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Содержание

THRESHOLD EIGENVALUES OF A TWO-CHANNEL MOLECULAR-RESONANCE MODEL

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Abstract: *in the present paper we consider a two-channel molecular-resonance model. This model can be represented as a bounded and self-adjoint* 2×2 *operator matrix* A *acting in the two-particle cut subspace of standard Fock space. It is associated with the Hamiltonian of a system consisting at most two particles, interacting via both a pair contact potentials* $(\mu_2 > 0)$ *and creation and annihilation operators* $(\mu_1 > 0)$. *We construct the Fredholm determinant corresponding to* A. *We find some necessary and sufficient conditions for that the number* $z = \min \sigma_{ess}(A)$ *to be an eigenvalue (threshold eigenvalue) of* A.

Keywords: *threshold eigenvalue, molecular-resonance model, coupling constant, nuclear space, molecular space, Hamiltonian, operator matrix, essential spectrum.*

ПОРОГОВЫЕ СОБСТВЕННЫЕ ЗНАЧЕНИЯ ДВУХКАНАЛЬНОЙ МОЛЕКУЛЯРНО-РЕЗОНАНСНОЙ МОДЕЛИ

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Аннотация: *в настоящей работе мы рассмотрим двухканальную молекулярно-резонансную модель. Этамодель представляется как ограниченная и самосопряженная* 2×2 *операторная матрица* A *действующая в двухчастичном обрезанном подпространстве стандартного фоковского пространства. Оно ассоциировано гамильтониановской системой, состоящей из не более чем двух частиц, взаимодействующих как с помощью парного контактного потенциала* $(\mu_2 > 0)$, *так и с помощью операторов рождения и уничтожения* $(\mu_1 > 0)$. *Построим определитель Фредгольма соответствующий к* A. *Найдем необходимые и достаточные условия для того, чтобы число* $z = \min \sigma_{ess}(A)$ *являлось собственным значением (пороговые собственное значение) оператора* A.

Ключевые слова: *пороговые собственное значение, молекулярно-резонансной модель, параметр взаимодействия, ядерное пространство, молекулярное пространство, Гамильтониан, операторная матрица, существенный спектр.*

We adopt the following conventions the present paper. Let C be the field of complex numbers, T^3 be the three-dimensional torus, the cube $\left(-\pi, \pi\right]^3$ with appropriately identified sides equipped with its Haar measure and $L_2\left(T^3\right)$ be the Hilbert space of square integrable (complex) functions defined on T^3. We consider a two-channel Hilbert space $H := H_0 \oplus H_1$ consisting of a one-dimensional “molecular” space $H_0 := C$ (channel 1) and a “nuclear” space (channel 2). The elements of H are represented as vectors $f=(f_0,f_1)$, where $f_0\in H_0$ and $f_1\in H_1$. The inner product

$$\langle f,g\rangle_H := f_0\overline{g_0}+\langle f_1,g_1\rangle_{H_1}$$

of any two elements $f=(f_0,f_1)$, $g=(g_0,g_1)\in H,$ is naturally defined via the inner products $f_0\overline{g_0}$ in H_0 and $\langle f_1,g_1\rangle_{H_1}$ in H_1, that is

$$\langle f_1,g_1\rangle_{H_1} := \int_{T^3} f_1(t)\overline{g_1(t)}dt.$$

As a Hamiltonian in the Hilbert space H we consider the 2×2 block operator matrix

$$A:=\begin{pmatrix} A_{00} & A_{01} \\ A_{01}^* & A_{11}\end{pmatrix}, \qquad (1)$$

where the matrix elements $A_{ij}:H_j\to H_i,\ i\le j,\ i,j=0,1$ are defined by

$$A_{00}f_0=\omega_0 f_0, \quad A_{01}f_1=\mu_1\int_{T^3}\upsilon_1(t)f_1(t)dt,,$$

$$(A_{11}f_1)(p)=\omega_1(p)f_1(p)-\mu_2 v_2(p)\mu_1\int_{T^3}\upsilon_2(t)f_1(t)dt.$$

Here $\mu_\alpha,\alpha=1,2$ are real numbers, $\omega_0\in R$ is a trial “molecular” energy, a vector $\upsilon_1\in H_1$ provides the coupling between the channels and A_{11} is the (self-adjoint) “nuclear Hamiltonian” in H_1. We assume that the functions $\upsilon_\alpha(\cdot),\alpha=1,2$ and $\omega_1(\cdot)$ are real-valued continuous functions on T^3.

Under these assumptions the operator A is bounded and self-adjoint.

It should be mentioned that the Hamiltonian (1) is a rather more general model than that one considered by Friedrichs [1] (for a some discussion of A_{11} see, e.g., [2,3] and the references cited therein).

