2020
APRIL
№.2 (51)
Part II

ISSN 2541-786X

EUROPEAN SCIENCE

HTTPS://SCIENTIFIC-PUBLICATION.COM

UNIVERSITY OF OXFORD

ESSENTIAL SPECTRUM OF A 2x2 OPERATOR MATRIX AND THE FADDEEV EQUATION (Dilmurodov E.B., Rasulov T.H.) p.7

FORMATION AND DEVELOPMENT OF COMPETITIVE SKILLS IN THE SUBJECTS OF “MASS CULTURE” IN CONTINUOUS EDUCATIONAL PROCESS (Tojiboyeva H.M) p.51

PROFESSIONAL ORIENTATION OF COMMUNICATIVE COMPETENCE OF STUDENTS (Kasimova Z.Kh.) p.53

EUROPEAN SCIENCE № 2(51) Part II 2020 ISSN 2541-786X

SCIENTIFIC PUBLISHING «PROBLEMS OF SCIENCE»

Содержание

INVESTIGATION OF THE SPECTRUM OF A DIAGONALIZABLE 4x4-OPERATOR MATRIX

Mustafoeva Z.E.[1], Rasulov T.H.[2] (Republic of Uzbekistan)
Email: Mustafoeva451@scientifictext.ru

[1]*Mustafoeva Zarinabonu Erkin qizi – Master Student;*
[2]*Rasulov Tulkin Husenovich – PhD in Mathematics, Head of Department,*
DEPARTMENT OF MATHEMATICS,
BUKHARA STATE UNIVERSITY,
BUKHARA, REPUBLIC OF UZBEKISTAN

Abstract: *in this paper we consider a* 4×4 *operator matrix* A *acting in the direct sum of four Hilbert spaces is considered. It is unitarily equivalent to* 2×2 *diagonal operator matrix* $\operatorname{diag}\{A^{(+)},A^{(-)}\}$*. Diagonal elements* $A^{(s)}, s=\pm$*, are acting in the four-particle cut subspace of Fock space. Essential and point spectrum of* A *are described using spectrum of more simple two* 4×4 *operator matrices* $A^{(s)}, s=\pm$*. We describe the location of the branches of essential spectrum of* A*. It is shown that the essential spectrum of* A *consists the union of at most 7 bounded closed intervals. An inclusion for the discrete spectrum* A *is obtained.*

Keywords: *operator matrix, standard Fock space, generalized Friedrichs model, essential spectrum.*

ИССЛЕДОВАНИЕ СПЕКТРА ОДНОЙ ДИАГОНАЛИЗИРУЕМОЙ 4X4-ОПЕРАТОРНОЙ МАТРИЦЫ

Мустафоева З.Э.[1], Расулов Т.Х.[2] (Республика Узбекистан)

[1]*Мустафоева Заринабону Эркин кизи – магисрант;*
[2]*Расулов Тулкин Хусенович – кандидат физико-математических наук, заведующий кафедрой,*
кафедра математики,
Бухарский государственный университет,
г. Бухара, Республика Узбекистан

Аннотация: *в данной работе рассматривается* 4×4*-операторная матрица* A*, действующая в прямой сумме четырех гильбертовых пространств. Оно унитарно эквивалентно к* 2×2 *диагональному операторному матрицу* $\operatorname{diag}\{A^{(+)};A^{(-)}\}$*. Диагональные элементы* $A^{(s)}, s=\pm$*, действуют в четырех-частичном урезанном подпространстве фоковского пространства. Существенный и точечный спектр оператора* A *описываются с помощью спектров двух более простых* 4×4*-операторных матриц* $A^{(s)}, s=\pm$*. Мы описываем местоположение ветвей существенного спектра оператора* A*. Установлено, что существенный спектр оператора* $A^{(s)}, s=\pm$ *состоит из объединения не более чем семь отрезков. Получено включение для дискретного спектра оператора* A*.*

Ключевые слова: *операторная матрица, стандартное пространство Фока, обобщенная модель Фридрихса, существенный спектр.*

In statistical physics [1], solid-state physics [2] and the theory of quantum fields [3], one considers systems, where the number of quasi-particles is bounded, but not fixed. Often, the number of particles can be arbitrary large as in cases involving photons, in other cases, such as scattering of spin waves on defects, scattering massive particles and chemical reactions, there are only participants at any given time, though their number can be change.

