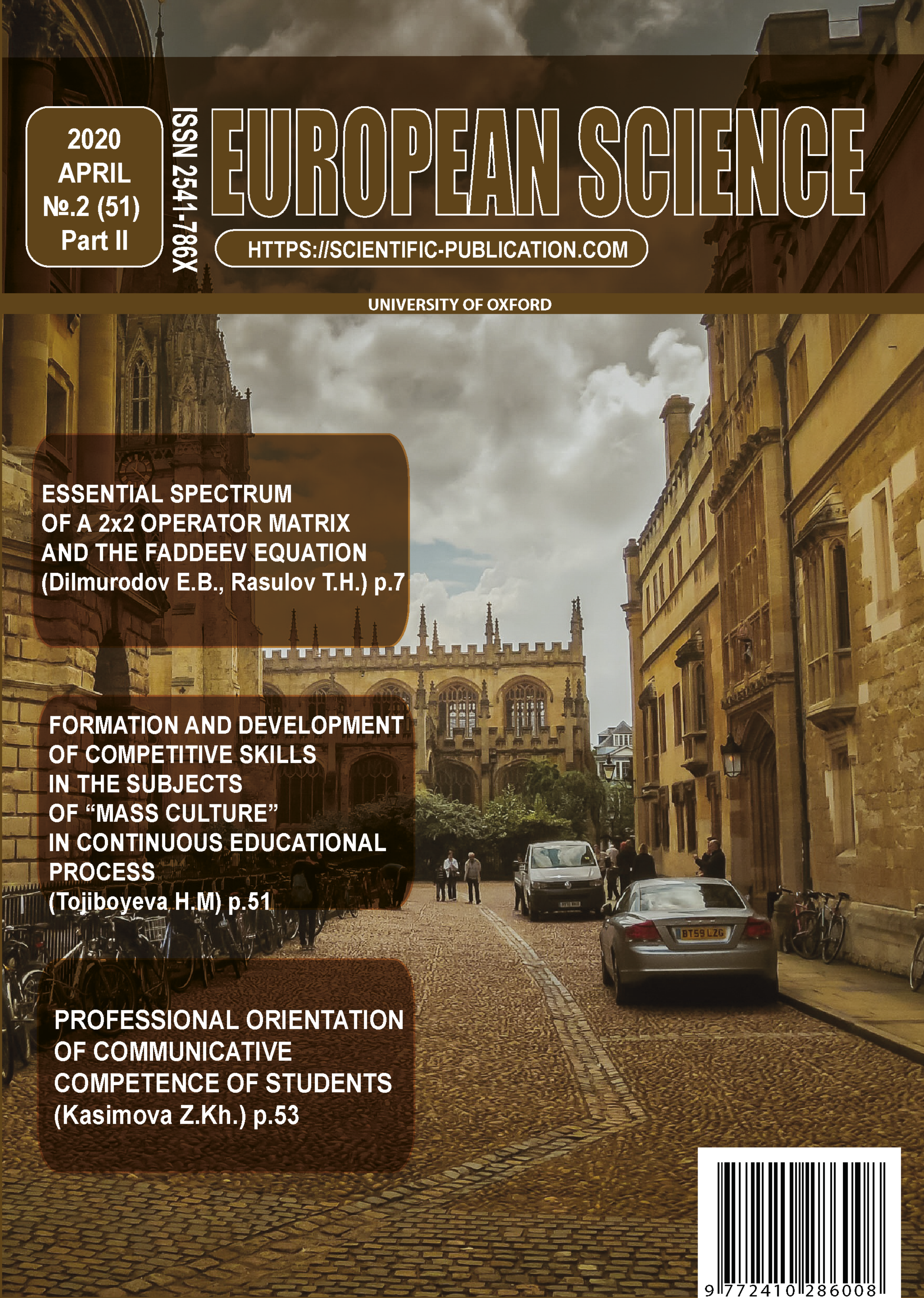

2020
APRIL
№.2 (51)
Part II

ISSN 2541-786X

EUROPEAN SCIENCE

HTTPS://SCIENTIFIC-PUBLICATION.COM

UNIVERSITY OF OXFORD

EUROPEAN SCIENCE № 2(51) Part II 2020 ISSN 2541-786X

ESSENTIAL SPECTRUM
OF A 2x2 OPERATOR MATRIX
AND THE FADDEEV EQUATION
(Dilmurodov E.B., Rasulov T.H.) p.7

