UCHBURCHAKLI PANJARADA ANIQLANGAN SHRYODINGER OPERATORINING XOSSALARI

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Annotatsiya. Maqolada uchburchakli panjarada aniqlangan diskret Shryodinger operatorining spektri o'rganilgan. Uchburchakli panjaradagi ikki zarrachali sistema hamiltonlarining spektral xossalari keltirilgan va bundan tashqari xos qiymatga ega bo'lish shartlari ko'rilgan va isbotlangan. Opertorning o'ng va chapdagi xos qiymatlari topilgan.

Kalit so'zlar. Shryodinger operatori, kompakt operator, muhim spektr, xos qiymat, xos funksiya.

Annotation. The spectrum of the discrete Schrödinger operator defined on a triangular lattice is studied in the article. Spectral properties of Hamiltonians of two-particle systems on a triangular lattice are given, and conditions for having eigenvalues are shown and proved. The right and left eigenvalues of the operator are found.

Keywords: Discrete Schrödinger operator, compact operator, essential spectrum, eigenvalue, eigenfunction.

$\mathbb{C}-$ bir o'lchamli kompleks sonlar fazosi bo'lsin. Ixtiyoriy $n \in N$ natural soni uchun $L_2[a,b]^n$ orqali $[a,b]^n$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Hilbert fazosini belgilaymiz. $\mathbb{T}^d$ bilan d o'lchamli tor, ya'ni

$$\mathbb{T}^{\mathrm{d}} = \underbrace{\mathbb{T}^1 \times \mathbb{T}^1 \times \ldots \times \mathbb{T}^1}_{d\ marta}$$

ni belgilaymiz. d o'lchamli tor $\mathbb{T}^{\mathrm{d}}$ da aniqlangan, Haar ma'nosida o'lchovga ega va

$$\int_{\mathbb{T}^d} |f(x)|^p dx < \infty$$

shartni qanoatlantiruvchi barcha $f\colon \mathbb{T}^{\mathrm{d}} \to \mathbb{C}$ funksiyalarning chiziqli fazosini qaraymiz, bunda integralda o'lchov Haar ma'nosida olinadi va p tayinlangan musbat son. Elementlarni qo'shish va songa ko'paytirish odatdagi funksiyalarni qo'shish va songa ko'paytirish kabi kiritiladi. Hosil bo'lgan fazo $L_p(\mathbb{T}^{\mathrm{d}})$ kabi belgilanadi. Demak,

$L_p(\mathbb{T}^d)$ fazoning elementlari $\mathbb{T}^d$ da aniqlangan va har bir oʻzgaruvchisi boʻyicha 2π davrga ega boʻlgan funksiyalardir [1].

Uchburchakli panjarada aniqlangan diskret Shryodinger operatorining impuls tasviri $L_2(\mathbb{T}^2)$ fazoda ushbu ko'rinishda ta'sir etadi:

$$H = H_0 + V$$

bu yerda, $H_0 - L_2(\mathbb{T}^2)$ fazoda aniqlangan koʻpaytirish operatori, ya'ni

$$(H_0 f)(x_1, x_2) = -\frac{1}{3}(\cos x_1 + \cos x_2 + \cos(x_1 - x_2))f(x_1, x_2)$$

va $V - L_2(\mathbb{T}^2)$ fazoda aniqlangan integral operator

$$Vf(x_1, x_2) = \mu_1 \int_{T^2} f(s_1, s_2)d(s_1, s_2) + \mu_2 \left[\int_{T^2} \cos(x_1 - s_1) f(s_1, s_2)d(s_1, s_2)d(x_1, x_2) + \int_{T^2} \cos(x_2 - s_2) f(s_1, s_2)d(s_1, s_2)d(x_1, x_2)\right].$$

Operatorlar nazariyasi elementlaridan foydalanib, H operatorning $L_2(\mathbb{T}^2)$ fazoda chiziqli, chegaralangan va oʻz-oʻziga qoʻshma operator boʻlishini koʻrish mumkin.

$L_2(\mathbb{T}^d)$ da $f(x) = f(-x)$ munosabatni qanoatlantiruvchi barcha funksiyalar to'plami bu fazoda yopiq qism fazoni tashkil qiladi. Biz uni juft funksiyalar fazosi deb ataymiz va $L_2^e(\mathbb{T}^d)$ deb belgilaymiz. Xuddi shunday, toq funksiyalar to'plami

$$L_2^o(\mathbb{T}^d) = \{f \in L_2(\mathbb{T}^d): f(x) = -f(-x)\}$$

ham $L_2(\mathbb{T}^d)$ da qism fazo hosil qiladi [2-4].

