

Holon Institute of Technology



# A B S T R A C T S

of the International Online Conference

## FRONTIER IN MATHEMATICS AND COMPUTER SCIENCE

October 12–15, 2020

TASHKENT

is proved. Applying a method of Green's function, we are able to find the solution of the problem in an explicit form. Moreover, decomposition and summation formulae, formulae of differentiation and some adjacent relations for Lauricella's hypergeometric functions in many variables were used in order to find the explicit solution for the formulated problem.

#### REFERENCES

1. Ergashev T.G., "Fundamental solutions for a class of multidimensional elliptic equations with several singular coefficients", Journal of Siberian Federal University. Mathematics and Physics, **13**, No. 1, 48–57 (2020).

## ON THE EIGENVALUES OF THE FRIEDRICHS MODEL WITH RANK $N$ PERTURBATIONS

Khayitova Kh.G.

*Bukhara State University, hayitova.hilola@mail.uz*

In the Hilbert space  $L_2(\mathbb{T}^d)$  of square integrable (complex) functions defined on  $d$ -dimensional torus  $\mathbb{T}^d$  we consider the Friedrichs model of the form

$$H := H_0 - \sum_{\alpha=1}^N \mu_\alpha V_\alpha,$$

where  $H_0$  is the multiplication operator by the function  $u(\cdot)$  on  $L_2(\mathbb{T}^d)$  :

$$(H_0 f)(x) = u(x) f(x)$$

and  $V_\alpha$  is the non-local interaction operator on  $L_2(\mathbb{T}^d)$  :

$$(V_\alpha f)(x) = v_\alpha(x) \int_{\mathbb{T}^d} v_\alpha(t) f(t) dt, \quad \alpha = \overline{1, N}.$$

Here  $N \in \mathbb{N}$  with  $N \geq 3$ ;  $\mu_\alpha > 0$ ,  $\alpha = \overline{1, N}$  are positive reals, the functions  $u(\cdot)$  and  $v_\alpha(\cdot)$ ,  $\alpha = \overline{1, N}$  are real-valued continuous functions on  $\mathbb{T}^d$ .

Under these assumptions the operator  $H$  is bounded and self-adjoint.

The perturbation  $v$  of the operator  $h_0(p)$  is a self-adjoint operator of rank  $\leq N$ . Therefore in accordance with the Weyl theorem about the invariance of the essential spectrum under the finite rank perturbations, the essential spectrum of the operator  $H$  coincides with the essential spectrum of  $H$ . Therefore

$$\sigma_{\text{ess}}(H) = \sigma(H_0) = [u_{\min}; u_{\max}],$$

where the numbers  $u_{\min}$  and  $u_{\max}$  are defined by

$$u_{\min} := \min_{x \in \mathbb{T}^d} u(x), \quad u_{\max} := \max_{x \in \mathbb{T}^d} u(x).$$

We define the analytic functions in  $\mathbb{C} \setminus [u_{\min}; u_{\max}]$  by

$$I_{\alpha\beta}(z) := \mu_\beta \int_{\mathbb{T}^d} \frac{v_\alpha(t)v_\beta(t)dt}{u(t) - z}, \quad \alpha, \beta = \overline{1, N} \quad \text{and} \quad \Delta(z) := \det(\delta_{\alpha\beta} - I_{\alpha\beta}(p; z))_{\alpha, \beta=1}^N,$$

where  $\delta_{ij}$  is the Kronecker delta.

Usually the function  $\Delta(\cdot)$  is called the Fredholm determinant associated with the operator  $H$  and it is easy to see that the number  $z \in \mathbb{C} \setminus [u_{\min}; u_{\max}]$  is an eigenvalue of  $H$  if and only if  $\Delta(z) = 0$ .

Let  $\text{mes}(\cdot)$  be the Lebesgue measure on  $\mathbb{R}^d$  and  $\text{supp}\{v_\alpha(\cdot)\}$  be the support of the function  $v_\alpha(\cdot)$ .

Main result of this work is the following theorem.

