Buxoro davlat universiteti
BUXORO, 200117, M.IQBOL ko'chasi, 11-uy, 2022

«AMALIY MATEMATIKA VA AXBOROT TEXNOLOGIYALARINING ZAMONAVIY MUAMMOLARI»
XALQARO ILMIY-AMALIY ANJUMAN

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АМАЛИЙ МАТЕМАТИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ЗАМОНАВИЙ МУАММОЛАРИ

ХАЛҚАРО ИЛМИЙ-АМАЛИЙ АНЖУМАН

МАТЕРИАЛЛАРИ

2022 йил, 11-12 май

БУХОРО – 2022

ЎЗБЕКИСТОН РЕСПУБЛИКАСИ
ОЛИЙ ВА ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ
ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ФАНЛАР АКАДЕМИЯСИ
В.И. РОМАНОВСКИЙ НОМИДАГИ МАТЕМАТИКА ИНСТИТУТИ
ЎЗБЕКИСТОН МИЛЛИЙ УНИВЕРСИТЕТИ
ТОШКЕНТ ДАВЛАТ ТРАНСПОРТ УНИВЕРСИТЕТИ
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QUARTIC NUMERICAL RANGE OF A TRIDIAGONAL 4× 4 OPERATOR MATRICES

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A block operator matrix is a matrix the entries of which are linear operators [1]. If the Hilbert space H is the product of four Hilbert spaces H_1, H_2, H_3 and H_4, that is, $H = H_1 \oplus H_2 \oplus H_3 \oplus H_4$, then every bounded linear operator $A \in L(H)$ has a block operator matrix representation

$$A := \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} \qquad (1)$$

with bounded linear operators $A_{ij} \in L(H_j, H_i)$, $i, j = \overline{1,4}$.

The operator A is a self-adjoint if and only if $A_{ij}^* = A_{ji}$ for all $i, j = \overline{1,4}$. The following generalization of the numerical range of A takes into account the block structure (1) of A with respect to the decomposition $H = H_1 \oplus H_2 \oplus H_3 \oplus H_4$.

Let $S_{H_k} := \{x \in H_k : \|x\| = 1\}$, $k = \overline{1,4}$ be the unit sphere in H_k and

$$S_H = S_{H_1} \times S_{H_2} \times S_{H_3} \times S_{H_4}.$$

Definition. For $f = (f_1, f_2, f_3, f_4) \in S_H$ we define 4× 4 matrix

$$A_f := \begin{pmatrix} (A_{11}f_1, f_1) & (A_{12}f_2, f_1) & (A_{13}f_3, f_1) & (A_{14}f_4, f_1) \\ (A_{21}f_1, f_2) & (A_{22}f_2, f_2) & (A_{23}f_3, f_2) & (A_{24}f_4, f_2) \\ (A_{31}f_1, f_3) & (A_{32}f_2, f_3) & (A_{33}f_3, f_3) & (A_{34}f_4, f_3) \\ (A_{41}f_1, f_4) & (A_{42}f_2, f_4) & (A_{43}f_3, f_4) & (A_{44}f_4, f_4) \end{pmatrix} \in M_4(\mathbb{C}).$$

Then the set

$$\mathcal{W}^4(A) := \bigcup_{f \in S_H} \sigma_p(A_f)$$

is called the quartic numerical range of A (with respect to the block operator matrix representation (1)).

The block numerical range for $n \times n$ operator matrices was introduced in [2] for bounded entries and in [3] for unbounded entries.

In this note we consider the case where $A_{ij} = 0$ if $|i - j| \neq 1$ and $A_{ij}^* = A_{ji}$ if $|i - j| = 1$ for $i, j = \overline{1,4}$. Our main results include a new formula for $\mathcal{W}^4(A)$ and an estimate for the bounds of A in terms of the quartic numerical range.

Let us introduce the following notations:

$$P(f) := |(A_{12}f_2, f_1)|^2 + |(A_{23}f_3, f_2)|^2 + |(A_{34}f_4, f_3)|^2,$$

$$Q(f) := |(A_{12}f_2, f_1)|^2 + |(A_{34}f_4, f_3)|^2;$$

$$E_1(f) := -\frac{\sqrt{2}}{2}\sqrt{P(f) + \sqrt{(P(f))^2 - 4Q(f)}}\,; \quad E_2(f) := -\frac{\sqrt{2}}{2}\sqrt{P(f) - \sqrt{(P(f))^2 - 4Q(f)}}\,;$$

$$E_3(f) := \frac{\sqrt{2}}{2}\sqrt{P(f) - \sqrt{(P(f))^2 - 4Q(f)}}\,;$$

$$E_4(f) := \frac{\sqrt{2}}{2}\sqrt{P(f) + \sqrt{(P(f))^2 - 4Q(f)}}\,.$$

The main result of this note is the following theorem.