FORMATION AND DEVELOPMENT
OF COMPETITIVE SKILLS
IN THE SUBJECTS
OF “MASS CULTURE”
IN CONTINUOUS EDUCATIONAL
PROCESS
(Tojiboyeva H.M) p.51

PROFESSIONAL ORIENTATION
OF COMMUNICATIVE
COMPETENCE OF STUDENTS
(Kasimova Z.Kh.) p.53

SCIENTIFIC PUBLISHING «PROBLEMS OF SCIENCE»

Содержание

CHARACTERISTIC PROPERTY OF THE FADDEEV EQUATION FOR THREE-PARTICLE MODEL OPERATOR ON A ONE-DIMENSIONAL LATTICE

Umirkulova G.H.[1], Rasulov T.H.[2] (Republic of Uzbekistan)
Email: Umirkulova451@scientifictext.ru

[1]Umirkulova Gulhayo Husniddin qizi – Master Student;
[2]Rasulov Tulkin Husenovich – PhD in Mathematics, Head of Department,
DEPARTMENT OF MATHEMATICS,
BUKHARA STATE UNIVERSITY,
BUKHARA, REPUBLIC OF UZBEKISTAN

Abstract: *in this paper a model operator (Hamiltonian) $H_{\mu\gamma}, \mu, \gamma > 0$ associated to a system of three quantum particles on a one-dimensional lattice that interact via non-local potentials is studies. We construct an analogue of the Faddeev type system of integral equations for the eigenfunctions of $H_{\mu\gamma}$ and give a characteristic property of this system of equations. We describe the essential spectrum of $H_{\mu\gamma}$. It is established that the essential spectrum of $H_{\mu\gamma}$ consists the union of at most three bounded closed intervals.*

Keywords: *model operator, non-local potentials, system of particles, the Faddeev equation, essential spectrum, Hilbert-Schmidt operator.*

ХАРАКТЕРИСТИЧЕСКИЕ СВОЙСТВА УРАВНЕНИЯ ФАДДЕЕВА ДЛЯ ТРЕХЧАСТИЧНОГО МОДЕЛЬНОГО ОПЕРАТОРА НА ОДНОМЕРНОЙ РЕШЕТКЕ

Умиркулова Г.Х.[1], Расулов Т.Х.[2] (Республика Узбекистан)

[1]Умиркулова Гулхаё Хусниддин кизи – магистрант;
[2]Расулов Тулкин Хусенович - кандидат физико-математических наук, заведующий кафедрой,
кафедра математики,
Бухарский государственный университет,
г. Бухара, Республика Узбекистан

Аннотация: *в этой работе изучен модельный оператор (Гамильтониан) $H_{\mu\gamma}, \mu, \gamma > 0$, ассоциированный с системой трех квантовых частиц на одномерной решетке и взаимодействующих с помощью парных нелокальных потенциалов. Построен аналог системы интегральных уравнений типа Фаддеева для собственных функций оператора $H_{\mu\gamma}$ и изучены его характеристические свойства. Описан существенный спектр оператора $H_{\mu\gamma}$. Установлено, что существенный спектр оператора $H_{\mu\gamma}$ состоит из объединения не более чем трех отрезков.*

Ключевые слова: *модельный оператор, нелокальные потенциалы, система частиц, уравнения Фаддеева, существенный спектр, оператор Гильберта-Шмидта.*

It is well-known that in the physical literature, local potentials, i.e., multiplication operators by a function, are typically used. But the potentials constructed, for example, in pseudo-potential theory [1] turn out to be non-local. Such for a periodic operator are given by the sum of local and a finite dimensional potentials. Non-local separable two-particle interactions have been used in nuclear physics and also many-particle problems because of the fact that the two-particle Schrödinger equation is very easily solvable for them, and leads to closed expressions for a large class of such interactions. They have also been used very systematically with Faddeev type equations for the three-particle problem. Their main feature is that the partial-wave t-matrix has a very simple form, and can be continued off the energy-shell in a straightforward manner, a feature which is most important, as is well known, in nuclear physics, and in the Faddeev type equations [2].

In the present paper the model operator (Hamiltonian) $H_{\mu\gamma}, \mu, \gamma > 0$ associated to a system of three particles on a one-dimensional lattice and interacting via non-local potentials is considered. One of the important problem in the spectral theory of such operators is to construct an analogue of the Faddeev type system of integral equations for the eigenfunctions of $H_{\mu\gamma}$ and to describe its essential spectrum.