$L_2(\mathbb{T}^d)$ da biror $L \in T^d$ uchun $f(x) = f(L - x)$ munosabatni qanoatlantiruvchi barcha funksiyalar to'plami ham bu fazoda yopiq qism fazoni tashkil qiladi. Biz uni $L_2(\mathbb{T}^d)$ fazodagi L −juft funksiyalar fazosi deb ataymiz va $L_{2,L}^e(\mathbb{T}^d)$ deb belgilaymiz. Xuddi shunday, $L -$ toq funksiyalar to'plami

$$L_{2,L}^o(\mathbb{T}^d) = \{f \in L_2(\mathbb{T}^d): f(x) = -f(L - x)\}$$

ham $L_2(\mathbb{T}^d)$ da qism fazo hosil qiladi.

$L_2^e(\mathbb{T}^2)$ va $L_2^o(\mathbb{T}^2)$ orqali mos holda $L_2(\mathbb{T}^2)$ fazodagi juft va toq funksiyalar toʻplamini belgilaymiz. Ravshanki, bu toʻplamlar $L_2(\mathbb{T}^2)$ fazoda qism fazolarni tashkil etadi.

Teorema. $L_2^e(\mathbb{T}^2)$ va $L_2^o(\mathbb{T}^2)$ fazolar H operator uchun invariant fazolar boʻladi, ya'ni

$$H: L_2^e(\mathbb{T}^2) \to L_2^e(\mathbb{T}^2)$$

va

$$H: L_2^o(\mathbb{T}^2) \to L_2^o(\mathbb{T}^2)$$

bo‘ladi. Hamda

a) $H \coloneqq H^e$ operator $L_2^e(\mathbb{T}^2)$ fazoda quyidagi ko‘rinishda tasvirga ega:

$$H^e f(x_1, x_2) = H_0 + \mu_1 \int_{\mathbb{T}^2} f(s_1, s_2)\big(d(s_1, s_2)\big) +$$

$$+\mu_2 \cos x_1 \int_{\mathbb{T}^2} \cos(s_1) f(s_1, s_2)\big(d(s_1, s_2)\big) +$$

$$+\mu_2 \cos x_2 \int_{\mathbb{T}^2} \cos(t_2) f(t_1, t_2)\big(d(t_1, t_2)\big).$$

b) $H \coloneqq H^o$ operator $L_2^o(\mathbb{T}^2)$ fazoda quyidagi ko‘rinishda tasvirga ega:

$$H_0 f(x_1, x_2) = H_0 - \mu_2 \sin x_1 \int_{\mathbb{T}^2} \sin(s_1) f(s_1, s_2)\big(d(s_1, s_2)\big) -$$

$$-\mu_2 \sin x_1 \int_{T^2} \sin(s_1) f(s_1, s_2)\big(d(s_1, s_2)\big)$$

Isbot. a) $H \coloneqq H^e$ operator $L_2^e(\mathbb{T}^2)$ fazoda aniqlangan bo‘lsin.

$$(Vf)(x_1, x_2)$$

$$= \mu_1 \int_{T^2} f(s_1, s_2) d(s_1, s_2) +$$

$$+ \mu_2 \left[\int_{T^2} \cos(x_1 - s_1) f(s_1, s_2) d(s_1, s_2)\right.$$

$$\left. + \int_{T^2} \cos(x_2 - s_2) f(s_1, s_2) d(s_1, s_2)\right] = g(x)$$

$$g(-x) = \mu_1 \int_{T^2} f(s_1, s_2) d(s_1, s_2) + \mu_2 \left[\int_{T^2} \cos(-x_1 - s_1) f(s_1, s_2) d(s_1, s_2)\right],$$

$$g(-x) = \mu_1 \int_{T^2} f(s_1, s_2)d(s_1, s_2) +$$

$$+\mu_2 \left[\int_{T^2} cos(x_1 + s_1) f(s_1, s_2)d(s_1, s_2) \right.$$

$$\left. + \int_{T^2} cos(x_2 + s_2) f(s_1, s_2)d(s_1, s_2) \right].$$

Hisoblashlarni soddalashtirish maqsadida

$$(s_1, s_2) = (-s_1, -s_2)', \qquad d(s_1, s_2) = -d(s_1, s_2)'$$

kabi belgilashlarni kiritamiz.

$$(Vf)(x_1, x_2) = \mu_1 \int_{T^2} f(-s_1, -s_2)\big(-d(s_1, s_2)\big) +$$

$$+\mu_2 \left[\int_{T^2} cos(x_1 - s_1) f(-s_1, -s_2)\big(-d(s_1, s_2)\big) \right.$$

$$\left. + \int_{T^2} cos(x_2\,, s_2) f(-s_1, -s_2)\big(-d(s_1, s_2)\big) \right] = \mu_1 \int_{T^2} f(s_1, s_2)d(s_1, s_2)$$

$$+$$

$$+\mu_2 \left[\int_{T^2} cos(x_1 - s_1) f(s_1, s_2)d(s_1, s_2) + \int_{T^2} cos(x_2 - s_2) f(s_1, s_2)d(s_1, s_2) \right]$$

$$= g(x).$$

Yuqoridagilardan ko‘rindiki, $g(-x) = g(x)$ bo‘ldi. Bu bizga $(Vf)(x_1, x_2)$ funksiyaning juft funksiya bo‘lishini isbotlaydi [4-10].