**Theorem 1.** *If  $\text{mes}\{\text{supp}v_\alpha(\cdot) \cap \text{supp}v_\beta(\cdot)\} = 0$  for all  $\alpha \neq \beta$ , then the number  $z \in \mathbb{C} \setminus [u_{\min}; u_{\max}]$  is an eigenvalue of  $H$  if and only if the number  $z$  is an eigenvalue of  $H_\alpha := H_0 - \mu_\alpha V_\alpha$  for some  $\alpha \in \{1, \dots, N\}$ ; moreover, the operator  $H$  has no more than  $N$  eigenvalues (counting multiplicities) lying on the l.h.s. of  $u_{\min}$  and has no eigenvalues on the r.h.s. of  $u_{\max}$ .*

## INTEGRATION OF THE LOADED KDV EQUATION IN THE CLASS OF RAPIDLY DECREASING COMPLEX-VALUED FUNCTIONS

Hoitmetov U.A.<sup>1</sup>, Musaeva F.K<sup>2</sup>

<sup>1</sup>*Khorezm Branch of the Romanovskiy Institute of Mathematics, x\_umid@mail.ru*

<sup>2</sup>*Urgench State University, m.feruza96@mail.ru*

In this paper, we study the loaded Korteweg-de Vries (KdV) equation, namely, consider the following equation:

$$u_t - 6uu_x + u_{xxx} = -\gamma(t)u(0, t)u_x(x, t) \quad (1)$$

here  $\gamma(t)$  is a given continuous function. Equation (1) is considered under the initial condition

$$u(x, 0) = u_0(x), \quad x \in R, \quad (2)$$

where the initial function  $u_0(x)$  is complex-valued and has the following properties:

1) for some  $\varepsilon > 0$

$$\int_{-\infty}^{\infty} |u_0(x)| e^{\varepsilon|x|} dx < \infty; \quad (3)$$

2) a non-self-adjoint operator  $L(0) := -\frac{d^2}{dx^2} + u_0(x)$ ,  $x \in R$  has equally  $N$  complex eigenvalues  $\lambda_1(0)$ ,  $\lambda_2(0)$ , ...,  $\lambda_N(0)$  with multiplicities  $m_1(0)$ ,  $m_2(0)$ , ...,  $m_N(0)$ , respectively, and has no spectral singularities.

We have to find a complex-valued sufficiently smooth function  $u(x, t)$  which sufficiently rapidly tends to its limits as  $x \rightarrow \pm\infty$ , including the case when

$$\int_{-\infty}^{\infty} \left| \frac{\partial^j u(x, t)}{\partial x^j} \right| e^{\varepsilon|x|} dx < \infty, \quad j = 0, 1, 2, 3. \quad (4)$$

$$u^{(j)}(x, 0) = u_0^{(j)}(x), \quad x \in \bar{b}_j^+, \quad j = 1, 2, \dots, n, \quad (4)$$

$$v^{(j)}(x, 0) = v_0^{(j)}(x), \quad x \in \bar{b}_j^-, \quad j = 1, 2, \dots, n, \quad (5)$$

$$y^{(j)}(x, 0) = y_0^{(j)}(x), \quad x \in \bar{b}_j^0, \quad j = 1, 2, \dots, (n-1) \quad (6)$$

граничные условия

$$u^{(1)}(0, t) = g_0^{(n)}(t), \quad v^{(1)}(0, t) = d_0^{(n)}(t), \quad t \geq 0, \quad (7)$$

$$u^{(n)}(L, t) = h_0^{(n)}(t), \quad v^{(n)}(L, t) = r_0^{(n)}(t), \quad t \geq 0 \quad (8)$$

на конечных связях соответственно.

В вершине решение удовлетворяет следующим условиям склейки (Кирхгофа):

$$u^{(j-1)}(L, t) = u^{(j)}(0, t) = y^{(j-1)}(0, t), \quad u_x^{(j-1)}(L, t) + u_x^{(j)}(0, t) + y_x^{(j-1)}(0, t) = 0; \quad (9)$$

$$v^{(j-1)}(L, t) = v^{(j)}(0, t) = y^{(j-1)}(L, t), \quad v_x^{(j-1)}(L, t) + v_x^{(j)}(0, t) + y_x^{(j-1)}(L, t) = 0, \quad j = \overline{2, n}.$$

Последние условия обычно называют условиями непрерывности и сохранения потока (Кирхгофа) на точке ветвления графов. Мы предполагаем, что исходные данные являются достаточно гладкими функциями и удовлетворяют условиям (7) - (9).