Let us state the problem. We denote by T^1 the one-dimensional torus. The operations addition and multiplication by real numbers elements of $T^1 \subset R$ should be regarded as operations on R modulo $2\pi Z$. For example, if

$x = \frac{3\pi}{5}, y = \frac{2\pi}{3} \in T^1$, then $x + y = -\frac{11\pi}{15}, 10x = 0 \in T^1$.

In the Hilbert space $L_2^S(T^2)$ of square-integrable symmetric (complex) functions defined on T^2, we consider the model operator:

$$H_{\mu,\gamma} := H_0 - \mu V_1 - \mu V_2 - \gamma V_3$$

where H_0 is the multiplication operator by the function $u(\cdot,\cdot)$:

$$(H_0 f)(x,y) = u(x,y) f(x,y)$$

and $V_\alpha, \alpha = 1,2,3$ are non-local interaction operators:

$(V_1 f)(x,y) = v_1(x) \int_{T^1} v_1(t) f(t,y) dt,$

$(V_2 f)(x,y) = v_1(y) \int_{T^1} v_1(t) f(x,t) dt,$

$$(V_3 f)(x,y) = \int_{T^1} v_2(t) f(t, x + y - t) dt.$$

Here $f \in L_2^S(T^2), \mu, \gamma$ are real positive numbers, the functions $v_\alpha(\cdot)$, $\alpha = 1,2$ are real-valued continuous functions on T^1 and the function $u(\cdot,\cdot)$ is a real-valued symmetric continuous function on T^2. By the definition, the operators $V_\alpha, \alpha = 1,2,3$ are partial integral operators with degenerate kernel of rank 1.

Under these assumptions on the parameters the operator $H_{\mu\gamma}$ is bounded and self-adjoint.

Note that Schrödinger operators of the form (1) associated with a system of three particles on a lattice were studied in many works, see e.g., [3-17]. In [3,4] the sufficient conditions for the finiteness and infiniteness of the discrete spectrum are found. In [5] it was proved that the essential spectrum of a three-particle Schrödinger operator on a lattice is the union of at most finitely many closed intervals even in the case where the corresponding two-particle Schrödinger operator on a lattice has an infinite number of eigenvalues. In [6] the Efimov effect for (1) was demonstrated when the parameter function $u(\cdot,\cdot)$ has a special form. Spectral properties of operator matrices, whose one of the diagonal elements have a form (1) were studied in [18-28].

The spectrum, the essential spectrum and the discrete spectrum of a bounded self-adjoint operator will be denoted by $\sigma(\cdot)$, $\sigma_{ess}(\cdot)$ and $\sigma_{disc}(\cdot)$, respectively.

For any $x \in T^1$ and μ, γ we define an analytic functions $\Delta_\mu^{(1)}(x;\cdot)$ and $\Delta_\gamma^{(2)}(x;\cdot)$ in $\setminus [m(x); M(x)]$ by

$$\Delta_\mu^{(1)}(x;\, z) := 1 - \mu \int_{T^1} \frac{v_1^2(t)dt}{u(x,t)-z}, \quad \Delta_\gamma^{(2)}(x;\, z) := 1 - \gamma \int_{T^1} \frac{v_2(t)dt}{u(t,x-t)-z},$$

where the numbers $m(x)$ and $M(x)$ are defined by

$m(x) := \min_{y \in T^1} u(x,y)$ and $M(x) := \max_{y \in T^1} u(x,y)$

Let σ_1 resp. σ_2 be the set of all complex numbers $z \in C \setminus [m(x);\ M(x)]$ such that the equality $\Delta_\mu^{(1)}(x;\, z) = 0$ resp. $\Delta_\gamma^{(2)}(x;\, z) = 0$ holds for some $x \in T^1$ and

$m := \min_{x,y \in T^1} u(x,y), \quad M := \max_{x,y \in T^1} u(x,y), \qquad := \sigma_1 \cup \sigma_2 \cup [m : M].$

