Buxoro davlat universiteti
BUXORO, 200117, M.IQBOL ko'chasi, 11-uy, 2022

@buxdu_uz @buxdu1 @buxdu1 www.buxdu.uz

«AMALIY MATEMATIKA VA AXBOROT TEXNOLOGIYALARINING ZAMONAVIY MUAMMOLARI»
XALQARO ILMIY-AMALIY ANJUMAN

TOSHKENT DAVLAT TRANSPORT UNIVERSITETI
Tashkent state transport university

BUXORO DAVLAT UNIVERSITETI

АМАЛИЙ МАТЕМАТИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ЗАМОНАВИЙ МУАММОЛАРИ

ХАЛҚАРО ИЛМИЙ-АМАЛИЙ АНЖУМАН

МАТЕРИАЛЛАРИ

2022 йил, 11-12 май

БУХОРО – 2022

ЎЗБЕКИСТОН РЕСПУБЛИКАСИ
ОЛИЙ ВА ЎРТА МАХСУС ТАЪЛИМ ВАЗИРЛИГИ
ЎЗБЕКИСТОН РЕСПУБЛИКАСИ ФАНЛАР АКАДЕМИЯСИ
В.И. РОМАНОВСКИЙ НОМИДАГИ МАТЕМАТИКА ИНСТИТУТИ
ЎЗБЕКИСТОН МИЛЛИЙ УНИВЕРСИТЕТИ
ТОШКЕНТ ДАВЛАТ ТРАНСПОРТ УНИВЕРСИТЕТИ
БУХОРО ДАВЛАТ УНИВЕРСИТЕТИ

Бухоро фарзанди, Беруний номидаги Давлат мукофоти лауреати, кўплаб ёш изланувчиларнинг ўз йўлини топиб олишида раҳнамолик қилган етук олим, физика-математика фанлари доктори Ғайбулла Назруллаевич Салиховнинг 90 йиллик юбилейларига бағишланади

АМАЛИЙ МАТЕМАТИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ЗАМОНАВИЙ МУАММОЛАРИ

ХАЛҚАРО ИЛМИЙ-АМАЛИЙ АНЖУМАН МАТЕРИАЛЛАРИ

2022 йил, 11-12 май

БУХОРО – 2022

Theorem 1 *(The propagation theorem).* If the function $f(A)$ can be propagated to ordinary power series by the Z^{-1} levels of the $F(Z)$ image, i.e.

$$F(Z) = C_0 \cdot Z^{-1} + C_1 \cdot Z^{-2} + \ldots + C_n \cdot Z^{-(n+1)} + \ldots \tag{4}$$

as, if it approaches F (Z) at $\{Z \in \mathbb{C}[m \times m] : |Z^{-1}| < RI\}$, then the original is found by the following formula:

$$f(A) = C_0 + C_1 \frac{A}{1!} + C_2 \frac{A^2}{2!} + \ldots C_n \frac{A^n}{n!} + \ldots \tag{5}$$

This series approaches for values $A > 0$ and at $A < 0$ is taken as $f(A) = 0$ **REFERENCES**

1. A K Gupta, D K Nagar, Matrix Variate Distributions,Monographs and surveys in pure and appleid mathematics, Chapman & Hall, Florida, USA 2000, pp.18-27.
2. Худайберганов Г., Кытманов А. М., Шаимкулов Б. А., Комплексный анализ в матричных областях, Красноярск, СФУ 2017.
3. Хуа Ло-кен, Гармонический анализ функций многих комплексных переменных в классических областях, М.: Изд-во иностр. лит., 1959.
4. Ражабов Ш.Ш., Задача коэффициентов каратеодоре в $\mathbb{C}[n \times n]$,Scientific Progress, Volume 2|ISSUE 6| ISSN: 2181-1601,Toshkent 2021, pp. 761-763.
5. Rajabov Sh.Sh., Matrix variable beta function and its properties, Science, research, development #16/7, Santa Monica (California), 2019, pp. 322-323.

BOUNDS OF THE ESSENTIAL SPECTRUM OF A THREE-PARTICLE MODEL HAMILTONIAN ON A 1D LATTICE

Rasulov T.H., Umirkulova G.H.

Bukhara State University, Bukhara, Uzbekistan

Let T be the one-dimensional torus and $L_2^{sym}(T^2)$ be the Hilbert space of square-integrable symmetric (complex) functions with domain T^2. We study the model Hamiltonian $H_{\mu,\lambda}$ defined by

$$H_{\mu,\lambda} := H_0 - \mu(V_1 + V_2) - \lambda V_3, \quad \mu, \lambda > 0, \tag{1}$$

in $L_2^{sym}(T^2)$, where H_0 is a non perturbed operator, i.e. the multiplication operator:

$$(H_0 f)(x,y) = u(x,y) f(x,y);$$

the operators V_α, $\alpha = 1,2,3$ are the partial integral operators of the form:

$$(V_1 f)(x,y) = v(y)\int_T v(t) f(x,t)dt,$$

$$(V_2 f)(x,y) = v(x)\int_T v(t) f(t,y)dt,$$

$$(V_3 f)(x,y) = \int_T f(t, x+y-t)dt.$$

Here $f \in L_2^{sym}(T^2)$, the function $v(\cdot)$ is a real-valued analytic function on T and the function $w(\cdot,\cdot)$ is a real-valued symmetric analytic function on T^2.

Under these assumptions the operator $H_{\mu,\lambda}$ is bounded and self-adjoint.