Recall that the study of systems describing N particles in interaction, without conservation of the number of particles is reduced to the investigation of the spectral properties of self-adjoint operators, acting in the cut subspace $H^{(N)}$ of Fock space, consisting of $n \le N$ particles. In [4] geometric and commutator techniques have been developed in order to find the location of the spectrum and to prove absence of singular continuous spectrum for Hamiltonians without conservation of the particle number.

In the present paper we consider a 4×4 operator matrix A. It is unitarily equivalent to the 2×2 diagonal operator matrix diag $\{A^{(+)},A^{(-)}\}$. The diagonal elements $A^{(s)}, s=\pm$ are acting in the four-particle cut subspace of standard Fock space. We investigate the essential, point and discrete spectrum of A.

Let C be the field of complex numbers, T^d be the d-dimensional torus, $L_2\left((T^d)^n\right)$ be the Hilbert space of square integrable (complex) functions defined on $\left(T^d\right)^n, n\in N$ and $F(L_2(T^d))$ be the standard Fock space over $L_2(T^d)$, where

$$F(L_2(T^d)):=C\oplus L_2(T^d)\oplus L_2((T^d)^2)\oplus\ldots.$$

Set $\Im:=C^2\otimes F(L_2(T^d))$. We write elements F of the space $\Im$ in the form

$$F=\{f_0^{(s)},f_1^{(s)}(k_1),f_2^{(s)}(k_1,k_2),\ldots,f_n^{(s)}(k_1,k_2,\ldots,k_n),\ldots\}$$

of functions of an increasing number of variables $(k_1,\ldots,k_n)\in(T^d)^n$, and a discrete variable $s=\pm$. The norm in $\Im$ is given by

$$\|F\|^2:=\sum_{s=\pm}\left|f_0^{(s)}\right|^2+\sum_{s,n}\int_{(T^d)^n}\left|f_n^{(s)}(k_1,\ldots,k_n)\right|^2dk_1\ldots dk_n .$$

Let $F^{(m)}(L_2(T^d)):=C\oplus L_2(T^d)\oplus L_2(T^d)^2)\oplus\ldots\oplus L_2((T^d)^m), m\in N$.

We consider the operator A acting the Hilbert space $\Im_3:=C^2\otimes F^{(3)}(L_2(T^d))$ and represented as a tridiagonal 4×4 operator matrix

$$A:=\begin{pmatrix} A_{00} & A_{01} & 0 & 0\\ A_{01}^* & A_{11} & A_{12} & 0\\ 0 & A_{12}^* & A_{22} & A_{23}\\ 0 & 0 & A_{23}^* & A_{33}\end{pmatrix}$$

with elements

$$A_{00}f_0^{(s)}=s\varepsilon f_0^{(s)}, A_{01}f_1^{(s)}=a\int_{T^d}v(q')f_1^{(-s)}(q')dq',$$

