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is proved. Applying a method of Green's function, we are able to find the solution of the problem in an explicit form. Moreover, decomposition and summation formulae, formulae of differentiation and some adjacent relations for Lauricella's hypergeometric functions in many variables were used in order to find the explicit solution for the formulated problem.

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ON THE EIGENVALUES OF THE FRIEDRICHS MODEL WITH RANK N PERTURBATIONS

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In the Hilbert space $L_2(\mathbb{T}^{\mathrm{d}})$ of square integrable (complex) functions defined on d-dimensional torus $\mathbb{T}^{\mathrm{d}}$ we consider the Friedrichs model of the form

$$H := H_0 - \sum_{\alpha=1}^{N} \mu_\alpha V_\alpha,$$

where H_0 is the multiplication operator by the function $u(\cdot)$ on $L_2(\mathbb{T}^{\mathrm{d}})$:

$$(H_0 f)(x) = u(x) f(x)$$

and V_α is the non-local interaction operator on $L_2(\mathbb{T}^{\mathrm{d}})$:

$$(V_\alpha f)(x) = v_\alpha(x) \int_{\mathbb{T}^{\mathrm{d}}} v_\alpha(t) f(t) dt, \quad \alpha = \overline{1, N}.$$

Here $N \in \mathbb{N}$ with $N \geq 3$; $\mu_\alpha > 0$, $\alpha = \overline{1, N}$ are positive reals, the functions $u(\cdot)$ and $v_\alpha(\cdot)$, $\alpha = \overline{1, N}$ are real-valued continuous functions on $\mathbb{T}^{\mathrm{d}}$.

Under these assumptions the operator H is bounded and self-adjoint.

The perturbation v of the operator $h_0(p)$ is a self-adjoint operator of rank $\leq N$. Therefore in accordance with the Weyl theorem about the invariance of the essential spectrum under the finite rank perturbations, the essential spectrum of the operator H coincides with the essential spectrum of H. Therefore

$$\sigma_{\text{ess}}(H) = \sigma(H_0) = [u_{\min}; u_{\max}],$$

where the numbers $u_{\min}$ and $u_{\max}$ are defined by

$$u_{\min} := \min_{x \in \mathbb{T}^{\mathrm{d}}} u(x), \quad u_{\max} := \max_{x \in \mathbb{T}^{\mathrm{d}}} u(x).$$

We define the analytic functions in $\mathbb{C} \setminus [u_{\min}; u_{\max}]$ by

$$I_{\alpha\beta}(z) := \mu_\beta \int_{\mathbb{T}^d} \frac{v_\alpha(t)v_\beta(t)dt}{u(t)-z}, \quad \alpha, \beta = \overline{1, N} \quad \text{and} \quad \Delta(z) := \det\left(\delta_{\alpha\beta} - I_{\alpha\beta}(p\,; z)\right)_{\alpha,\beta=1}^N,$$

where δ_{ij} is the Kronecker delta.

Usually the function $\Delta(\cdot)$ is called the Fredholm determinant associated with the operator H and it is easy to see that the number $z \in \mathbb{C} \setminus [u_{\min}; u_{\max}]$ is an eigenvalue of H if and only if $\Delta(z) = 0$.

Let mes$(\cdot)$ be the Lebesgue measure on $\mathbb{R}^d$ and supp$\{v_\alpha(\cdot)\}$) be the support of the function $v_\alpha(\cdot)$.

Main result of this work is the following theorem.

Theorem 1. *If* mes$\{\text{supp}v_\alpha(\cdot) \cap \text{supp}v_\beta(\cdot)\} = 0$ *for all* $\alpha \neq \beta$, *then the number* $z \in \mathbb{C} \setminus [u_{\min}; u_{\max}]$ *is an eigenvalue of* H *if and only if the number* z *is an eigenvalue of* $H_\alpha := H_0 - \mu_\alpha V_\alpha$ *for some* $\alpha \in \{1, ..., N\}$; *moreover, the operator* H *has no more than* N *eigenvalues* (*counting multiplicities*) *lying on the l.h.s. of* $u_{\min}$ *and has no eigenvalues on the r.h.s. of* $u_{\max}$.

