METODS OF ASSEMBLING OF THE DEGREE SERIES OF SEVERAL VARIABLES OF THE COMPLEX NUMBERS

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Annotation. Here we shall describe some of the simplest examples of domains in the space C^n. As usual, a domain is an open connected set, where openness means that along with any point of it the set also contains a neighborhood of that point, and connectedness of an open set D means that, for any points $z', z'' \epsilon D$ there exists a continuous are

$$\gamma: [0,1] \to d$$

for which

$$\gamma(0) = z' \text{ and } \gamma(1) = z''.$$

Key words: *degree series, several variables, complex numbers, point, function.*

In this section, we introduce double series and we shall give the definition of their convergence and divergence. Then we study relationship between double and iterated series, and we give a sufficient condition for equality of iterated series.

Definition. A function $l: \mathrm{C}^n \to \mathrm{C}$ is said to be R-linear (respectively C-linear) if:

(a) $l(z' + z'') = l(z') + l(z'')$ for all $z', z'' \epsilon \mathrm{C}^n$,

(b) $l(\lambda z) = \lambda l(z)$ for all $z \epsilon C^n$ and all $\lambda \epsilon R$ (respectively all $\lambda \epsilon C$).

Any R −linear function in C^n has the form

$$l(z) = \sum_{v=1}^{n} (a_v z_v + b_v \bar{z_v}), \qquad a_v, b_v \epsilon c,$$

and any C-linear function has the form

$$l(z) = \sum_{v=1}^{n} a_v z_v, \qquad a_v \epsilon C$$

An R −linear function l is C −linear if and only if

$$l(iz) = il(z.)$$

Definition. A function $f: u \to c$, where U is a neighborhood of a point $z \epsilon R$, is said to be R-*differentiable* (respectively, C-*differentiable*) at z if

$$f(z+h)=f(z)+l(h)+o(h),$$

where l is some R-linear (respectively C −linear) function, and $o(h)/|h| \to 0$ as $h \to 0$.

The function l is called the differential of f at z and is denoted by the symbol *df*. Setting

$$h=dz=dx+idy,$$

where

$$dz=(dz_1,\dots,dz_n)$$

is a complex vector and

$$dx=(dx_1,\dots,dx_n) \quad \text{and} \quad dy=(dy_1,\dots,dy_n)$$

are real vectors, we can, in the general case of R −differentiability, write the differential in the form [1-9]

$$df=\sum_{v=1}^{n}\left(\frac{\partial f}{\partial x_v}dx_v+\frac{\partial f}{\overline{\partial y_v}}\overline{dy_v}\right)$$

or, after passing to complex coordinates, in the form

$$df=\sum_{v=1}^{n}\left(\frac{\partial f}{\partial z_v}dx_v+\frac{\partial f}{\overline{\partial z_v}}\overline{dz_v}\right) \qquad (1)$$

where we have introduced the notations

$$\frac{\partial f}{\partial z_v}=\frac{1}{4}\left(\frac{\partial f}{\partial x_v}-i\frac{\partial f}{\partial y_v}\right),$$

$$\frac{\partial f}{\overline{\partial z_v}}=\frac{1}{2}\left(\frac{\partial f}{\partial x_v}+i\frac{\partial f}{\partial y_v}\right),$$

$$v=1,2,\dots,n.$$

The first sum in (1) is denoted by the symbol ∂f and the second by the symbol $\overline{\partial f}$, so that

$$\partial=\sum_{v=1}^{n}\frac{\partial}{\partial z_v}dz_v, \qquad \bar{\partial}=\sum_{v=1}^{n}\frac{\partial}{\overline{\partial z_v}}d\bar{z}_v, \quad v=1,2,\dots,n.$$

Theorem. *For a function f that is R-differentiable at a point* $z\epsilon C^n$ *to be C-differentiable at z it is necessary and sufficient that the Cauchy-Riemann conditions*

$$\bar{\partial}f=0$$

hold.

