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ESSENTIAL SPECTRUM  
OF A  $2 \times 2$  OPERATOR MATRIX  
AND THE FADDEEV EQUATION  
(Dilmurodov E.B., Rasulov T.H.) p.7

FORMATION AND DEVELOPMENT  
OF COMPETITIVE SKILLS  
IN THE SUBJECTS  
OF "MASS CULTURE"  
IN CONTINUOUS EDUCATIONAL  
PROCESS  
(Tojiboyeva H.M) p.51

PROFESSIONAL ORIENTATION  
OF COMMUNICATIVE  
COMPETENCE OF STUDENTS  
(Kasimova Z.Kh.) p.53



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# Содержание

<b>PHYSICO-MATHEMATICAL SCIENCES .....</b>	<b>7</b>
<i>Dilmurodov E.B., Rasulov T.H. (Republic of Uzbekistan) ESSENTIAL SPECTRUM OF A 2x2 OPERATOR MATRIX AND THE FADDEEV EQUATION / Дилмуров Э.Б., Расулов Т.Х. (Республика Узбекистан) СУЩЕСТВЕННЫЙ СПЕКТР ОДНОЙ 2Х2 ОПЕРАТОРНОЙ МАТРИЦЫ И УРАВНЕНИЕ ФАДДЕЕВА.....</i>	<i>7</i>
<i>Tosheva N.A., Rasulov T.H. (Republic of Uzbekistan) MAIN PROPERTY OF REGULARIZED FREDHOLM DETERMINANT CORRESPONDING TO A FAMILY OF 3x3 OPERATOR MATRICES / Тошева Н.А., Расулов Т.Х. (Республика Узбекистан) ОСНОВНЫЕ СВОЙСТВА РЕГУЛЯРИЗОВАННОГО ОПРЕДЕЛИТЕЛЯ ФРЕДГОЛЬМА, СООТВЕТСТВУЮЩИЕ СЕМЕЙСТВУ 3x3 ОПЕРАТОРНЫХ МАТРИЦ.....</i>	<i>11</i>
<i>Bahronov B.I., Rasulov T.H. (Republic of Uzbekistan) STRUCTURE OF THE NUMERICAL RANGE OF FRIEDRICH'S MODEL WITH RANK TWO PERTURBATION / Бахронов Б.И., Расулов Т.Х. (Республика Узбекистан) СТРУКТУРА ЧИСЛОВОЙ ОБЛАСТИ ЗНАЧЕНИЙ МОДЕЛИ ФРИДРИХСА С ДВУМЕРНЫМ ВОЗМУЩЕНИЕМ .....</i>	<i>15</i>