We introduce the following vector space

$$L_2^{(2)}(T^1) := \{(f_1, f_2) : f_\alpha \in L_2(T^1), \alpha = 1,2\}.$$

In our analysis of the essential and discrete spectrum of $H_{\mu\gamma}$ the crucial role is played by the Faddeev-Newton-type operator matrix $T(z), z \in C\setminus$ acting on $L_2^{(2)}(T^1)$ as

$$T(z) := \begin{pmatrix} T_{11}(z) & T_{12}(z) \\ T_{21}(z) & 0 \end{pmatrix}$$

with the entries $T_{ij}(z) : L_2(T^1) \to L_2(T^1),\ i,j = 1,2$:

$$(T_{11}(z) g_1)(x) = \frac{\mu v_1(x)}{\Delta_\mu^{(1)}(x;z)} \int_{T^1} \frac{v_1(t) g_1(t) dt}{u(x,t) - z};$$

$$(T_{12}(z) g_2)(x) = \frac{\gamma}{\Delta_\mu^{(1)}(x;z)} \int_{T^1} \frac{v_1(t-x) g_2(t) dt}{u(t,t-x) - z};$$

$$(T_{21}(z) g_1)(x) = \frac{\mu}{\Delta_\gamma^{(2)}(x;z)} \int_{T^1} \frac{v_1(x-t)(v_2(t) + v_2(x-t)) g_1(t) dt}{u(t,t-x) - z}.$$

We note that for each $z \in C\setminus$ the entries $T_{ij}(z)$ belong to the Hilbert-Schmidt class and therefore, $T(z)$ is a compact operator.

The following theorem is an analog of the well-known Faddeev's result for the operator $H_{\mu\gamma}$ and establishes a connection between eigenvalues of $H_{\mu\gamma}$ and $T(z)$.

Theorem 1. *The number $z \in C\setminus\Sigma$ is an eigenvalue of the operator $H_{\mu\gamma}$ if and only if the number $\lambda = 1$ is an eigenvalue of the operator $T(z)$. Moreover, the eigenvalues z and 1 have the same multiplicities.*

We point out that the matrix equation $T(z)g = g$ is an analogue of the Faddeev type system of integral equations for eigenfunctions of $H_{\mu\gamma}$.

The following theorem describes the location of the essential spectrum of $H_{\mu\gamma}$.

Theorem 2. *The essential spectrum of $H_{\mu\gamma}$ is coincide with the Σ, that is, $\sigma_{ess}(H_{\mu\gamma}) = \Sigma$. Moreover, the set Σ consists no more than three bounded closed intervals.*

In the following we introduce the new subsets of the essential spectrum of $H_{\mu\gamma}$: the sets $\sigma_1 \cup \sigma_2$ and $[m\colon M]$ are called two- and three-particle branches of the essential spectrum of $H_{\mu\gamma}$, respectively.

References / Список литературы

1. *Heine V., Cohen M., Weaire D.* The Pseudopotential Concept // Academic Press. New York-London, 1970. 558 P.
2. *Newton R.G.* Scattering Theory of Waves and Particles // Springer-Verlag. New York, 1982. 745 P.
3. *Albeverio S., Lakaev S.N., Muminov Z.I.* On the number of eigenvalues of a model operator associated to a system of three-particles on lattices // Russ. J. Math. Phys., 14:4, 2007. Pp. 377-387.
4. *Albeverio S., Lakaev S.N., Djumanova R.Kh.* The essential and discrete spectrum of a model operator associated to a system of three identical quantum particles // Rep. Math. Phys. 63:3, 2009. Pp. 359-380.
5. *Albeverio S., Lakaev S.N., Muminov Z.I.* On the structure of the essential spectrum for the three-particle Schrödinger operators on lattices // Math. Nachr. 280:7, 2007. Pp. 699-716.
6. *Rasulov T.Kh.* Asymptotics of the discrete spectrum of a model operator associated with the system of three particles on a lattice // Theor. Math. Phys. 163:1, 2010. Pp. 429-437.
7. *Rasulov T.Kh.* Essential spectrum of a model operator associated with a three particle system on a lattice // Theor. Math. Phys., 166:1, 2011. Pp. 81-93.
8. *Rasulova Z.D.* Investigations of the essential spectrum of a model operator associated to a system of three particles on a lattice // J. Pure and App. Math.: Adv. Appl. 11:1, 2014. Pp. 37-41.
9. *Rasulova Z.D.* On the spectrum of a three-particle model operator // J. Math. Sci.: Adv. Appl. 25, 2014. Pp. 57-61.
10. *Muminov M.I., Rasulov T.H.* Universality of the discrete spectrum asymptotics of the three-particle Schrödinger operator on a lattice// Nanosystems: Physics, Chemistry, Mathematics, 6:2, 2015. Pp. 280-293.
11. *Rasulov T.Kh., Rasulova Z.D.* On the spectrum of a three-particle model operator on a lattice with non-local potentials // Siberian Electronic Mathematical Reports. 12, 2015. Pp. 168-184.
12. *Rasulov T.H.* Number of eigenvalues of a three-particle lattice model Hamiltonian // Contemporary Analysis and Applied Mathematics. 2:2, 2014. Pp. 179-198.
13. *Rasulov T.H., Rasulova Z.D*. Essential and discrete spectrum of a three-particle lattice Hamiltonian with non-local potentials // Nanosystems: Physics, Chemistry, Mathematics, 5:3, 2014. Pp. 327-342.
14. *Rasulov T.Kh., Mukhitdinov R.T.* The finiteness of the discrete spectrum of a model operator associated with a system of three particles on a lattice // Russian Math., 58:1, 2014. Pp. 52-59.
15. *Rasulov T.Kh.* Structure of the essential spectrum of a model operator associated to a system of three particles on a lattice // J. Samara State Tech. Univ., Ser. Phys. and Math. Sci. 27:2, 2012. Pp. 34-43.
16. *Rasulov T.Kh.* On the essential spectrum of a model operator associated with the system of three particles on a lattice // J. Samara State Tech. Univ., Ser. Phys. and Math. Sci. 24:3, 2011. P. 42-51.
17. *Rasulov T.Kh., Rakhmonov A.A.* The Faddeev equation and location of the essential spectrum of a three-particle model operator // J. Samara State Tech. Univ., Ser. Phys. and Math. Sci. 23:2, 2011. Pp. 166-176.