We see from (i) that

$$df(ih)=i\partial f(h)-i\bar{\partial}f(h)$$

and

$$id f(h) = i\partial f(h) + \overline{i\partial} f(h).$$

Therefore the condition of C −differentiability

$$df(ih) = idf(h)$$

is equivalent to the condition

$$\bar{\partial} f(h) = 0 \quad \text{for all} \quad h \epsilon C^n.$$

The Cauchy-Riemann conditions

$$\bar{\partial} f = 0$$

are equivalent to a system of n complex equations

$$\frac{\partial f}{\partial \bar{z}_v} = \frac{1}{2}\left(\frac{\partial f}{\partial x_v} + i\frac{\partial f}{\partial y_v}\right) = 0,$$

or the equivalent system of 2n real equations

$$\frac{\partial u}{\partial x_v} = \frac{\partial v}{\partial y_v}, \qquad \frac{\partial u}{\partial y_v} = -\frac{\partial v}{\partial x_v}, \qquad v = 1,2,..,n.$$

Where

$$u = Re\, f \quad \text{and} \quad v = Re\, f.$$

For $n > 1$ this system is overdetermined (contains $2n$ equations relative to two unknown functions) and this circum-stance is a principal difference of multidimensional complex analysis from the one-dimensional theory [6-18].

Definition. A function *f* is said to be holomorphic at a point $z\epsilon C^n$ if it is C-differentiable in some neighborhood of this point. On an open set the concepts of C-differentiability and holomorphy are the same.

We remark that in the definition of holomorphy on an arbitrary (not necessarily open) set M there is a subtlety, which is seen from the following example.

Example. Suppose the set $M \subset C^2$ consists of two closed balls

$$\overline{B_1} = \left\{|x - (0,1)| \le \frac{1}{2}\right\} \text{ and } \overline{B_2} = \left\{|x + (0,1)| \le \frac{1}{2}\right\},$$

joined by the line segment

$$L = \left\{x_1 = 0, x_2 = x_1, |x_2| \le \frac{1}{2}\right\}.$$

On *M* we define the function

$$\begin{cases} z_1, & z\epsilon\overline{B_1} \\ 0, & z\epsilon l \\ -z_1, & z\epsilon\overline{B_2}. \end{cases}$$

It is obviously continuous on *M*, and for each point $z_0 \epsilon M$ one can contract a neighborhood U_{z^0} into which *f* extends as a holomorphic function. In fact, for any point of $\overline{B_1}$, including the point

$$\left(0,\frac{1}{2}\right)$$

of intersection of $\overline{B_1}$ and L, we can take as such neighborhoods balls that do not intersect $\overline{B_2}$, and extend f into them by setting it equal to z_1. For points of $\overline{B_2}$ we make an analogous construction, only we set $f(z) = -z_1$. Finally, for interior points of L we take balls that do not contain the ends of this line segment and set $f = 0$ in them. However, from the uniqueness theorem that we shall prove in subsection 5, it follows that f cannot be extended to a holomorphic function in any connected neighborhood Ω of the whole set M. In fact, it follows from this theorem that there does not exists a function that is holomorphic in Ω that is equal to z_1 in one ball from Ω and to $-z_1$ in the other.

From this example we see that it is necessary to distinguish functions that are locally holomorphic on a set, i.e., functions that can be extended locally to a holomorphic function at each point of the set, and globally holomorphic functions, which extend to functions that are holomorphic in a neighborhood of the whole set. Later on, as a rule, in speaking about the holomorphy of a function on a set, we shall have global holomorphy in mind [13-25].