<i>Umirkulova G.H., Rasulov T.H. (Republic of Uzbekistan) CHARACTERISTIC PROPERTY OF THE FADDEEV EQUATION FOR THREE-PARTICLE MODEL OPERATOR ON A ONE-DIMENSIONAL LATTICE / Умиркулова Г.Х., Расулов Т.Х. (Республика Узбекистан) ХАРАКТЕРИСТИЧЕСКИЕ СВОЙСТВА УРАВНЕНИЯ ФАДДЕЕВА ДЛЯ ТРЕХЧАСТИЧНОГО МОДЕЛЬНОГО ОПЕРАТОРА НА ОДНОМЕРНОЙ РЕШЕТКЕ .....</i>	<i>19</i>
<i>Mustafoeva Z.E., Rasulov T.H. (Republic of Uzbekistan) INVESTIGATION OF THE SPECTRUM OF A DIAGONALIZABLE 4x4-OPERATOR MATRIX / Мустафоева З.Э., Расулов Т.Х. (Республика Узбекистан) ИССЛЕДОВАНИЕ СПЕКТРА ОДНОЙ ДИАГОНАЛИЗИРУЕМОЙ 4Х4-ОПЕРАТОРНОЙ МАТРИЦЫ .....</i>	<i>23</i>
<i>Merajov N.I., Rasulov T.H. (Republic of Uzbekistan) DESCRIPTION OF THE POINT SPECTRUM OF A 3x3 TRIDIAGONAL OPERATOR MATRIX WITH FREDHOLM OPERATORS / Меражов Н.И., Расулов Т.Х. (Республика Узбекистан) ОПИСАНИЕ ТОЧЕЧНОГО СПЕКТРА ТРИДАГОНАЛЬНОГО 3Х3 ОПЕРАТОРНОЙ МАТРИЦЫ С ФРЕДГОЛЬМСКИМИ ОПЕРАТОРАМИ .....</i>	<i>27</i>
<i>Nematova Sh.B., Rasulov T.H. (Republic of Uzbekistan) THRESHOLD EIGENVALUES OF A TWO-CHANNEL MOLECULAR-RESONANCE MODEL / Нематова Ш.Б., Расулов Т.Х. (Республика Узбекистан) ПОРОГОВЫЕ СОБСТВЕННЫЕ ЗНАЧЕНИЯ ДВУХКАНАЛЬНОЙ МОЛЕКУЛЯРНО-РЕЗОНАНСНОЙ МОДЕЛИ .....</i>	<i>31</i>
<b>TECHNICAL SCIENCES.....</b>	<b>35</b>
<i>Mansurova Sh.P. (Republic of Uzbekistan) QUESTIONS FEATURES OF DESIGNING AIR CURTAIN / Мансурова Ш.П. (Республика Узбекистан) ВОПРОСЫ ОСОБЕННОСТИ ПРОЕКТИРОВАНИЯ ВОЗДУШНЫХ ЗАВЕС .....</i>	<i>35</i>