18. *Rasulov T.Kh.* Investigation of the spectrum of a model operator in a Fock space // Theoret. and Math. Phys. 161:2, 2009. Pp. 1460-1470.
19. *Rasulov T.Kh.* Study of the essential spectrum of a matrix operator // Theoret. and Math. Phys. 164:1, 2010. Pp. 883-895.
20. *Muminov M.I., Rasulov T.H.* The Faddeev equation and essential spectrum of a Hamiltonian in Fock Space // Methods Funct. Anal. Topology 17:1, 2011. Pp. 47-57.
21. *Muminov M.I., Rasulov T.H.* Infiniteness of the number of eigenvalues embedded in the essential spectrum of a 2x2 operator matrix // Eurasian Mathematical Journal. 5:2, 2014. Pp. 60-77.
22. *Muminov M.I., Rasulov T.H.* Embedded eigenvalues of an Hamiltonian in bosonic Fock space // Comm. in Mathematical Analysis. 17:1, 2014. Pp. 1-22.
23. *Muminov M.I., Rasulov T.H.* On the eigenvalues of a 2x2 block operator matrix // Opuscula Mathematica. 35:3, 2015. Pp. 369-393.
24. *Rasulov T.H.* On the finiteness of the discrete spectrum of a 3x3 operator matrix // Methods of Functional Analysis and Topology, 22:1. 2016. Pp. 48-61.
25. *Rasulov T.H.* The finiteness of the number of eigenvalues of an Hamiltonian in Fock space // Proceedings of IAM, 5:2. 2016. Pp. 156-174.

НАУЧНОЕ ИЗДАНИЕ

ИЗДАТЕЛЬСТВО
«ПРОБЛЕМЫ НАУКИ»

АДРЕС РЕДАКЦИИ:
153008, РФ, Г. ИВАНОВО, УЛ. ЛЕЖНЕВСКАЯ, Д. 55, 4 ЭТАЖ
ТЕЛ.: +7 (910) 690-15-09.

HTTPS://SCIENTIFIC-PUBLICATION.COM
E-MAIL: INFO@P8N.RU

ТИПОГРАФИЯ:
ООО «ПРЕССТО».
153025, Г. ИВАНОВО, УЛ. ДЗЕРЖИНСКОГО, Д. 39, СТРОЕНИЕ 8

ИЗДАТЕЛЬ:
ООО «ОЛИМП»
УЧРЕДИТЕЛЬ: ВАЛЬЦЕВ СЕРГЕЙ ВИТАЛЬЕВИЧ
117321, Г. МОСКВА, УЛ. ПРОФСОЮЗНАЯ, Д. 140