Theorem. For the quartic numerical range $\mathcal{W}^4(A)$ of A we have

$$\mathcal{W}^4(A) = \cup_{k=1}^4 \cup_{f \in S_H} \{E_k(f)\}.$$

Moreover, for the lower and upper bounds of A the following estimates are holds:

$$\min\sigma(\mathrm{A}) \geq \inf\mathcal{W}^4(\mathrm{A}) = \inf\bigcup_{\mathrm{f}\in\mathrm{S_H}}\{\mathrm{E_1(f)}\};$$

$$\max\sigma(\mathrm{A}) \leq \sup\mathcal{W}^4(\mathrm{A}) = \sup\bigcup_{\mathrm{f}\in\mathrm{S_H}}\{\mathrm{E_4(f)}\}.$$

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THE GENERALIZED FRACTIONAL DIFFERENTIAL EQUATION OF LAGUERRE TYPE

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Abstract: In this paper, we present the Generalized Differential Equation of Laguerre Type writing with the generalized N-derivative and its associated Generalized Differential Equation, and we solve that using the generalized Laplace Transform and the power series method.

Key words: Generalized derivatives and integral, Laguerre Equation, Associated Laguerre Equation.

THE SPECTRUM OF THE DISCRETE SCHRÖDINGER OPERATOR WITH TWO-RANK PERTURBATION

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The discrete Schrödinger operators have attracted considerable attention for both combinatorial Laplacians and quantum graphs; for some recent summaries refer to [1], [2] and the references therein. Particularly, eigenvalue behavior of discrete Schrödinger operators with small rank potentials are discussed in [3], [4], [5].

The one-particle discrete Schrödinger operator $h_{\lambda\mu}$ in the momentum representation is defined by

$$h_{\lambda\mu} = h_0 - V_{\lambda\mu},,$$

where the non-perturbed operator h_0 acts on $L^2(\mathbb{T})$ as multiplication operator by the function $e(p)$:

$$(h_0 f)(p) = e(p)f(p), \quad f \in L^2(\mathbb{T}), \ \ p \in \mathbb{T},$$

where

$$e(p) = 1 - \cos p, \ \ p \in \mathbb{T}, \ \ \mathbb{T} = (-\pi, \pi].$$

The potential operator of the form

$$\left(V_{\lambda\mu}f\right)(p) = \frac{\mu}{2\pi}\int_{\mathbb{T}} f(q)\,dq + \frac{\lambda}{2\pi}\int_{\mathbb{T}} e^{ix_0(p-q)}f(q)\,dq, \quad f \in L^2(\mathbb{T}), \ \ p \in \mathbb{T}, \ \ x_0 \in \mathbb{Z}.$$

For any $\lambda, \mu \in \mathbb{C}$, we define Fredholm determinant as a analytic function in $z \in \mathbb{C}\backslash[e_{min}, e_{max}]$ as

$$H_z(\lambda,\mu) = \big(\lambda - \gamma(z)\big)\big(\mu - \gamma(z)\big) - \xi(z)$$

where

$$\gamma(z) = \frac{a(z)}{a^2(z) - b^2(z)}, \qquad \xi(z) = \frac{b^2(z)}{\left(a^2(z) - b^2(z)\right)^2}.$$

and

$$a(z) = \frac{1}{2\pi}\int_{\mathbb{T}} \frac{1}{e(q) - z}dq, \qquad b(z) = \frac{1}{2\pi}\int_{\mathbb{T}} \frac{e^{ix_0 q}}{e(q) - z}dq.$$

Lemma. The number $z \in \mathbb{C}\backslash[e_{min}, e_{max}]$ is an eigenvalue of $h_{\lambda\mu}$ if only if $H(\lambda,\mu,z) = 0$.

We introduce the continuation of the function $H(\lambda,\mu,z)$ at the point $z = 0$ as follows

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