Решение задачи построено так называемым методом Фокаса, которое является обобщением метода преобразования Фурье. При этом, задача сведена к системе алгебраических уравнений, относительно преобразования Фурье неизвестных значений решения в вершинах графа [1–5].

#### ЛИТЕРАТУРА

1. Fokas A.S. A Unified Approach to Boundary Value Problems. *Society for Industrial and Applied Mathematics*. 2008. Pp. 352.
2. Berkolaiko G. An elementary introduction to quantum graphs. *arXiv:1603.07356v2 [math-ph] 17 Dec 2016*.
3. Khudayberganov G., Sobirov Z.A., Eshimbetov M.R. Unified transform (Fokas) method for the Schrodinger equation on simple metric graph. *Journal of Siberian Federal University. Mathematics and Physics*. 2019 12(4), 412-420.
4. Khudayberganov G., Sobirov Z.A., Eshimbetov M.R. The Fokas' unified transformation method for heat equation on general star graph. *Uzbek Mathematical Journal.*, 2019. № 1., Pp. 73–81.
5. Sheils N.E. Interface Problems using the Fokas Method. *A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy. University of Washington* 2015. Pp. 210.

## CONTENTS

|     |  |    |
|-----|--|----|
| 1.  | Abdishukurova G.M., Narmanov A.Ya. Diffeomorphisms of Foliated Manifolds   | 4  |
| 2.  | Abdullaev D., Aripov M., Rakhmanov O. Mobile Applications for Medical Systems  | 5  |
| 3.  | Abduxakimov S.X. Thermodynamic Formalism for Critical Circle Maps  | 6  |
| 4.  | Adkhamova A.Sh. Smoothness of Generalized Solution of Boundary Value Problem For Control System with Delay                                       | 7  |
| 5.  | Agranovsky Mark. Generalized Funk-Radon Transforms   | 8  |
| 6.  | Akhatov A.R., Saidaliyev B.M. Digitizing Agriculture Using Gat Technologies  | 9  |
| 7.  | Akhmedov O.S., Abuganiyev A.A. On the Proof of Existence of Periodic Solution for the Holling's Predator-Prey System                             | 10 |
| 8.  | Aktamov Kh.S. The Problems Of Integral Geometry for Family of Curves on the Plane  | 11 |
| 9.  | Alimov A. Positive Izometries of the Orlicz Ideals of Compact Operators  | 12 |
| 10. | Alimov Sh.A., Komilov N.M. On the Control Problem Associated with Heat Transfer Process  | 13 |
| 11. | Aliyev N., Muminov Z. On the Spectrum of the Three-Particle Hamiltonian on a Two Dimensional Lattice   | 15 |
| 12. | Aminov B., Chilin V. Hermitian Operators in Non commutative Atomic Orlicz Spaces   | 16 |
| 13. | Anarova Sh. A., Ibrokhimova Z. E., Mirgaziev J. U. Investigation of Fractal Ring Structures Based on the R - Function Method (RFM)               | 17 |
| 14. | Anarova Sh.A., Ismoilov Sh.M. Mathematical Modeling of the Stress-Strain State of Rods Under Spatial Load Considering Temperature                | 18 |
| 15. | Apakov Yu.P. Boundary Problems for Mixed Parabolic-Hyperbolic Equations with Parallel Planes of Changing Type                                    | 19 |
| 16. | Apushkinskaya D.E. Boundary Point Principle and Divergence-Type Equations  | 20 |
| 17. | Aripov M., Norov A.M. About One of the Methods of Intellectual Processing of Uzbek Texts   | 21 |
| 18. | Ashurov R. Uniqueness and Existence for Inverse Problem of Determining an Order of Time-Fractional Derivative of Subdiffusion and Wave Equations | 22 |
| 19. | Asrakulova D.S., Boboraximova M.I. On Periodic Solutions of Matematical Models of Two River Branches   | 23 |
| 20. | Asrakulova D.S., Boboraximova M.I. On Periodic Solutions of Mathematical Models of Three River Branches  | 24 |
| 21. | Atamuratov A.A., Kamolov Kh.Q. Polynomially Convex Sets on Regular Parabolic Manifolds   | 25 |
| 22. | Ayupov Sh.A., Khudoyberdiyev A.Kh, Yusupov B.B. Local and 2-Local Derivations of Solvable Leibniz Algebras with Abelian Nilradicals              | 26 |
| 23. | Ayupov Sh.A., Kudaybergenov K.K. Ring Isomorphisms of Murray–Von Neumann Algebras  | 27 |
| 24. | Azamov A.A., Begaliyev A.O. On Connectivity Domain of a Solution Existence   |    |