$$(A_{11}f_1^{(s)})(p)=(s\varepsilon+w(p))f_1^{(s)}(p),$$

$$(A_{12}f_2^{(s)})(p)=\alpha\int_{T^d}v(q')f_2^{(-s)}(p,q')dq' ,$$

$$(A_{22}f_1^{(s)})(p,q)=(s\varepsilon+w(p)+w(q))f_2^{(s)}(p,q),$$

$$(A_{23}f_3^{(s)})(p)=\alpha\int_{T^d}v(q')f_3^{(-s)}(p,q,q')dq'$$

$$(A_{33}f_3^{(s)})(p,q,t)=(s\varepsilon+w(p)+w(q)+w(t))f_3^{(s)}(p,q,t),$$

$$\{f_0^{(s)},f_1^{(s)},f_2^{(s)},f_3^{(s)},s=\pm\}\in\Im_3$$

Here A_{ij} is the adjoint operator to A_{ij} for $i<i$ and the norm of element

$f=\{f_0^{(s)},f_1^{(s)},f_2^{(s)},f_3^{(s)},s=\pm\}\in\Im_3$ is defined by

$$\|f\|^2=\sum_{s=\pm}(\left|f_0^{(s)}\right|^2+\int_{T^d}\left|f_1^{(s)}(p)\right|^2dp+\int_{(T^d)^2}\left|f_2^{(s)}(p,q)\right|^2dpdq+\int_{(T^d)^3}\left|f_3^{(s)}(p,q,t)\right|^2dpdqdt)\ \varepsilon$$

is a real number, $v(\cdot)$ and $w(\cdot)$ are the real-valued continuous functions on T^d, and $\alpha>0$ is the coupling constant.

To investigate the spectral properties of A we introduce the following two bounded self-adjoint operators $A^{(s)}, s=\pm$, which acts in $F^{(3)}(L_2(T^d))$ as

$$A := \begin{pmatrix} \widetilde{A}_{00}^{(s)} & \widetilde{A}_{01} & 0 & 0 \\ \widetilde{A}_{01}^{*} & \widetilde{A}_{11}^{(s)} & \widetilde{A}_{12} & 0 \\ 0 & \widetilde{A}_{12}^{*} & \widetilde{A}_{22}^{(s)} & \widetilde{A}_{23} \\ 0 & 0 & \widetilde{A}_{23}^{*} & \widetilde{A}_{33}^{(s)} \end{pmatrix}$$

with the entries

$$\widetilde{A}_{00}^{(s)} f_0 = s\varepsilon f_0, \ \widetilde{A}_{01} f_1 = \alpha \int_{T^d} v(q') f_1(q') dq',$$

$$(\widetilde{A}_{11}^{(s)} f_1)(p) = (-s\varepsilon + w(p)) f_1(p), \ (\widetilde{A}_{12} f_{12})(p) = \alpha \int_{T^d} v(q') f_2(p,q') dq',$$

$$(\widetilde{A}_{22}^{(s)} f_2)(p,q) = (-s\varepsilon + w(p) + w(q)) f_2(p,q),$$

$$(\widetilde{A}_{23} f_3)(p,q) = \alpha \int_{T^d} v(q') f_3(p,q,q') dq',$$

$$(\widetilde{A}_{33}^{(s)} f_3)(p,q,t) = (-s\varepsilon + w(p) + w(q) + w(t)) f_3(p,q,t),$$

$$(f_0, f_1, f_2, f_3) \in F^{(3)}(L_2(T^d)).$$

Using simple calculations we obtain the following equalities

$$(\widetilde{A}_{01}^{*} f_0)(p) = \alpha v(p) f_0, \ (\widetilde{A}_{12}^{*} f_1)(p,q) = \alpha v(q) f_1(p),$$

$$(\widetilde{A}_{23}^{*} f_2)(p,q,t) = \alpha v(t) f_2(p,q), \ (f_0, f_1, f_2) \in F^{(2)}(L_2(T^d)).$$

We remark that the operators $\widetilde{A}_{01}$, $\widetilde{A}_{12}$ and $\widetilde{A}_{23}$, resp $\widetilde{A}_{01}^{*}$, $\widetilde{A}_{12}^{*}$ and $\widetilde{A}_{23}^{*}$ are called annihilation resp. creation operators, respectively. In this paper we consider the case, where the number of annihilations and creations of the particles of the considering system is equal to 1. It means that $\widetilde{A}_{ij} \equiv 0$ for all $|i-j| > 1$. The spectral properties of 2x2, 3x3 and 4x4 operator matrices are studied in many works, see for example, [5-23]. Spectral inclusion property for diagonally dominant unbounded operator matrices is investigated in [24].