Molecules are usually treated as purely Coulombic systems, while the strong interaction between their nuclear constituents is assumed to play a negligible role. If there is no coupling between the channels, i.e. for $\upsilon_1=0$, the spectrum of A consists of the spectrum of A_{11} and the addition discrete eigenvalue ω_0. A nontrivial coupling between the channels will, in general, shift the eigenvalue ω_0 into an unphysical sheet of the energy plane. The resulting perturbed energy appears as a resonance, i.e., as a pole of the analytic continuation of the resolvent $(A-z)^{-1}$ taken between suitable states (see [4]). In [5], the Hamiltonian (1) was studied in order to exhibit explicitly the mechanism leading to the enhancement of fusion probability in case of a narrow near-threshold nuclear resonance.

Let the operator A_0 acts in H as

$$A_0:=\begin{pmatrix} 0 & 0 \\ 0 & A_{11}^0\end{pmatrix} \text{ with } (A_{11}^0 f_1)(p)=\omega_1(p)f_1(p).$$

The perturbation $A-A_0$ of the operator A_0 is a self-adjoint operator of rank 3, and thus, according to the Weyl theorem, the essential spectrum of the operator A coincides with the essential spectrum of A_0. It is evident that $\sigma_{ess}(A_0)=[m;M]$, where the numbers m and M are defined by

$$m:=\min_{x\in T^3}\omega_1(p),\quad M:=\max_{x\in T^3}\omega_1(p).$$

This yields $\sigma_{ess}(A)=[m;M]$.

For any fixed $\mu_\alpha, \alpha=1,2$ we define an analytic function $\Delta_{\mu_1\mu_2}(\cdot)$ (the Fredholm determinant associated with the operator A) in $C\setminus[m;M]$ by

$$\Delta_{\mu_1\mu_2}(z):=\Delta^{(1)}_{\mu_1}(z)\Delta^{(2)}_{\mu_2}(z)+\mu_1^2\mu_2(\Delta^{(3)}(z))^2,$$

where the functions $\Delta^{(1)}_{\mu_1}(\cdot),\Delta^{(2)}_{\mu_2}(\cdot)$ and $\Delta^{(3)}(\cdot)$ are defined by

$$\Delta^{(1)}_{\mu_1}(z):=\omega_0-z-\mu_1^2\int_{T^3}\frac{\upsilon_1^2(t)dt}{\omega_1(t)-z},$$

$$\Delta^{(2)}_{\mu_2}(z):=1-\mu_2\int_{T^3}\frac{\upsilon_2^2(t)dt}{\omega_1(t)-z},\quad \Delta^{(3)}(z):=\int_{T^3}\frac{\upsilon_1(t)\upsilon_2(t)dt}{\omega_1(t)-z}.$$

By the Birman-Schwinger principle and the Fredholm theorem we conclude that the operator A has an eigenvalue $z\in C\setminus[m;M]$ if and only if $\Delta_{\mu_1\mu_2}(z)=0$.

Suppose that the function $\omega_1(\cdot)$ has an unique non-degenerate minimum at the point $p_0\in T^3$, the functions $\omega_1(\cdot)$, $v_\alpha, \alpha=1,2$ have continuous partial derivatives up to order 3 inclusive at some neighborhood of the point $p_0\in T^3$ and Lebesques measure of $\sup p\{\upsilon_1(\cdot)\}\cap\sup p\{\upsilon_2(\cdot)\}$ is equal to zero, where $\mathrm{supp}\{\upsilon_\alpha(\cdot)\}$ of the function $\upsilon_\alpha(\cdot)$. For the case $\omega_0>m$ we set

$$\mu_1^0:=\left((\omega_0-m)\left(\int_{T^3}\frac{\upsilon_1^2(t)dt}{\omega_1(t)-m}\right)^{-1}\right)^{1\backslash 2},\quad \mu_2^0:=\left(\int_{T^3}\frac{\upsilon_2^2(t)dt}{\omega_1(t)-m}\right)^{-1}.$$

Main result of the paper is the following theorem

Theorem 1. The number $z=m$ is an eigenvalue of A with multiplicity 2 if and only if $\mu_\alpha=\mu_\alpha^0$ and $\upsilon_\alpha(p_0)=0$ for $\alpha=1,2$.

This result plays important role in the investigations of the essential and discrete spectrum of the corresponding 2×2 (see e.g., [6-13]), 3×3 (see e.g., [14-23]) and 4×4 (see e.g., [24, 25]) operator matrices in the cut subspaces of standart Fock space.