Endilikda $(H_0 f)(x_1, x_2)$ operatorning juft yoki juft emasligini tekshiramiz:

$$(H_0 f)(x_1, x_2) = -\frac{1}{3}(cos\, x_1 + cos\, x_2 + cos(x_1 - x_2))f(x_1, x_2) = k(x)$$

$$k(-x) = -\frac{1}{3}(cos(-x_1) + cos(-x_2) + cos(-x_1 + x_2))f(-x_1, -x_2),$$

$$k(-x) = -\frac{1}{3}(cos\, x_1 + cos\, x_2 + cos(x_1 - x_2))f(x_1, x_2) = k(x)$$

Yuqoridagilarni inobatga olsak, demak $(Hf)(x_1, x_2)$ operator juft ekan.

$$Hf(x) = H_0 + \mu_1 \int_{T^2} f(s_1, s_2)d(s_1, s_2) +$$

$$+\mu_2\left[\int_{T^2} \cos x_1 \cos s_1 f(s_1,s_2)d(s_1,s_2) + \int_{T^2} \sin x_1 \sin s_1 f(s_1,s_2)d(s_1,s_2)\right] +$$

$$+\mu_2\left[\int_{T^2} \cos x_2 \cos s_2\, f(s_1,s_2)d(s_1,s_2) + \int_{T^2} \sin x_2 \sin s_2\, f(s_1,s_2)d(s_1,s_2)\right] =$$

$$= \langle f(x) - juft\ bo'lsa | f(-x) = f(x)\rangle = H_0 + \mu_1 \int_{T^2} f(s_1,s_2)d(s_1,s_2) +$$

$$+\mu_2\left[\cos x_1 \int_{T^2} \cos s_1 f(s_1,s_2)d(s_1,s_2) + \sin x_1 \int_{T^2} \sin s_1 f(s_1,s_2)d(s_1,s_2)\right] +$$

$$\mu_2\left[\cos x_2 \int_{T^2} \cos s_2\, f(s_1,s_2)d(s_1,s_2) + \sin x_2 \int_{T^2} \sin s_2\, f(s_1,s_2)d(s_1,s_2)\right].$$

$$= H_0 + \mu_1 \int_{T^2} f(-t_1,-t_2)(-d(t_1,t_2)) +$$

$$+\mu_2 \cos x_1 \int_{T^2} \cos(-t_1) f(-t_1,-t_2)(-d(t_1,t_2)) +$$

$$+\mu_2 \sin x_1 \int_{T^2} \sin(-t_1) f(-t_1,-t_2)(-d(t_1,t_2)) +$$

$$+\mu_2 \cos x_2 \int_{T^2} \cos(-t_2) f(-t_1,-t_2)\big(-d(t_1,t_2)\big) +$$

$$+\mu_2 \sin x_2 \int_{T^2} \sin(-t_2) f(-t_1,-t_2)(-d(t_1,t_2)) =$$

Shunday qilib, oxirgi tenglikda simmetrik oraliqda toq funksiyaning integrali nolga teng bo‘lishini inobatga olsak $(Hf)(x_1,x_2)$ operatorning ko‘rinishi quyidagicha bo‘ladi:

$$(Hf)(x_1,x_2) = H_0 + \mu_1 \int_{T^2} f(t_1,t_2)\big(d(t_1,t_2)\big) +$$

$$+\mu_2 \cos x_1 \int_{T^2} \cos(t_1) f(t_1,t_2)\big(d(t_1,t_2)\big) +$$

$$+\mu_2 \cos x_2 \int_{T^2} \cos(t_2) f(t_1,t_2)\big(d(t_1,t_2)\big).$$

Bu esa yuqorida keltirilgan teoremaning (a) bandini isbotlaydi.