Denote by $\sigma(\cdot), \sigma_{\text{ess}}(\cdot), \sigma_{\text{pp}}(\cdot)$ and $\sigma_{\text{disc}}(\cdot)$, respectively, the spectrum, the essential spectrum, the point and the discrete spectrum of a bounded self-adjoint operator.

The following theorem describes the connection between spectra of A and $A^{(s)}, s = \pm$.

Theorem 1. The equality $\sigma(A) = \sigma(A^{(+)}) \cup \sigma(A^{(-)})$ holds. Moreover,

$$\sigma_{\text{ess}}(A) = \sigma_{\text{ess}}(A^{(+)}) \cup \sigma_{\text{ess}}(A^{(-)}), \ \sigma_{\text{P}}(A) = \sigma_{\text{p}}(A^{(+)}) \cup \sigma_{\text{p}}(A^{(-)}).$$

The set $\sigma_{\text{ess}}(A^{(s)}), s = \pm$ consists at most seven bounded closed.

Since the part of $\sigma_{\text{disc}}(A^{(s)})$ can be located in $\sigma_{\text{ess}}(A^{(-s)})$ we have the inclusion

$$\sigma_{\text{disc}}(A) \subseteq \sigma_{\text{disc}}(A^{(+)}) \cup \sigma_{\text{disc}}(A^{(-)}), \qquad (1)$$

$$\sigma_{\text{disc}}(A) = \{\sigma_{\text{disc}}(A^{(+)}) \cup \sigma_{\text{disc}}(A^{(-)})\} \setminus \sigma_{\text{ess}}(A) \qquad (2)$$

More exactly

$$\sigma_{\text{disc}}(A) = \bigcup_{s=\pm} \{\sigma_{\text{disc}}(A^{(s)}) \setminus \sigma_{\text{ess}}(A^{(-s)})\}.$$

We remark that for $s = \pm$ the operator $A^{(s)}$ has more simple structure than A, and hence, Theorem 1 and relations (1), (2) plays important role in the next investigations of the spectrum of A.