The sum and product of functions that are holomorphic at a point $z\epsilon C^n$ are also holomorphic at this point, so that the set of all functions holomorphic at the point z forms a ring, which is denoted by the symbol Π_z. The ring of functions that are holomorphic in a domain $D \subset C^n$ is denoted by the symbol $\Pi(D)$. From the Cauchy-Riemann conditions we see that a function f that is holomorphic in a neighborhood $U \subset C^n$ of the point z^0 is holomorphic with respect to each variable z_v for fixed remaining variables (in a neighborhood of the point z_v^0 on the complex line

$$\{z: z_\mu = z_\mu^0, \mu \neq v\}.$$

Further, since the system splits into equations in each of which the derivative with respect to only one of the variables occurs, it seems that the condition of holomorphy with respect to a set of variables does not impose any constraints on the dependence on the remaining variables. However, some constraint is nevertheless established: it turns out that a function that is holomorphic in U with respect to each variable z_v separately is certainly R −differentiable with respect to all the variables jointly and then by Theorem 1 is holomorphic in U.

Pluriharmonic functions. We start with a simple remark: if the function

$$f = u + iv$$

is holomorphic at a point $z\epsilon C^n$, then the function

$$\bar{f} = u - iv$$

is R −differentiable in a neighborhood of this point and, for any $v = 1,2,..,n$,

$$\frac{\partial \bar{f}}{\partial z_v} = \frac{1}{2}\left(\frac{\partial \bar{f}}{\partial x_v} - i\frac{\partial \bar{f}}{\partial y_v}\right) = \frac{\overline{\partial f}}{\partial \bar{z}_v} = 0$$

there. Such functions $\bar{f}$ are termed antiholomorphic at the point z.

Suppose f is holomorphic at a point $z \epsilon \mathrm{C}^n$. Then by the preceding remark we have

$$\partial u/\partial z_v = \frac{1}{2}\partial f/\partial z_v$$

for its real part

$$u = \frac{1}{2}(f + \bar{f})$$

in a neighborhood of z. We will also use the fact that the partial derivatives of a holomorphic function are also holomorphic. It follows from this that, for any

$$\mu, v = 1,2,..,n,$$

$$\frac{\partial^2 u}{\partial z_\mu \partial \bar{z}_v} = \frac{\partial}{\partial \bar{z}_v}\left(\frac{\partial u}{\partial z_\mu}\right) = 0. \qquad (2)$$

Separating the real and imaginary parts of the operator in the left-hand side of:

$$\frac{\partial}{\partial \bar{z}_\mu}\frac{\partial}{\partial z_v} = \frac{1}{4}\left(\frac{\partial^2}{\partial x_\mu \partial x_v} + \frac{\partial^2}{\partial y_\mu \partial y_v}\right) + \frac{i}{4}\left(\frac{\partial^2}{\partial x_\mu \partial y_v} - \frac{\partial^2}{\partial x_v \partial y_\mu}\right),$$

we find that condition (2) splits into n^2 second-order partial equations:

$$\frac{\partial^2 u}{\partial x_\mu \partial x_v} + \frac{\partial^2 u}{\partial x_\mu \partial x_v} = 0, \qquad \frac{\partial^2 u}{\partial x_\mu \partial y_v} - \frac{\partial^2 u}{\partial x_v \partial y_\mu} = 0 \qquad (3)$$

$$(\mu, v = 1,2,\ldots,n).$$

If we use the operators ∂ and $\bar{\partial}$ introduced in (2) of subsection (3), then the system (2) can be rewritten as a single condition

$$\bar{\partial}\partial u = 0. \qquad (4)$$

Definition. A function u of class C^2 in a domain $D \subset C^n$ that satisfies the condition (4) at each point of D is said to be *pluriharmonic* in D.

Pluriharmonic functions are connected with holomorphic functions of several variables in the same way that harmonic functions (in R^2) are connected with holomorphic functions of one variable. Namely, we have the following two theorems:

Theorem. *The real and imaginary parts of a function f that are holomorphic in a domain $D \subset C^n$ are pluriharmonic in D.*

Proof. For the real part $u = Ref$ the theorem has already been proved. Since if $f\epsilon\varsigma$(D), so is $-if$, and since

$$Im\, f = Re(-if),$$

the theorem also holds for the imaginary part of f. The converse is also true, but generally only locally.

Theorem. *For any function u that is pluriharmonic in a neighborhood U of the point*

$$(x^0, y^0)\epsilon R^{2n},$$

there exists a function f, holomorphic at

$$z^0 = x^0 + iy^0,$$

whose real (of imaginary) part is equal to u.