References / Список литературы

1. *Friedrichs K.O.* Uber die Spectralzerlegung einee Integral operators // Math. Ann., 115:1, 1938. Pp. 249-272.
2. *Abdullaev Zh.I., Ikromov I.A., Lakaev S.N.* Embedded eigenvalues and resonances of a generalized Friedrichs model // Theor. Math. Phys. 103:1. 1995. Pp. 390-398.
3. *Muminov M.E.* Expression for the number of eigenvalues of a Friedrichs model // Math. Notes, 82:1-2, 2007. Pp. 67-74.
4. *Reed M., Simon B.* Methods of modern mathematical physics. IV: Analysis of Operators // Academic Press, New York, 1979.

5. *Motovilov A.K., Sandhas W., Belyaev Y.B.* Perturbation of a lattice spectral band by a nearby resonance // J. Math. Phys. 42, 2001. Pp. 2490-2506.
6. *Muminov M.I., Rasulov T.H.* On the eigenvalues of a 2×2 block operator matrix // Opuscula Mathematica. 35:3, 2015. Pp. 369-393.
7. *Muminov M.I., Rasulov T.H.* Infiniteness of the number of eigenvalues embedded in the essential spectrum of a 2×2 operator matrix // Eurasian Mathematical Journal. 5:2, 2014. Pp. 60-77.
8. *Rasulov T.H., Dilmurodov E.B.* Eigenvalues and virtual levels of a family of 2×2 operator matrices // Methods of Functional Analysis and Topology, 25:1, 2019. Pp.273-281.
9. *Rasulov T.H., Dilmurodov E.B.* Investigations of the numerical range of a operator matrix. J. Samara State Tech. Univ., Ser. Phys. and Math. Sci. 35:2, 2014. Pp. 50-63.
10. *Rasulov T.H., Dilmurodov E.B.* Threshold analysis for a family of 2×2 operator matrices // Nanosystems: Physics, Chemestry, Mathematics, 10, 2019. № 6. Pp. 616-622.
11. *Rasulov T.H., Dilmurodov E.B.* Threshold effects for a family of 2×2 operator matrices // Journal of Global Research in Mathematical Archives, 6, 2019. № 10, Pp. 4-8.
12. *Dilmurodov E.B.* On the virtual levels of a family matrix operators of order 2 // Scientific reports of Bukhara State University. 2019. № 1. Pp. 42-46.
13. *Rasulov T.Kh., Dilmurodov E.*B. Estimates for quadratic numerical range of a operator matrix // Uzbek Math. Zh., 2015. № 1. Pp. 64-74.
14. *Rasulov T.H.* On the finiteness of the discrete spectrum of a 3×3 operator matrix // Methods of Functional Analysis and Topology, 22:1, 2016. Pp. 48-61.
15. *Rasulov T.Kh.* Discrete spectrum of a model operator in Fock space // Theor. Math. Phys., 153:2, 2007. Pp. 1313-1321.
16. *Rasulov T.Kh.* On the number of eigenvalues of a matrix operator // Siberian Math. J. 52:2, 2011. Pp. 316-328.
17. *Muminov M.I., Rasulov T.Kh.* An eigenvalue multiplicity formula for the Schur complement of a 3×3 block operator matrix // Siberian Math. J., 56:4, 2015. Pp. 878-895.
18. *Muminov M., Neidhardt H., Rasulov T.* On the spectrum of the lattice spin-boson Hamiltonian for any coupling: 1D case // Journal of Mathematical Physics, 56, 2015. 053507.
19. *Rasulov T.Kh.* Branches of the essential spectrum of the lattice spin-boson model with at most two photons // Theoretical and Mathematical Physics, 186:2, 2016. 251-267.
20. *Rasulov T., Tosheva N.* New branches of the essential spectrum of a family of 3×3 operator matrices // Journal of Global Research in Math. Archive. 6:9, 2019. Pp. 18-21.
21. *Muminov M.I., Rasulov T.H.* Embedded eigenvalues of an Hamiltonian in bosonic Fock space // Comm. in Mathematical Analysis. 17:1, 2014. Pp. 1-22.
22. *Rasulov T.H.* The finiteness of the number of eigenvalues of an Hamiltonian in Fock space // Proceedings of IAM, 5:2, 2016. Pp. 156-174.
23. *Rasulov T.Kh.* Study of the essential spectrum of a matrix operator // Theoret. and Math. Phys. 164:1, 2010. Pp. 883-895.
24. *Rasulov T.H.* Investigations of the essential spectrum of a Hamiltonian in Fock space // Appl. Math. Inf. Sci. 4:3, 2010. Pp. 395-412.
25. *Rasulov T.H., Muminov M.I., Hasanov M.* On the spectrum of a model operator in Fock space // Methods Funct. Anal. Topology 15:4, 2009. Pp. 369-383.

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