|     |  |    |
|-----|--|----|
| 49. | <b>Golberg Anatoly.</b> Nonlinear Beltrami Equation  | 56 |
| 50. | <b>Goldman M.L., Bakhtigareeva E.G., Haroske D.</b> Differential Properties of Generalized Bessel Potentials   | 57 |
| 51. | <b>Goldman M.L., Bakhtigareeva E.G.</b> Nontriviality Conditions for Generalized Morrey-Type Spaces  | 58 |
| 52. | <b>Hasanov A., Ergashev T.G.</b> Holmgren Problem for Elliptic Equation with Several Singular Coefficients   | 59 |
| 53. | <b>Khayitova Kh.G.</b> On the Eigenvalues of the Friedrichs Model with Rank $N$ Perturbations  | 60 |
| 54. | <b>Hoitmetov U.A., Musaeva F.K.</b> Integration of the Loaded KDV Equation in the Class of Rapidly Decreasing Complex-Valued Functions   | 61 |
| 55. | <b>Ibaydullayev T.T., Holboyev A.G.</b> On a Differential Game on the 1-Skeleton of Simplex Graph with Slower Pursuers   | 63 |
| 56. | <b>Ibodullaeva N.M., Ismoilov Sh.Sh., Artykbaev A.</b> Non-Euclidean Geometry and Existence of the Solution of the Monge-Ampere Equation   | 64 |
| 57. | <b>Irgashev B.Yu.</b> Boundary Value Problem for Degenerate Equation with Fractional Derivative  | 65 |
| 58. | <b>Kalandarov T.S.</b> 2-Local Automorphisms on Algebras of Continuous Functions   | 66 |
| 59. | <b>Karimov J.J.</b> The Invariant Measures on Symbolic Spaces  | 67 |
| 60. | <b>Kasimov Sh.G., Babaev M.M.</b> On the Solvability of a Mixed Problem with a Lagging Argument in Time and Related to Powers of the Laplace Operator with Nonlocal Boundary Conditions in Sobolev Classes | 69 |
| 61. | <b>Kasimov Sh.G., Xaitboyev G.S.</b> A Multidimensional Analog of the A.N. Tikhonov Theorem on Calculating Values of a Function With Respect To Approximately Given Fourier Coefficients                   | 70 |
| 62. | <b>Khalkhuzhaev A., Pardabaev M.</b> Spectral Properties of Perturbed Discrete Bilaplacian   | 71 |
| 63. | <b>Khasanov A.B., Allanazarova T.J.</b> Integration of the Nonlinear Modified Korteweg-De Vries Equation with a Loaded Term  | 72 |
| 64. | <b>Khayitkulov B.Kh.</b> Approximate Solution of the Nonstationary Convection - Diffusion Problem Based on the Optimal Selection of the Location of Heat Sources   | 75 |
| 65. | <b>Kholikov D.K.</b> Non-Local Problem for the Loaded Equation of the Third Order  | 76 |
| 66. | <b>Khudayberganov G., Abdullayev J. Sh.</b> The Boundary Morera Theorem for Unbounded Realization of the Lie Ball  | 77 |
| 67. | <b>Khujakulov J.R., Abdullaev O.Kh., Sobirov Z.A.</b> On Inverse Source Problem for Time Fractional Diffusion Equation on Simple Metric Graph  | 80 |
| 68. | <b>Khusanbaev Ya.M., Kudratov Kh.E.</b> Inequalities for Moments of Branching Process in a Varying Environment   | 81 |
| 69. | <b>Khuzhayorov B., Fayziev B.M., Begmatov T.I.</b> A Mathematical Model of Two-Component Suspension Filtration in a Porous Medium with "Charging" Effect   | 83 |
| 70. | <b>Khuzhayorov B.Kh., Usmonov A.I.</b> Mathematical Modeling of Anomalous Solute Transport in a Cylindrical Porous Media with a Fractal Structure Taking   |    |