Proof. Instead of

$$du = \sum\left(\frac{\partial u}{\partial x_v}dx_v + \frac{\partial u}{\partial y_v}dy_v\right)$$

we consider the so-called conjugate differential

$$* \, du = \sum_{v=1}^{n}\left(-\frac{\partial u}{\partial y_v}dx_v + \frac{\partial u}{\partial x_v}dy_v\right).$$

The form first $w =* du$ is closed in U, since, because u is pluriharmonic, its coefficients are of class C^1 and

$$dw = \sum_{\mu,v=1}^{n}\left(\frac{\partial^2 u}{\partial x_v \partial y_\mu} - \frac{\partial^2 u}{\partial x_\mu \partial y_v}\right)\left(dx_\mu \wedge dx_v + dy_\mu \wedge dy_v\right)$$
$$+ \sum_{\mu,v=1}^{n}\left(\frac{\partial^2 u}{\partial x_\mu \partial x_v} + \frac{\partial^2 u}{\partial y_\mu \partial y_v}\right)dx_\mu \wedge dy_v = 0.$$

But every closed form is locally exact, so that in a neighborhood of z^0 there is a function v such that $* \, du = dv$: it is expressed by the integral

$$v(z) = \int_{z^0}^{z} * \, du,$$

which does not depend on the path since $* \, du$ is closed. But then in a neighborhood of z^0

$$\frac{\partial v}{\partial x_v} = -\frac{\partial u}{\partial y_v}, \qquad \frac{\partial v}{\partial y_v} = \frac{\partial u}{\partial x_v},$$

and consequently the function

$$f = u + iv$$

is of class C^2 and satisfies the Cauchy-Riemann equations of the preceding subsection; by Theorem 1 of subsection it is holomorphic at z^0 and

$$u = \mathrm{Re}\, f.$$

Adding these equations for

$$v = 1,2, \dots, n,$$

we will find that the Laplacian of the function u with respect to the variables

$$x_1, y_1, \dots, x_n, y_n$$

satisfies

$$\Delta u = \sum_{v=1}^{n} \left(\frac{\partial^2 u}{\partial x_v^2} + \frac{\partial^2 u}{\partial y_v^2} \right) = 0.$$

Consequently, *pluriharmonic functions form a subclass of the class of harmonic functions* in the space R^{2n} (obviously proper for $n > 1$). There naturally arises the question of the determination of a pluriharmonic function in a domain $D \subset R^{2n}$ from given boundary values (Dirichlet problem). This question is not solved as simply as in the case of harmonic functions [20-32]. We illustrate the difficulties that arise using the example of one of the simplest domains, the polydisc

$$\mathrm{U} = \{z \epsilon \mathrm{C}^n \colon |z_v| < 1\}.$$

Since, a pluriharmonic function is harmonic with respect to each variable

$$z_v = x_v + i y_v$$

in the disc

$$\{|z_v| < 1\},$$

we can successively apply the Poisson integral from subsection 2 of the Appendix to Part 1. For any $z \epsilon u$ we obtain

$$u(z) = \int_0^{2\pi} p(\varsigma_1, z_1) dt_1 \dots . \int_0^{2\pi} u(\varsigma) p(\varsigma_n, z_n) dt_n,$$

where

$$\varsigma_v = e^{u_v}, \qquad \varsigma = (\varsigma_1, \dots, \varsigma_n),$$

and

$$p_n(\varsigma_v, z_v) = \frac{1}{2\pi} \frac{1 - |z_v|^2}{|\varsigma_v - z_v|^2}$$

in the Poisson kernel. Denoting by

$$p_n(\varsigma, z) = \prod_{v=1}^{n} p(\varsigma_v, z_v)$$

the n −dimensional Poisson kernel, by

$$q_n = [0,2\pi] \times \dots \times [0,2\pi] \text{ (n times)}$$

the n −dimensional cube, and by

$$dt = dt_1 \dots . dt_n$$

the volume element, we rewrite the Poisson integral formula in the following abbreviated form:

$$u(z) = \int u(\varsigma) p_n(\varsigma, z) dt.$$

The right-hand side of this formula includes only the values of u on the *skeleton* Γ of the polydisc, i.e., on the n-dimensional part of the boundary ∂U. From this it is clear that one cannot arbitrarily give the values of a pluriharmonic function u on the whole boundary of the polydisc. If in the right-hand side we substitute the values of some function $u(\varsigma)$ that is continuous on Γ, then the function $u(z)$ defined in U by this formula will, as is not hard to check, satisfy the equations for all $v = 1,2,..,n$.

Simplest properties of holomorphic functions. Here we establish a number of elementary properties of holomorphic functions of several variables that are analogous to properties of functions of one variable. For brevity we shall denote by

$$U = \{z \epsilon \mathrm{C}^n : |z_v - a_v| < r_v, \ \ v = 1,2,..,n\}$$

the polydisc with center a and vector radius

$$r = (r_1, \dots, r_n).$$

We denote by

$$\varphi(u) \cap \mathrm{C}(\overline{\mathrm{U}})$$

the set of functions that are holomorphic in U and continuous in $\overline{\mathrm{U}}$.

Theorem. *Any function*

$$f \epsilon \varphi(u) \cap C(\overline{U})$$

at any point $z \epsilon u$ *is represented by a multiple Cauchy integral*

$$f(z) = \frac{1}{(2\pi i)^n} \int \frac{f(\varsigma) d\varsigma_1 \dots d\varsigma_n}{(\varsigma_1 - z_1) \dots (\varsigma_n - z_n)},$$

where Γ *is the skeleton of U, i.e., the product of the boundary circles*

$$\gamma_v = \{|z_v - a_v| = r_v\}, \quad v = 1,2,..,n.$$

Proof. Let $'z$ and $'U$ be the projections of z and U into C^{n-1}; since for any $'z \epsilon 'u$ the function

$$f(z) = f('z, z_n)$$

is holomorphic in z_n in the disc

$$\{|z_v - a_v| < r_v\}$$

and is continuous in its closure, then by the Cauchy integral formula from theorem

$$f(z)=\frac{1}{2\pi i}\int\frac{f('z,\varsigma_n)}{\varsigma_n-z_n}d\varsigma_n.$$

Remark. As we see from the proof of Theorem 1, in order for a function f to be representable by a multiple Cauchy integral it is sufficient for f to be holomorphic in each variable z_v in the disc $\{|z_v-a_v|<r_v\}$ and continuous with respect to the set of all variables in U. As in the case of one variable, from the Cauchy integral representation of a function we deduce the possibility of expanding it in a power series. For this we expand the kernel of the integral in a multiple geometric progression:

$$\frac{1}{\varsigma-z}=\frac{1}{\varsigma-a}\times\frac{1}{\left(1-\frac{z_1-a_1}{\varsigma_1-a_1}\right)\dots\left(1-\frac{z_n-a_n}{\varsigma_n-a_n}\right)}=\frac{1}{\varsigma-a}\sum_{k=1}^{\infty}\left(\frac{z-a}{\varsigma-a}\right)^k,$$

where

$$k=(k_1,k_2,\dots,k_n)$$

is an integer vector,

$$|k|=k_1+k_2+\dots+k_n,$$

and

$$\left(\frac{z-a}{\varsigma-a}\right)^k=\left(\frac{z_1-a_1}{\varsigma_1-a_1}\right)^{k_1}\dots\left(\frac{z_n-a_n}{\varsigma_n-a_n}\right)^{k_n}.$$

This expansion can be rewritten in the form

$$\frac{1}{\varsigma-z}=\sum_{|k|=0}^{\infty}\frac{(z-a)^k}{(\varsigma-a)^{k+1}},$$

where

$$k+1=(k_1+1,\dots,k_n+1);$$

for any $z\epsilon U$ it converges absolutely and uniformly in ς on Γ. Multiplying it by the continuous (and hence bounded) function $f(\varsigma)/(2\pi i)^n$ on Γ and integrating term-by-term over Γ, we will obtain the desired assertion:

Theorem. *If*

$$f\epsilon\varphi(u)\cap C(\bar{U}),$$

then at each point $z\epsilon u$ it is represented as a multiple power series

$$f(z)=\sum_{|k|=0}^{\infty}c_k(z-a)^k$$

with coefficients

$$c_k = \frac{1}{(2\pi i)^n} \int \frac{f(\varsigma)d\varsigma}{(\varsigma - a)^{k-1}}.$$

Remark. Any function $f\epsilon\varphi(u)$ can be represented as the sum of a series at each point $z\epsilon U$. For the proof it suffices to remark that the point z belongs to some polydisc

$$U' \subset\subset U$$

and to apply Theorem 2 to U'.

References

1. Xayitova X. Chiziqli tenglamalarni o'qitishda «aqliy hujum» va «kichik guruhlarda ishlash» metodlaridan foydalanish //центр научных публикаций (buxdu. uz). – 2021. – Т. 8. – №. 8.
2. Gafurovna K. K. Some applications of the derivative of a function //Лучшие интеллектуальные исследования. – 2024. – Т. 19. – №. 3. – С. 7-12.
3. G'afurovna X. X. et al. Olmos panjaradagi diskret shryodinger operatorining spektri //Journal of new century innovations. – 2023. – Т. 29. – №. 2. – С. 120-125.
4. Xayitova X. Funksional tenglamalar yechishning ayrim metodlari //центр научных публикаций (buxdu. uz). – 2024. – Т. 45. – №. 45.
5. Xayitova X. Tanlash usuli bilan kombinatorika masalalarni yechish metodikasi: tanlash usuli bilan kombinatorika masalalarni yechish metodikasi //центр научных публикаций (buxdu. uz). – 2022. – Т. 25. – №. 25.
6. Boboyeva M.N. Maktab matematika darslarida misol-masalalar yechish orqali turli kasblarga oid ma'lumotlarni singdirish. Science and Education 2:8 (2021), 496-504 b.
7. Boboyeva M.N. Differensial hisobning iqtisodda qo'llanilishini takomillashtirish istiqbollari. Science and Education 2:8 (2021),476-485 b.
8. Boboyeva M.N. "Matritsalar haqida tushuncha va ular ustida amallar" mavzusini ayrim interfaol metodlardan foydalanib o'qitish. Pedagogik mahorat Maxsus son (2021), 38-42 b.
9. Boboyeva M.N. Increasing creative activity of students by application of methods of analysis and synthesis in mathematics lessons. ResearchJet Journal of Analysis and Inventions. 3:05 (2022), p.67-75.
10. Бобоева М.Н. "Чизиқли тенгламалар системаси" мавзусини ўқитишда муаммоли таълим технологияси ва "зинама-зина" методини қўллаш. Pedagogik akmeologiya. Maxsus son (2022) 67-74 b.

