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«AMALIY MATEMATIKA VA AXBOROT TEXNOLOGIYALARINING ZAMONAVIY MUAMMOLARI»
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АМАЛИЙ МАТЕМАТИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ЗАМОНАВИЙ МУАММОЛАРИ

ХАЛҚАРО ИЛМИЙ-АМАЛИЙ АНЖУМАН

МАТЕРИАЛЛАРИ

2022 йил, 11-12 май

БУХОРО – 2022

ЎЗБЕКИСТОН РЕСПУБЛИКАСИ
ОЛИЙ ВА ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ
ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ФАНЛАР АКАДЕМИЯСИ
В.И. РОМАНОВСКИЙ НОМИДАГИ МАТЕМАТИКА ИНСТИТУТИ
ЎЗБЕКИСТОН МИЛЛИЙ УНИВЕРСИТЕТИ
ТОШКЕНТ ДАВЛАТ ТРАНСПОРТ УНИВЕРСИТЕТИ
БУХОРО ДАВЛАТ УНИВЕРСИТЕТИ

Бухоро фарзанди, Беруний номидаги Давлат мукофоти лауреати, кўплаб ёш изланувчиларнинг ўз йўлини топиб олишида раҳнамолик қилган етук олим, физика-математика фанлари доктори Ғайбулла Назруллаевич Салиховнинг 90 йиллик юбилейларига бағишланади

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Assume $\{\mathbf{x}^{(n)} \in S^{m-1} : n = 0,1,2,\ldots\}$ is the trajectory of the initial point $\mathbf{x} \in S^{m-1}$, where $\mathbf{x}^{(n+1)} = V(\mathbf{x}^{(n)})$ for all $n = 0,1,2,\ldots$, with $\mathbf{x}^{(0)} = \mathbf{x}$.

Definition 1. A point $\mathbf{x} \in S^{m-1}$ is called a fixed point of a QSO V if $V(\mathbf{x}) = \mathbf{x}$.

Definition 2. A QSO V is called regular if for any initial point $\mathbf{x} \in S^{m-1}$, the limit

$$\lim_{n\to\infty} V(\mathbf{x}^{(n)})$$

exists.

Definition 3. A continuous function $\phi : \operatorname{int} S^{m-1} \to R$ for an operator V if the limit $\lim_{n\to\infty}\phi\left(V^n(\mathbf{x})\right)$ exists and finite for all $\mathbf{x} \in S^{m-1}$.

Definition 4.[2] A fixed point $\mathbf{x}^*$ is called hyperbolic if its Jacobian $D_{\mathbf{x}}V(\mathbf{x}^*)$ has no eigenvalues on the unit circle.

Definition 5.[2] A hyperbolic fixed point $\mathbf{x}^*$ is called:

i) attracting if all the eigenvalues of the Jacobian $D_{\mathbf{x}}V(\mathbf{x}^*)$ are less than 1 in absolute value;

ii) repelling if all the eigenvalues of the Jacobian $D_{\mathbf{x}}V(\mathbf{x}^*)$ are greater than 1 in absolute value;

iii) a saddle otherwise.

Consider the following two strictly non-Volterra QSOs on the two-dimensional simplex

$$V : \begin{cases} x_1' = \frac{1}{3}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + 2x_1x_2, \\ x_2' = \frac{1}{3}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + 2x_2x_3, \\ x_3' = \frac{1}{3}x_1^2 + \frac{1}{3}x_2^2 + \frac{1}{3}x_3^2 + 2x_3x_1. \end{cases} \tag{3}$$

Lemma 1. The center $\mathbf{x}$ is a unique and attracting point of the QSO (3).

Lemma 2. The function $\phi(\mathbf{x}) = |x_1 - x_2| \cdot |x_2 - x_3| \cdot |x_3 - x_1|$ is a Lyapunov function for the operator (3).

Lemma 3. $\lim_{n\to\infty} V^n\left(\mathbf{x}^{(0)}\right) = \mathbf{c}$ for any initial point $\mathbf{x}^{(0)} \in S^2$.

Theorem. a) The QSO (3) has a unique fixed point $\mathbf{c} = (1/3, 1/3, 1/3)$;

b) The fixed point $\mathbf{c}$ is an attracting point;

c) For any $\mathbf{x}^{(0)} \in S^2$, the trajectory $\{\mathbf{x}^{(n)}\}$ tends to the fixed point $\mathbf{c}$;

d) The QSO (3) is a regular transformation.