|     |  |     |
|-----|--|-----|
|     | Into Account Nonequilibrium Adsorption Phenomena   | 84  |
| 71. | <b>Kim V.A., Parovik R.I.</b> Forced Oscillations of the Duffing Oscillator with Variable Memory   | 85  |
| 72. | <b>Komilova N. J., Tulakova Z. R.</b> Dirichlet Problem for Multidimensional Helmholtz Equation with Two Singular Coefficients                                       | 87  |
| 73. | <b>Kriheli Boris, Levner Eugene.</b> An Artificial-Intelligence Algorithm for Improved Diagnostics of Viral Infections   | 88  |
| 74. | <b>Kroyter M.</b> The Sign Problem In Lattice Field Theory: Going Beyond Lefschetz Thimbles  | 89  |
| 75. | <b>Kudaybergenov K.K.</b> Derivations of Murray-Von Neumann Algebras   | 89  |
| 76. | <b>Kuliev K., Eshimova M.</b> Generalized Hardy Inequality with a Special Kernel and Weights   | 91  |
| 77. | <b>Kuznetsov M.</b> Algorithm of Optimization of Fractionated Radiotherapy Within Its Combination With Antiangiogenic Therapy by Means of Mathematical Modeling      | 92  |
| 78. | <b>Kytmanov A.M.</b> Transcendental Systems of Equations   | 92  |
| 79. | <b>Lakaev S.N., Abdukhakimov S.Kh.</b> The Existence and Location of Eigenvalues of the Two Particle Discrete Schrödinger Operators                                  | 93  |
| 80. | <b>Lakaev S.N., Khamidov Sh.I.</b> The Number and Location of Eigenvalues of the Two Particle Discrete Schrödinger Operators   | 94  |
| 81. | <b>Liiko V.</b> Smoothness of Generalized Solutions of Mixed Problem for Elliptic Equations Near Boundaries of Subdomains  | 95  |
| 82. | <b>Lolaev M.Ya.</b> An Interval Method in Feature Engineering to Maximize the Accuracy of Classification Models  | 96  |
| 83. | <b>Madrakhimov Sh.F., Makharov K.T.</b> Analysis of the Aging Rate Using Data Mining Methods   | 97  |
| 84. | <b>Makhmudov J.M., Kaytarov Z.D., Abdiyeva H.S.</b> Mathematical Model of Anomalous Transport of Multi-Species Contaminant in Porous Media with Fractal Structure    | 98  |
| 85. | <b>Makhmudov K.O.</b> Nonstationary Maxwell Equations  | 99  |
| 86. | <b>Maksimova A.G., Lazareva G.G.</b> Numerical Solution of the Lamé Equation with a Crack Defined on the Boundary  | 100 |
| 87. | <b>Maltseva S.V., Louis A.K.</b> An Iterative Method for Solving the Problem of Recovering a Vector Field by Limited Data  | 100 |
| 88. | <b>Mamanazarov A.O.</b> A Nonlocal Problem for a Parabolic-Hyperbolic Equation with Singular Coefficients  | 101 |
| 89. | <b>Marakhimov A.R., Khudaybergenov K.K.</b> Adaptive Activation Functions for Artificial Neural Networks   | 102 |
| 90. | <b>Matyakubov A.S., Raupov D.R.</b> Blow-Up Solutions of a Parabolic System Not in Divergence Form with Variable Density: Explicit Estimates and Asymptotic Behavior | 103 |
| 91. | <b>Mirzaev A.I.</b> The Importance of Choosing the first Step for finding an Informative Feature Set   | 104 |
| 92. | <b>Mozokhina A.S.</b> Influence of Pumping Activity of Lymphatic Vessels on The Lymph Propagation in the Human Lymphatic System                                      | 106 |