Throughout this note we assume that there exists a finite subset $\Lambda \subset T$ such that the function $u(\cdot,\cdot)$ has non-degenerate minima at the points of $\Lambda \times \Lambda$. Set

$$m := \min_{x,y \in T} u(x,y) \quad \text{and} \quad M := \max_{x,y \in T} u(x,y).$$

One can easily seen that [1] if $v(x') = 0$ for all $x' \in \Lambda$, then the integral

$$\int_T \frac{v^2(t)dt}{u(x,t) - m}$$

is positive and finite for any $x \in T$. Under this assumption we define

$$\mu_0 := \left(\max_{x\in T} \int_T \frac{v^2(t)dt}{u(x,t)-m} \right)^{-1}.$$

We introduce two bounded and self-adjoint operators $H_\mu^{(1)}$ and $H_\lambda^{(2)}$ (so-called channel operators). They act in $L_2(T^2)$ by

$$H_\mu^{(1)} = H_0 - \mu V_1, \quad H_\lambda^{(2)} = H_0 - \lambda V_3.$$

Set

$$E_{\mu,\lambda} := \min\{\xi : \xi \in \sigma_{\text{ess}}(H_{\mu,\lambda})\}.$$

Then $E_{\mu,\lambda} \in \sigma_{\text{ess}}(H_{\mu,\lambda})$ is called the lower bound of the essential spectrum of $H_{\mu,\lambda}$.

The main result of the present note is the following theorem.

Theorem 1. *For the essential spectrum of $H_{\mu,\lambda}$ we have*

$$\sigma_{\text{ess}}(H_{\mu,\lambda}) = \sigma(H_\mu^{(1)}) \cup \sigma(H_\lambda^{(2)}).$$

For the lower bound $E_{\mu,\lambda}$ the following assertions hold:

(i) If $v(x') \neq 0$ for some $x' \in \Lambda$, then for all $\mu, \lambda > 0$ we have $E_{\mu,\lambda} < m$;

(ii) Let $v(x') = 0$ for all $x' \in \Lambda$.

(ii$_1$) For any $\mu > \mu_0$ and $\lambda > 0$ we have $E_{\mu,\lambda} < m$;

(ii$_2$) For any $\mu \leq \mu_0$ and $\lambda > 0$ we have $E_{\mu,\lambda} = \min \sigma(H_\lambda^{(2)})$.

Moreover, $\max(\sigma(H_{\mu,\lambda})) = M$ for any $\mu, \lambda > 0$.

This result plays a key role in the analysis of the discrete spectrum of $H_{\mu,\lambda}$. In [1] the discrete spectrum of $H_{\mu,0}$ was discussed.

REFERENCE

1. T.H.Rasulov. Number of eigenvalues of a three-particle lattice model Hamiltonian. Contemporary Analysis and Applied Mathematics. 2:2 (2014), pp. 179–198.

CUBIC NUMERICAL RANGE OF 3 × 3 BLOCK OPERATOR MATRICES

Rasulov T.H., Sharipova M.Sh.
Bukhara State University

Block operator matrices are matrices the entries of which are linear operators between Banach or Hilbert spaces. They arise in various areas of mathematics and its applications. Let $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ be complex Hilbert spaces, and consider $\mathcal{H} := \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3$. With respect to this decomposition, every bounded linear operator $\mathcal{A} \in L(\mathcal{H})$ has a 3×3 block operator matrix representation

$$\mathcal{A} := \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{12}^* & A_{22} & A_{23} \\ A_{13}^* & A_{23}^* & A_{33} \end{pmatrix} \tag{1}$$

with bounded linear entries $A_{ij} \in L(\mathcal{H}_j, \mathcal{H}_i)$, $i,j = 1,2,3$ such that $A_{ii}^* = A_{ii}, i = 1,2,3$. In the following we denote by

$$S_{\mathcal{H}_1\oplus\mathcal{H}_2\oplus\mathcal{H}_3} := S_{\mathcal{H}_1} \times S_{\mathcal{H}_2} \times S_{\mathcal{H}_3} = \{(f_1 f_2 f_3)^t \in \mathcal{H} : \|f_i\| = 1, i = 1,2,3\}$$

the product of the unit spheres $S_{\mathcal{H}_i}$ in $\mathcal{H}_i$; we also write S^3 or $S_{\mathcal{H}}$ instead of $S_{\mathcal{H}_1\oplus\mathcal{H}_2\oplus\mathcal{H}_3}$ if the decomposition $H = \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3$ is clear (note the slight difference in notation between $S_{\mathcal{H}_1\oplus\mathcal{H}_2\oplus\mathcal{H}_3}$ and the unit sphere $S_{\mathcal{H}_1\oplus\mathcal{H}_2\oplus\mathcal{H}_3}$ in $H_1 \oplus H_2 \oplus H_3$). In this case $S^3 := \{f = (f_1\, f_2\, f_3)^t \in H : \|f_i\| = 1, i = 1,2,3\}$.

Definition 1. For $f = (f_1\, f_2\, f_3)^t \in S_{\mathcal{H}_1\oplus\mathcal{H}_2\oplus\mathcal{H}_3}$ we introduce the 3×3 matrix

$$\mathcal{A}_f = \begin{pmatrix} (A_{11}f_1, f_1) & (A_{12}f_2, f_1) & (A_{13}f_3, f_1) \\ (A_{12}^*f_1, f_2) & (A_{22}f_2, f_2) & (A_{23}f_2, f_3) \\ (A_{13}^*f_1, f_3) & (A_{23}^*f_2, f_3) & (A_{33}f_3, f_3) \end{pmatrix} \in M_3(C).$$

Then the set

МУНДАРИЖА