The double degree series and their domain of convergence

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This research considers two traditional important questions, which are interesting, at least to most mathematicians. The first question arises in the theory of the degree series of variable of the complex numbers, which concerns the relationship, if any, and their the domains of convergence. The second question arises in the theory of the degree series of several variables of the complex numbers. In this research we introduce double series and we give the definition of their convergence and divergence. We discuss the geometrical structure of following double degree series

$$\sum_{n+m=0}^{\infty} a_{nm}x^n y^m \tag{1}$$

Abel's theorem. If double degree series has absolute converges in point

$$(x_0, y_0)x_0 \neq 0, y_0 \neq 0$$

then, in direct rectangle,

$$T = \{(x,y) : |x| < |x_0|, |y| < |y_0|\}$$

it will have absolute and uniformly converges. Abel theorem needs convergence of the degree series of several variables for $x_0 \neq 0$ points. In other words its enough that point has conditional convergence. But not for the double degree series.

We have some examples of the degree series which hasn't any converges except of $(1,1)$ and $(0,0)$ points. It we need the convergence at line in $(x_0, 0)$ or $(0, y_0)$ points, we'll have the convergence of (1) line in $|x| < |x_0|$ or $|y| < |y_0|$ on direct line.

Counting on Abel theorem we'll have following theorem:

The domain of convergence of the double degree series will be symmetric to the OX and OY axle. Following examples show the variety of convergence of the double series. For example,

$$\sum_{n=0}^{\infty}(x^2 + y^2)^n$$

the domain of convergence of this degree series consist of

$$B = \{(a,y) : |x^2 + y^2| < 1.\}$$

And the domain of convergence of this degree series:

$$\sum_{n=0}^{\infty}(x^2 - y^2)^n$$

consists of

$$G = \{(a,y) : |x^2 - y^2| < 1.\}$$

This domain of convergence $x^2 - y^2 = 1$ and $y^2 - x^2 < 1$ limited by hyperboloid.

All symmetric domains to OX or OY axles have converges with some degree series, in other words there is always some degree series which has converges only in requested domain and has no converges out of this domain. The purpose of this dissertation is to define and study a class of convergent double series and which will be of use in accelerating their convergence.

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