<i>Ustemirov Sh.R. (Republic of Uzbekistan) ANALYSIS OF REVERSE WATER SUPPLY SYSTEMS AND PROBLEMS OF WATER QUALITY OF INDUSTRIAL ENTERPRISES / Устемиров Ш.Р. (Республика Узбекистан) АНАЛИЗ СИСТЕМ ОБОРОТНОГО ВОДОСНАБЖЕНИЯ И ПРОБЛЕМ КАЧЕСТВА ВОДЫ ПРОМЫШЛЕННЫХ ПРЕДПРИЯТИЙ.....</i>	39
<b>AGRICULTURAL SCIENCES.....</b>	<b>42</b>
<i>Isaeva L.B., Sanoev H.A. (Republic of Uzbekistan) DYNAMICS OF SOIL HUMIDITY IN THE ROOT TREE OF A PLANT / Исаева Л.Б., Саноев Х.А. (Республика Узбекистан) ДИНАМИКА ВЛАЖНОСТИ ПОЧВЫ В КОРНЕВОМ СТВОЛЕ РАСТЕНИЯ.....</i>	42
<b>ECONOMICS .....</b>	<b>45</b>
<i>Makarenko V.V., Zaporozhtseva E.N. (Russian Federation) FINANCIAL STATEMENTS AS THE MAIN SOURCE OF INFORMATION ON THE FINANCIAL POSITION OF THE ENTERPRISE / Макаренко В.В., Запорожцева Е.Н. (Российская Федерация) БУХГАЛТЕРСКАЯ ОТЧЁТНОСТЬ КАК ОСНОВНОЙ ИСТОЧНИК ИНФОРМАЦИИ О ФИНАНСОВОМ ПОЛОЖЕНИИ ПРЕДПРИЯТИЯ.....</i>	45
<b>PHILOLOGICAL SCIENCES.....</b>	<b>49</b>
<i>Karimov Z.A. (Republic of Uzbekistan) PHILOSOPHICAL ANALYSIS OF LIFESTYLE AND REPRODUCTIVE NOTIONS / Каримов З.А. (Республика Узбекистан) ФИЛОСОФСКИЙ АНАЛИЗ ОБРАЗА ЖИЗНИ И РЕПРОДУКТИВНЫХ ПОНЯТИЙ.....</i>	49
<b>PEDAGOGICAL SCIENCES.....</b>	<b>51</b>
<i>Tojiboyeva H.M. (Republic of Uzbekistan) FORMATION AND DEVELOPMENT OF COMPETITIVE SKILLS IN THE SUBJECTS OF "MASS CULTURE" IN CONTINUOUS EDUCATIONAL PROCESS / Тожибоева Х.М. (Республика Узбекистан) ФОРМИРОВАНИЕ И РАЗВИТИЕ КОНКУРЕНТНЫХ НАВЫКОВ В СУБЪЕКТАХ «МАССОВОЙ КУЛЬТУРЫ» В НЕПРЕРЫВНОМ ОБРАЗОВАТЕЛЬНОМ ПРОЦЕССЕ .....</i>	51
<i>Kasimova Z.Kh. (Republic of Uzbekistan) PROFESSIONAL ORIENTATION OF COMMUNICATIVE COMPETENCE OF STUDENTS / Касимова З.Х. (Республика Узбекистан) ПРОФЕССИОНАЛЬНАЯ НАПРАВЛЕННОСТЬ КОММУНИКАТИВНОЙ КОМПЕТЕНТНОСТИ СТУДЕНТОВ .....</i>	53
<i>Kakhkhorov S.K., Mirzoyev D.P. (Republic of Uzbekistan) RESEARCHING COMMUTATION DEVICES / Каххоров С.К., Мирзоев Д.П. (Республика Узбекистан) ИЗУЧЕНИЕ КОММУТАЦИОННЫХ УСТРОЙСТВ.....</i>	56
<i>Kakhkhorov S.K., Jamilov Yu.Yu. (Republic of Uzbekistan) OPPORTUNITIES OF THE FORMATION OF STUDENTS' COMPETENCE ON ALTERNATIVE ENERGY USING TRAINING SOFTWARE DEVICES / Каххоров С.К., Жамилов Ю.Ю. (Республика Узбекистан) ВОЗМОЖНОСТИ ФОРМИРОВАНИЯ КОМПЕТЕНТНОСТИ У СТУДЕНТОВ ПО АЛЬТЕРНАТИВНОЙ ЭНЕРГИИ С ИСПОЛЬЗОВАНИЕМ ПРОГРАММНЫХ СРЕДСТВ ОБУЧЕНИЯ.....</i>	61
<i>Rasulova Z.D. (Republic of Uzbekistan) DIDACTIC BASIS OF DEVELOPING CREATIVE THINKING OF FUTURE TEACHERS / Расулова З.Д.</i>	