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EXISTENCE OF THE EIGENVALUES OF A TENSOR SUM OF THE FRIEDRICHS MODELS WITH RANK 2 PERTURBATION

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Let T^1 be the one-dimensional torus. In the Hilbert space $L_2^s(T^2)$ of square-integrable symmetric (complex) functions defined on T^2, we consider the model operator:

$$H_{\mu,\lambda} := H_{0,0} - \mu(V_{11} + V_{12}) + \lambda(V_{21} + V_{22}), \quad \mu, \lambda > 0, \tag{1}$$

where $H_{0,0}$ is the multiplication operator:

$$(H_{0,0} f)(x,y) = (u(x) + u(y)) f(x,y),$$

and V_{ij}, $i, j = 1,2$ are non-local interaction operators:

$$(V_{i1} f)(x,y) = \upsilon_i(x) \int_{T^1} \upsilon_i(t) f(t,y) dt, \quad (V_{i2} f)(x,y) = \upsilon_i(y) \int_{T^1} \upsilon_i(t) f(x,t) dt.$$

Here, $f \in L_2^s(T^2)$, the functions $u(\cdot)$ and $\upsilon_i(\cdot)$, $i = 1,2$ are real-valued continuous functions on T^1.

Under these assumptions, the operator $H_{\mu,\lambda}$ is bounded and self- adjoint.

To study the spectral properties of the model operator $H_{\mu,\lambda}$, we introduce a Friedrichs model $h_{\mu,\lambda}$ with rank 2 perturbation, acting on $L_2(T^1)$ by the rule:

$$h_{\mu,\lambda} := h_{0,0} - \mu k_1 + \lambda k_2,$$

where the operators $h_{0,0}$ and $k_i(\cdot)$, $i = 1,2$ are defined as

$$(h_{0,0} g)(x) = u(x) g(x), \ (k_i g)(x) = \upsilon_i(x) \int_{T^1} \upsilon_i(t) g(t) dt, \ i = 1,2.$$

From the definitions of $H_{\mu,\lambda}$ and $h_{\mu,\lambda}$ we obtain the representation

$$H_{\mu,\lambda} = h_{\mu,\lambda} \otimes I + I \otimes h_{\mu,\lambda},$$

where I is an identity operator on $L_2(T^1)$.

Therefore, by theorem on the spectrum of the tensor sum of two operators the equality

$$\sigma(H_{\mu,\lambda}) = \sigma(h_{\mu,\lambda}) + \sigma(h_{\mu,\lambda})$$

holds.

Let $\mathrm{supp}\{\upsilon_\alpha(\cdot)\}$ be the support of the function $\upsilon_\alpha(\cdot)$ and $\mathrm{mes}(\Omega)$ be the Lebesgue measure of the measurable set $\Omega \subset T^1$ and

$$m := \min_{x \in T^1} u(x), \ M := \max_{x \in T^1} u(x).$$

Assume that the function $u(\cdot)$ has the non-degenerate global minimum at the points $x_1, x_2, ..., x_m \in T^1$ and the non-degenerate global maximum at the points $y_1, y_2, ..., y_n \in T^1$.

Main result of the note is the following theorem.

Theorem. Suppose that

$$\mathrm{mes}(\mathrm{supp}\{\upsilon_1(\cdot)\} \cap \mathrm{supp}\{\upsilon_2(\cdot)\}) = 0$$

and $\upsilon_1(x_i) \neq 0$, $\upsilon_2(y_i) \neq 0$ for some $i \in \{1,...,m\}$, $j \in \{1,...,n\}$.

(a) For all values of $\mu, \lambda > 0$ the operator $h_{\mu,\lambda}$ has a two simple eigenvalues $E_\mu^{(1)} < m$ and $E_\lambda^{(2)} > M$.

(b) For any $\mu, \lambda > 0$ the numbers $2E_\mu^{(1)}$ and $2E_\lambda^{(2)}$ are simple eigenvalues of $H_{\mu,\lambda}$. Moreover

$$\sigma_{\mathrm{ess}}(H_{\mu,\lambda}) = [E_\mu^{(1)} + m; E_\mu^{(1)} + M] \cup [2m; 2M] \cup [E_\lambda^{(2)} + m; E_\lambda^{(2)} + M]$$

$$\sigma_{\mathrm{pp}}(H_{\mu,\lambda}) = \{2E_\mu^{(1)}; E_\mu^{(1)} + E_\lambda^{(2)}; 2E_\lambda^{(2)}\}.$$

(c) For any fixed . . and $b > M$, there are two numbers $\mu_0 = \mu_0(a) > 0$ and $\lambda_0 = \lambda_0(b) > 0$, respectively, such that the numbers $2a$, $a + b$ and $2b$ are eigenvalues of H_{μ_0,λ_0}.

(d) For any $c \in [2m; 2M]$ there exist two numbers $\mu_1 = \mu_1(c) > 0$ and $\lambda_1 = \lambda_1(c) > 0$ such that the number c is an eigenvalue of H_{μ_1,λ_1}.

МУНДАРИЖА