] and other papers.

Note that for the case where we consider perturbation of self-adjoint and non-self-adjoint operators, then the results are quit different. For example if we consider rank one perturbation of the self-adjoint operator then the discrete spectrum can change only to one. Although embedding eigenvalues can change finite number for the case where the associated kernel is smooth, even can arise infinitely many embedding eigenvalues if the perturbation one dimensional function is sufficiently "bad".

Let $R(u)$ be the range of the function $u(\cdot)$.

The following Lemma can be proved by very standard arguments.

Lemma 1. *The number $z \notin R(u)$ is an eigenvalues of the operator A with the eigenfunction*

$$f(x) := \frac{\varphi(x)}{u(x) - z}$$

if and only if the following relation

$$1 - \int_0^{2\pi} \frac{\varphi(y)\psi(y)dy}{u(y) - z} = 0 \qquad (2)$$

is fulfilled.

The main result of the present paper is the following Lemma.

Lemma 2. *For any fixed $m \in N$ there exist the functions $u(\cdot)$, $\varphi(\cdot)$ and $\psi(\cdot)$ such that the non-self-adjoint operator A has at least m eigenvalues lying out-side of the essential spectrum.*

Proof. By Lemma 1 it is enough to find the functions $u(\cdot)$, $\varphi(\cdot)$ and $\psi(\cdot)$ satisfying the equality (2). Now we choose

$$u(y) := e^{iy},\ \varphi(y) := ie^{iy} \text{ and } \psi(y) := P(e^{iy})/Q(e^{iy}),$$

where P, Q are polynomial which have no roots on the unite circle.

If we choose $P(z) = C$ with $|C| >> 1$ and $Q(z) := z^N - \varepsilon$ with $|\varepsilon| < 1$, then it is easy to see that for $|z| > 1$ the equation (2) has exactly N roots. Because for such z we have

$$\int_0^{2\pi} \frac{\varphi(y)\psi(y)dy}{u(y)-z} = \frac{P(z)}{Q(z)}.$$

If $|z|<1$ then it is easy to see that

$$\int_0^{2\pi} \frac{\varphi(y)\psi(y)dy}{u(y)-z} \equiv 0.$$

Therefore the equation (2) has no any solution for $|z|<1$.

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