2-hol. $H \coloneqq H^o$ operator $L_2^0(T^2)$ fazoda aniqlangan bo‘lsin. U holda toq funksiya xossasiga ko‘ra, quyidagi ifoda o‘rinli bo‘ladi:

$$(Vf)(x_1,x_2) =$$

$$= \mu_1 \int_{T^2} f(s_1,s_2)d(s_1,s_2) +$$

$$+ \mu_2 \left[\int_{T^2} \cos(x_1 - s_1) f(s_1,s_2)d(s_1,s_2) \right.$$

$$\left. + \int_{T^2} \cos(x_2 - s_2) f(s_1,s_2)d(s_1,s_2) \right] = g(x)$$

$$g(-x) = \mu_1 \int_{T^2} f(s_1,s_2)d(s_1,s_2) +$$

$$+ \mu_2 \left[\int_{T^2} \cos(-x_1 - s_1) f(s_1,s_2)d(s_1,s_2) \right.$$

$$\left. + \int_{T^2} \cos(-x_2 - s_2) f(s_1,s_2)d(s_1,s_2) \right] =$$

$$\langle (s_1,s_2) = (-s_1,-s_2) | d(s_1,s_2) = -d(s_1,s_2) \rangle$$

$$= \mu_1 \int_{T^2} f(-s_1,-s_2)(-d(s_1,s_2)) +$$

$$+ \mu_2 \left[\int_{T^2} \cos(-x_1 + s_1) f(-s_1,-s_2)\big(-d(s_1,s_2)\big) \right.$$

$$\left. + \int_{T^2} \cos(-x_2 + s_2) f(-s_1,-s_2)(-d(s_1,s_2)) \right].$$

$$g(-x) = -\mu_1 \int_{T^2} f(s_1,s_2)d(s_1,s_2) -$$

$$-\mu_2 \left[\int_{T^2} cos(x_1 - s_1) f(s_1,s_2)\big(d(s_1,s_2)\big) \right.$$

$$\left. - \int_{T^2} cos(x_2 - s_2) f(s_1,s_2)d(s_1,s_2) \right] = -g(x).$$

Yuqoridagilardan ko‘rinadiki, $g(-x) = -g(x)$ bo‘ldi. Bu bizga $(Vf)(x_1,x_2)$ funksiyaning toq funksiya bo‘lishini isbotlaydi. Endi $(H_0 f)(x_1,x_2)$ operatorni quyidagicha ifodalaymiz:

$$(H_0 f)(x_1,x_2) = -\frac{1}{3}(cos\, x_1 + cos\, x_2 + cos(x_1 - x_2))f(x_1,x_2) = k(x)$$

$$k(-x) = -\frac{1}{3}(cos(-x_1) + cos(-x_2) + cos(-x_1 + x_2))f(-x_1,-x_2),$$

$$k(-x) = \frac{1}{3}(cos\, x_1 + cos\, x_2 + cos(x_1 - x_2))f(x_1,x_2) = -k(x)$$

Yuqoridagilarni inobatga olsak, demak $(Hf)(x_1,x_2)$ operator toq ekan.

$$Hf(x) = H_0 + \mu_1 \int_{T^2} f(s_1,s_2)d(s_1,s_2) +$$

$$+\mu_2 \left[\int_{T^2} cos\, x_1\, cos\, s_1 f(s_1,s_2)d(s_1,s_2) + \int_{T^2} sin\, x_1\, sin\, s_1 f(s_1,s_2)d(s_1,s_2) \right] +$$

$$+\mu_2 \left[\int_{T^2} cos\, x_2\, cos\, s_2\, f(s_1,s_2)d(s_1,s_2) + \int_{T^2} sin\, x_2\, sin\, s_2\, f(s_1,s_2)d(s_1,s_2) \right].$$

Shunday qilib, oxirgi tenglikda simmetrik oraliqda toq funksiyaning integrali nolga teng bo‘lishini inobatga olsak $H^o f(x_1,x_2)$ operatorning ko‘rinishi quyidagicha bo‘ladi:

$$H^o f(x_1,x_2) = H_0 - \mu_2 \sin x_1 \int_{T^2} \sin(t_1) \mathrm{f}(t_1,t_2)\big(\mathrm{d}(t_1,t_2)\big) -$$

$$-\mu_2 \sin x_1 \int_{T^2} \sin(t_1) \mathrm{f}(t_1,t_2)\big(\mathrm{d}(t_1,t_2)\big).$$

Teoremaning b-bandi ham isbotlandi.

Xulosa. Ushbu maqolada ikki o'lchamli panjaradagi hamiltonlarining spektral xossalari o'rganilgan, jumladan Shryodinger operatorining chiziqli, chegaralangan va oʻz-oʻziga qoʻshmaligi va muhim va diskret spektr mavjudligini isbotlangan va xos qiymatlari to ʻplami keltirilgan [10-18]. Undan tashqari, operatorning spektral xossalarini ifodalovchi teorema tavsiflangan.

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