11. Sayliyeva, Gulrux Rustam Qizi. "Diskret matematika va matematik mantiq fanining «predikatlar mantig'i» bobi mavzularini tushuntirishda samarali yondashuv va undagi zamonaviy usul va metodlar." *Scientific progress* 2.1 (2021): 552-558.
12. Sayliyeva GRQ Diskret matematika va matematik mantiq fanida bul funktsiyalarni jegalkin ko'phadlariga yo'nalish mavzusini materiallarda "matematik domino" metodidan yuklash //Scientific progress. – 2021. – T. 2. – Yo'q. 2. – 773-780-betlar.
13. Sayliyeva G. Ehtimollar nazariyasi va matematik statistika fanidan "Ta'riflar, teoremalar, isbotlar, formulalar, misollar" usulidan foydalanish // ilmiy nashirlar markazi (buxdu. uz). – 2021. – T. 8. – Yo'q. 8.
14. Sayliyeva G. Diskret matematika va matematik mantiq" fanining amaliyot darslarida o'tilgan mavzuni mustahkamlashda "g'oyaviy charxpalak","charxpalak" texnologiyasi va "assotsatsiyalar" metodlariidan foydalanish //центр научных публикаций (buxdu. uz). – 2021. – T. 7. – №. 7.
15. Sayliyeva G. Talabalarning o'qitilayotgan fanlarga qiziqishini oshirishda foydalaniladigan samarali pedagogik metodlar //ilmiy nashriyotlar markazi (buxdu. uz). – 2023. – T. 44. – Yo'q. 44.
16. Jumayeva C. Основы и способы развития речемыслительной деятельности школьников при обучении математике //центр научных публикаций (buxdu. uz). – 2024. – T. 45. – №. 45.
17. Jumayeva C. Local inner derivations on four-dimensional lie algebras //центр научных публикаций (buxdu. uz). – 2024. – T. 45. – №. 45.
18. Jumayeva C. "Jegalkin ko 'phadi" mavzusini o 'qitishda interfaol metodlarni qo 'llash //центр научных публикаций (buxdu. uz). – 2023. – T. 44. – №. 44.
19. Jumayeva C. Ba'zi to 'rt o 'lchamli li algebralarining lokal ichki differensiallashlari //центр научных публикаций (buxdu. uz). – 2023. – T. 44. – №. 44.
20. qizi Jumayeva C. I. et al. Mulohazalar hisobida isbotlanuvchi formulalar: nazariya, amaliyot va tahlil //Science and Education. – 2024. – T. 5. – №. 4. – C. 455-461.
21. Абдуллаева М.А. Применение метода "Рыбий скелет" при решении задач арифметических прогрессии// Центр научных публикаций (buxdu. uz), 8:8 (2022), с. 1156-1166.
22. 2. M. Abdullayeva, "Чала квадрат тенглама" мавзусини ўқитишда "Бумеранг" технологияси// центр научных публикаций (buxdu. uz), 8:8 (2021), с. 1651-1660.

23. M. Abdullayeva, Aniq integralning tatbiqlari mavzusini o'qitishda" Charxpalak" texnologiyasi// центр научных публикаций (buxdu. uz), 8:8 (2021), с. 1410-1421.
24. M. Abdullayeva, "Determinant va ularning xossalari. Determinant tushunchasi va uni hisoblash" mavzusini o'qitishda svetofor metodini qo'llash// центр научных публикаций (buxdu. uz), 8:8 (2021), с. 1661-1670.
25. Abdullayeva M.A. Turli maqsadlarga javob beruvchi testlar orqali talabaning bilim, malaka va ko'nikmalarini nazorat qilish// Science and Education, 5:4 (2024), 445-454.
26. Rasulov, X. (2021). Краевая задача для одного нелинейного уравнения смешанного типа. Центр научных публикаций (buxdu.Uz), 7(7).
27. Rasulov, R. X. R. (2021). Гиперболик типдаги тенглама учун Коши масаласи. Центр научных публикаций (buxdu.Uz), 7(7).
28. Rasulov, R. X. R. (2022). О краевых задачах для уравнений эллиптического типа с линией искажения. Центр научных публикаций (buxdu.Uz), 8(8).
29. Rasulov, R. X. R. (2022). Бузилиш чизиғига эга бўлган квазичизиқли аралаш типдаги тенглама учун Трикоми масаласига ўхшаш чегаравий масала ҳақида. Центр научных публикаций (buxdu.Uz), 18(18).
30. Rasulov, X. (2022). Краевые задачи для квазилинейных уравнений смешанного типа с двумя линиями вырождения. Центр научных публикаций (buxdu.Uz), 8(8).
31. Rasulov, X. (2022). Об одной краевой задаче для нелинейного уравнения эллиптического типа с двумя линиями вырождения. Центр научных публикаций (buxdu.Uz), 18(18).
32. Rasulov, X. (2022). О динамике одной квадратичной динамической системы с непрерывным временем. Центр научных публикаций (buxdu.Uz), 18(18).