(Республика Узбекистан) ДИДАКТИЧЕСКИЕ ОСНОВЫ РАЗВИТИЯ У БУДУЩИХ УЧИТЕЛЕЙ КРЕАТИВНОГО МЫШЛЕНИЯ .....	65
<i>Ochilov Z.S., Hayitov O.A. (Republic of Uzbekistan) INNOVATIVE FIELDS OF CREATIVE ACTIVITY OF PROFESSOR ADIBA SHARIPOVA / Очилов З.С., Хайитов О.А. (Республика Узбекистан) ИННОВАЦИОННЫЕ СФЕРЫ ТВОРЧЕСКОЙ ДЕЯТЕЛЬНОСТИ ПРОФЕССОРА АДИБЫ ШАРИПОВОЙ .....</i>	69
<i>Safarova D.S. (Republic of Uzbekistan) PEDAGOGY OF COOPERATION AND EDUCATION DEVELOPMENT / Сафарова Д.С. (Республика Узбекистан) ПЕДАГОГИКА СОТРУДНИЧЕСТВА И РАЗВИТИЕ ОБРАЗОВАНИЯ .....</i>	71
<i>Tukboeva D.Z. (Republic of Uzbekistan) SOURCES OF FORMATION OF ECONOMIC CULTURE YOUNG PEOPLE IN THE WORKS OF EAST ENCYCLOPEDIISTS SCIENTISTS / Тукбоева Д.З. (Республика Узбекистан) ИСТОКИ ФОРМИРОВАНИЯ ЭКОНОМИЧЕСКОЙ КУЛЬТУРЫ У МОЛОДЁЖИ В ТРУДАХ УЧЁНЫХ-ЭНЦИКЛОПЕДИСТОВ ВОСТОКА .....</i>	73
<i>Ashrapov R.R. (Republic of Uzbekistan) THE CULTURE OF BOOK READING IN THE FORMATION OF THE SOCIO-SPIRITUAL IMAGE OF YOUTH / Ашрапов Р.Р. (Республика Узбекистан) КУЛЬТУРА КНИГОЧТЕНИЯ В ФОРМИРОВАНИИ СОЦИАЛЬНО-ДУХОВНОГО ОБЛИКА МОЛОДЕЖИ .....</i>	75
<i>Sharopova N.B. (Republic of Uzbekistan) INTERACTIVE TECHNIQUES FOR TEACHING RUSSIAN LANGUAGE / Шаропова Н.Б. (Республика Узбекистан) ИНТЕРАКТИВНЫЕ ПРИЁМЫ ПРИ ОБУЧЕНИИ РУССКОМУ ЯЗЫКУ .....</i>	77
<i>Yusupova I.B. (Republic of Uzbekistan) SELF-KNOWLEDGE AND SELF-APPROVAL - KEY COMPONENTS OF THE MODERN PERSONALITY / Юсупова И.Б. (Республика Узбекистан) САМОПОЗНАНИЕ И САМОУТВЕРЖДЕНИЕ – КЛЮЧЕВЫЕ СОСТАВЛЯЮЩИЕ СОВРЕМЕННОЙ ЛИЧНОСТИ .....</i>	79
<i>Jabborova D.F. (Republic of Uzbekistan) INNOVATIVE TEACHING IMPROVEMENT TECHNOLOGIES / Жабборова Д.Ф. (Республика Узбекистан) ИННОВАЦИОННЫЕ ТЕХНОЛОГИИ СОВЕРШЕНСТВОВАНИЯ ОБУЧЕНИЯ .....</i>	81
<i>Imomova G.F. (Republic of Uzbekistan) LANGUAGE INTERACTION - AN IMPORTANT FACTOR FOR THE DEVELOPMENT OF PUPILS / Имомова Г.Ф. (Республика Узбекистан) ВЗАИМОДЕЙСТВИЕ ЯЗЫКОВ – ВАЖНЫЙ ФАКТОР РАЗВИТИЯ УЧЕНИКОВ .....</i>	83
<i>Nematova N.K. (Republic of Uzbekistan) MODERN TRENDS FOR FORMING ECONOMIC KNOWLEDGE IN A STUDENTING YOUTH / Нематова Н.К. (Республика Узбекистан) СОВРЕМЕННЫЕ ТЕНДЕНЦИИ ФОРМИРОВАНИЯ ЭКОНОМИЧЕСКИХ ЗНАНИЙ У УЧАЩЕЙСЯ МОЛОДЁЖИ .....</i>	85
<i>Kurbanova M.A., Kurbonova N.A. (Republic of Uzbekistan) POSSIBILITIES OF USING THE EDUCATIONAL COMPUTER PROGRAM IN MATHEMATICAL EDUCATION OF PRESCHOOLERS / Курбонова М.А., Курбонова Н.А. (Республика Узбекистан) ВОЗМОЖНОСТИ ИСПОЛЬЗОВАНИЯ УЧЕБНОЙ КОМПЬЮТЕРНОЙ ПРОГРАММЫ В МАТЕМАТИЧЕСКОМ ОБРАЗОВАНИИ ДОШКОЛЬНИКОВ .....</i>	87