References / Список литературы

1. *Minlos R.A., Spohn H.* The three-body problem in radioactive decay: the case of one atom and at most two photons // American Mathematical Society Translations-Series 2, 177, 1996. P. 159-193.
2. *Mogilner A.I.* Hamiltonians in solid state physics as multiparticle discrete Schrodinger operators: problems and results // Advances in Sov. Math. 5 (1991), pp. 139-194.
3. *Friedrichs K.O.* Perturbation of spectra in Hilbert space // AMS, Providence, Rhode Island, 1965.
4. *Sigal I.M., Soffer A., Zielinski L.* On the spectral properties of Hamiltonians without conservation of the particle number // J. Math. Phys. 43(4) (2002), Pp. 1844-1855.
5. *Rasulov T.H.* On the finiteness of the discrete spectrum of a 3x3 operator matrix // Methods of Functional Analysis and Topology, 22:1 (2016), Pp. 48-61.
6. *Muminov M.I., Rasulov T.H.* On the eigenvalues of a 2x2 block operator matrix // Opuscula Mathematica. 35:3 (2015), Pp. 369-393.
7. *Rasulov T.Kh.* Discrete spectrum of a model operator in Fock space // Theor. Math. Phys., 153:2, 2007. Pp. 1313-1321.
8. *Rasulov T.Kh.* On the number of eigenvalues of a matrix operator // Siberian Math. J. 52:2, 2011. Pp. 316-328.
9. *Muminov M.I., Rasulov T.Kh.* An eigenvalue multiplicity formula for the Schur complement of a 3x3 block operator matrix // Siberian Math. J., 56:4, 2015. Pp. 878–895.
10. *Muminov M., Neidhardt H., Rasulov T.* On the spectrum of the lattice spin-boson Hamiltonian for any coupling: 1D case // Journal of Mathematical Physics, 56, 2015. 053507.
11. *Rasulov T.Kh.* Branches of the essential spectrum of the lattice spin-boson model with at most two photons // Theoretical and Mathematical Physics, 186:2, 2016. 251-267.
12. *Rasulov T., Tosheva N.* New branches of the essential spectrum of a family of 3x3 operator matrices // Journal of Global Research in Math. Archive. 6:9, 2019. Pp. 18-21.
13. *Muminov M.I., Rasulov T.H.* Embedded eigenvalues of an Hamiltonian in bosonic Fock space // Comm. in Mathematical Analysis. 17:1, 2014. Pp. 1-22.
14. *Rasulov T.H.* The finiteness of the number of eigenvalues of an Hamiltonian in Fock space // Proceedings of IAM. 5:2, 2016. Pp. 156-174.
15. *Muminov M.I., Rasulov T.H.* Infiniteness of the number of eigenvalues embedded in the essential spectrum of a 2x2 operator matrix // Eurasian Mathematical Journal. 5:2, 2014. Pp. 60-77.
16. *Rasulov T.Kh.* Study of the essential spectrum of a matrix operator // Theoret. and Math. Phys. 164:1, 2010. Pp. 883-895.
17. *Rasulov T.H.* Investigations of the essential spectrum of a Hamiltonian in Fock space // Appl. Math. Inf. Sci. 4:3, 2010. Pp. 395-412.
18. *Rasulov T.H., Dilmurodov E.B.* Eigenvalues and virtual levels of a family of 2x2 operator matrices // Methods of Functional Analysis and Topology. 25:1, 2019. Pp. 273-281.
19. *Rasulov T.H., Dilmurodov E.B.* Investigations of the numerical range of a operator matrix. J. Samara State Tech. Univ., Ser. Phys. and Math. Sci. 35:2 (2014), Pp. 50-63.
20. *Rasulov T.H., Dilmurodov E.B.* Threshold analysis for a family of 2x2 operator matrices // Nanosystems: Physics, Chemestry, Mathematics. 10, 2019. № 6 Pp. 616-622.
21. *Rasulov T.H., Dilmurodov E.B.* Threshold effects for a family of 2x2 operator matrices // Journal of Global Research in Mathematical Archives. 6, 2019. № 10. Pp. 4-8.
22. *Dilmurodov E.B.* On the virtual levels of a family matrix operators of order 2 // Scientific reports of Bukhara State University, 2019. № 1. Pp. 42-46.
23. *Rasulov T.Kh., Dilmurodov E.B.* Estimates for quadratic numerical range of a operator matrix // Uzbek Math. Zh., 2015. № 1. Pp. 64-74.
24. *Rasulov T., Tretter C.* Spectral inclusion for diagonally dominant unbounded operator matrices // Rocky Mountain J. Math., 2018. № 1. 279-324.

НАУЧНОЕ ИЗДАНИЕ

ИЗДАТЕЛЬСТВО
«ПРОБЛЕМЫ НАУКИ»

АДРЕС РЕДАКЦИИ:
153008, РФ, Г. ИВАНОВО, УЛ. ЛЕЖНЕВСКАЯ, Д. 55, 4 ЭТАЖ
ТЕЛ.: +7 (910) 690-15-09.

HTTPS://SCIENTIFIC-PUBLICATION.COM
E-MAIL: INFO@P8N.RU

ТИПОГРАФИЯ:
ООО «ПРЕССТО».
153025, Г. ИВАНОВО, УЛ. ДЗЕРЖИНСКОГО, Д. 39, СТРОЕНИЕ 8

ИЗДАТЕЛЬ:
ООО «ОЛИМП»
УЧРЕДИТЕЛЬ: ВАЛЬЦЕВ СЕРГЕЙ ВИТАЛЬЕВИЧ
117321, Г. МОСКВА, УЛ. ПРОФСОЮЗНАЯ, Д. 140