INTEGRATION OF THE LOADED KDV EQUATION IN THE CLASS OF RAPIDLY DECREASING COMPLEX-VALUED FUNCTIONS

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In this paper, we study the loaded Korteweg-de Vries (KdV) equation, namely, consider the following equation:

$$u_t - 6uu_x + u_{xxx} = -\gamma(t)u(0,t)u_x(x,t) \tag{1}$$

here $\gamma(t)$ is a given continuous function. Equation (1) is considered under the initial condition

$$u(x,0) = u_0(x), \;\; x \in R, \tag{2}$$

where the initial function $u_0(x)$ is complex-valued and has the following properties:

1) for some $\varepsilon > 0$

$$\int_{-\infty}^{\infty} |u_0(x)|\, e^{\varepsilon|x|}dx < \infty; \tag{3}$$

2) a non-self-adjoint operator $L(0) := -\dfrac{d^2}{dx^2} + u_0(x),\ x \in R$ has equally N complex eigenvalues $\lambda_1(0)$, $\lambda_2(0)$, ..., $\lambda_N(0)$ with multiplicities $m_1(0)$, $m_2(0)$, ..., $m_N(0)$, respectively, and has no spectral singularities.

We have to find a complex-valued sufficiently smooth function $u(x,\, t)$ which sufficiently rapidly tends to its limits as $x \to \pm\infty$, including the case when

$$\int_{-\infty}^{\infty} \left|\frac{\partial^j u(x,t)}{\partial x^j}\right| e^{\varepsilon|x|}dx < \infty, \quad j = 0,\, 1,\, 2,\, 3. \tag{4}$$

$$u^{(j)}(x,\,0)=u_0^{(j)}(x),\;\,x\in\overline{b}_j^+,\,j=1,2,...,n, \tag{4}$$

$$v^{(j)}(x,\,0)=v_0^{(j)}(x),\;\,x\in\overline{b}_j^-,\,j=1,2,...,n, \tag{5}$$

$$y^{(j)}(x,\,0)=y_0^{(j)}(x),\;\,x\in\overline{b}_j^0,\,j=1,2,...,(n-1) \tag{6}$$

граничные условия

$$u^{(1)}(0,t)=g_0^{(n)}(t),\;\,v^{(1)}(0,t)=d_0^{(n)}(t),\,t\geq 0, \tag{7}$$

$$u^{(n)}(L,t)=h_0^{(n)}(t),\;\,v^{(n)}(L,t)=r_0^{(n)}(t),\,t\geq 0 \tag{8}$$

на конечных связях соответственно.

В вершине решение удовлетворяет следующим условиям склейки (Кирхгофа):

$$u^{(j-1)}\left(L,t\right)=u^{(j)}\left(0,t\right)=y^{(j-1)}\left(0,t\right),u_x^{(j-1)}\left(L,t\right)+u_x^{(j)}\left(0,t\right)+y_x^{(j-1)}\left(0,t\right)=0; \tag{9}$$

$$v^{(j-1)}\left(L,t\right)=v^{(j)}\left(0,t\right)=y^{(j-1)}\left(L,t\right),v_x^{(j-1)}\left(L,t\right)+v_x^{(j)}\left(0,t\right)+y_x^{(j-1)}\left(L,t\right)=0,j=\overline{2,n}.$$

Последние условия обычно называют условиями непрерывности и сохранения потока (Кирхгофа) на точке ветвления графов. Мы предполагаем, что исходные данные являются достаточно гладкими функциями и удовлетворяют условиям (7) - (9).

Решение задачи построена так называемым методом Фокаса, которое является обобщением метода преобразования Фурье. При этом, задача сведена к системе алгебраических уравнений, относительно преобразования Фурье неизвестных значений решения в вершинах графа [1–5].

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