**STRUCTURE OF THE NUMERICAL RANGE OF FRIEDRICH'S MODEL  
WITH RANK TWO PERTURBATION**  
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**Abstract:** In this paper we consider a Friedrichs model  $H$  with rank two perturbations. It is associated with a system of two quantum particles on  $d$ -dimensional lattice. We investigate the structure of the closure of numerical range  $W(H)$  of this operator in detail by terms of its parameters for all dimensions  $d$  of the torus  $T^d$ . For  $d \geq 3$  and  $\alpha = 1, 2$  we find the critical value  $\mu_\alpha^0$  of the parameter  $\mu_\alpha > 0$  such that for all  $\mu_\alpha \in (0; \mu_\alpha^0]$  there is no eigenvalues outside of the essential spectrum of  $H$  and for all  $\mu_\alpha > \mu_\alpha^0$  there are two eigenvalues of  $H$ . Moreover, we find the conditions, which guarantees that the set  $W(H)$  is closed.

**Keywords:** Friedrichs model, perturbation, quantum particles, numerical range, spectrum, threshold eigenvalues, virtual level.

**СТРУКТУРА ЧИСЛОВОЙ ОБЛАСТИ ЗНАЧЕНИЙ МОДЕЛИ  
ФРИДРИХСА С ДВУМЕРНЫМ ВОЗМУЩЕНИЕМ**  
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**Аннотация:** в работе рассматривается модель Фридрихса  $H$  с двумерным возмущением. Оно ассоциировано с системой двух квантовых частиц на  $d$ -мерной решетке. Структура замыкания числовой области значений  $W(H)$  этого оператора подробно исследована в терминах его параметров при всех размерностях  $d$  тора  $T^d$ . Для  $d \geq 3$  и  $\alpha = 1, 2$  находим критическое значение  $\mu_\alpha^0$  параметра  $\mu_\alpha > 0$  такое, что при всех  $\mu_\alpha \in (0; \mu_\alpha^0]$  не существуют собственные значения, лежащие вне существенного спектра оператора  $H$  и для любых  $\mu_\alpha > \mu_\alpha^0$  оператор  $H$  имеет два собственных значения. Найдем условие, гарантирующее замкнутость множества  $W(H)$ .

**Ключевые слова:** модель Фридрихса, возмущения, квантовые частицы, поля значений, спектр, пороговые собственные значения, виртуальный уровень.

In a well-known monograph [1], Friedrichs considered the operator of multiplication by an independent variable as an unperturbed operator and chose a perturbation to be given by an integral operator. This model was subsequently called the model of the perturbation theory of the continuous spectrum because the continuous (essential) spectrum of a self-adjoint operator is unchanged under compact perturbation. The present paper is devoted to the study the numerical range of a Friedrichs model, where as a perturbation chosen an integral operator with rank two.

First, we recall the definition of numerical range. Let  $L$  be a complex Hilbert space with inner product  $(\cdot; \cdot)$  and  $H$  be a linear operator in  $L$  with domain  $D(H) \subset L$ . Then the numerical range of an  $L$  is the subset of the complex numbers  $C$ , given by  $W(H) := \{(Hx, x) : x \in D(H), \|x\| = 1\}$ . It was first studied by O.Toeplitz in [2]. In [3] F.Hausdorff showed that indeed the set  $W(H)$  is convex. We recall that the numerical range of a bounded linear operator satisfies the so-called *spectral inclusion property*

$$W(H) \subset \sigma_p(H), \quad \overline{W(H)} \subset \sigma(H)$$

for the point spectrum  $\sigma_p(H)$  (or set of eigenvalues) and the spectrum  $\sigma(H)$  of  $H$ ; note that  $W(H)$  is closed if  $\dim L < \infty$ .

The notion of numerical range is generalized by the different ways, see for example [4-7]. One important use of  $W(H)$  is to bound the spectrum  $\sigma(H)$ .

For positive integer number  $d$  let  $T^d$  be the  $d$ -dimensional torus, the cube  $(-\pi, \pi]^d$  with appropriately identified sides equipped with its Haar measure and  $L_2(T^d)$  be the Hilbert space of square integrable (complex) functions defined on  $T^d$ .

Let us consider a so-called Friedrichs model  $H$  acting on the Hilbert space  $L_2(T^d)$

$$H := H_0 - \mu_1 V_1 + \mu_2 V_2, \quad (1)$$

where the operators  $H_0$  and  $V_\alpha$ ,  $\alpha = 1, 2$  are defined by

$$(H_0 f)(p) = u(p)f(p), \quad (V_\alpha f)(p) = \mu_\alpha v_\alpha(p) \int_{T^d} v_\alpha(t)f(t)dt, \quad \alpha = 1, 2.$$

Here  $\mu_\alpha > 0$ ,  $\alpha = 1, 2$  are positive reals,  $u(\cdot)$  and  $v_i(\cdot)$ ,  $i = 1, 2$  are real-valued continuous functions on  $T^d$ . Under these assumptions, operator  $H$  defined by (1) is bounded and self-adjoint.

Notice that when we study the model operators associated to a system of three particles on a  $d$ -dimensional lattice [8-19] and interacting via non-local potentials, the role of a twoparticle discrete Schrödinger operator is played the Friedrichs model. Such type operators are also important in the investigations of the essential spectrum and the number of eigenvalues, located inside (in the gap, in the below of the bottom) of the essential spectrum of operator matrices [20-25].

It is clear that the essential spectrum of the operator  $H$  coincides with the spectrum of  $[m; M]$ , where the numbers  $m$  and  $M$  are defined by

$$m := \min_{p \in T^d} u(p), \quad M := \max_{p \in T^d} u(p).$$

Throughout this paper, we assume that the function  $u(\cdot)$  has the continuous partial derivatives up to the third-order inclusive on  $T^d$ , in addition it has an unique non-degenerate minimum at the point  $p_1 \in T^3$  and an unique non-degenerate maximum at the point  $p_2 \in T^3$ .

For the case  $d \geq 3$  it is easy to check that the integrals

$$\int_{T^d} \frac{v_1^2(t)dt}{u(t) - m}, \quad \int_{T^d} \frac{v_2^2(t)dt}{M - u(t)}, \quad (2)$$

are positive and finite. In this case we introduce the following quantities:

$$\mu_1^0 := \left( \int_{T^d} \frac{v_1^2(t)dt}{u(t) - m} \right)^{-1}, \quad \mu_2^0 := \left( \int_{T^d} \frac{v_2^2(t)dt}{M - u(t)} \right)^{-1}.$$

One can show that under the condition

$$mes(\text{supp } \{\psi_1(\cdot)\} \cap \text{supp } \{\psi_2(\cdot)\}) = 0. \quad (3)$$

for any  $\mu_\alpha \leq \mu_\alpha^0$ ,  $\alpha = 1, 2$  the operator  $H$  has no eigenvalues lying in the outside of  $[m; M]$  and for any  $\mu_\alpha > \mu_\alpha^0$ ,  $\alpha = 1, 2$  the operator  $H$  has two simple eigenvalues  $E_1 \in (-\infty; m)$  and  $E_2 \in (M; \infty)$ .

The following theorem describes the numerical range  $W(H)$  of  $H$ .

**Theorem 1.** Let  $d \geq 3$  and the condition (3) be fulfilled.

1) If  $\mu_\alpha \leq \mu_\alpha^0$  for any  $\alpha = 1, 2$ , then  $\overline{W(H)} = [m; M] = \sigma(H)$ ;

2) If  $\mu_1 \leq \mu_1^0$  and  $\mu_2 > \mu_2^0$ , then  $\overline{W(H)} = [m; E_2] \subset \sigma(H)$ ;

3) If  $\mu_1 > \mu_1^0$  and  $\mu_2 \leq \mu_2^0$ , then  $\overline{W(H)} = [E_1; M] \subset \sigma(H)$ ;

4) If  $\mu_\alpha > \mu_\alpha^0$  for any  $\alpha = 1, 2$ , then  $W(H) = [E_1; E_2] \subset \sigma(H)$ .

A simple calculations show that if  $d = 1, 2$  and  $v_\alpha(p_\alpha) \neq 0$ ,  $\alpha = 1, 2$ , then the integrals (2) are infinite. Therefore, for any  $\mu_\alpha > 0$ ,  $\alpha = 1, 2$  the operator  $H$  has two simple eigenvalues  $E_1 \in (-\infty; m)$  and  $E_2 \in (M; \infty)$ . From these fact it follows that  $W(H) = [E_1; E_2] \subset \sigma(H)$ .

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