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Andijon, 28 mart 2022 yil

МИНИСТЕРСТВОВЫСШЕГОИСРЕДНЕГОСПЕЦИАЛЬНОГООБРАЗОВАНИЯ
РЕСПУБЛИКИ УЗБЕКИСТАН
АНДИЖАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

Сборник материалов Республиканской научно-практической конференции

ТЕОРЕТИЧЕСКИЕ ОСНОВЫ И ПРИКЛАДНЫЕ ЗАДАЧИ СОВРЕМЕННОЙ МАТЕМАТИКИ
II

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THEORETICAL FOUNDATIONS AND APPLIED PROBLEMS OF MODERN MATHEMATICS
II

Andijan, March 28, 2022

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UMUMLASHGAN FRIDRIXS MODELLARI OILASI XOS QIYMATLARINING SONI VA JOYLASHUV O’RNI

Xayitova Xilola
Buxoro davlat universiteti

$\mathbb{T}^{\boldsymbol{d}}$ orqali $\boldsymbol{d}$ o’lchamli torni, $\mathbb{C}$ orqali bir o‘lchamli kompleks fazoni va $\boldsymbol{L_2}\left(\mathbb{T}^{\boldsymbol{d}}\right)$ orqali $\mathbb{T}^{\boldsymbol{d}}$ to‘plamda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymat qabul qiluvchi) funksiyalarning Gilbert fazosini belgilaymiz.

Ixtiyoriy $\boldsymbol{f}=(\boldsymbol{f_0},\boldsymbol{f_1}),\boldsymbol{g}=(\boldsymbol{g_0},\boldsymbol{g_1})\in\mathbb{C}\oplus\boldsymbol{L_2}\left(\mathbb{T}^{\boldsymbol{d}}\right)$ elementlar uchun ularning skalyar ko’paytmasi

$$(\boldsymbol{f},\boldsymbol{g})=\boldsymbol{f_0}\overline{\boldsymbol{g_0}}+\int\limits_{\mathbb{T}^{\boldsymbol{d}}}\boldsymbol{f_1}(\boldsymbol{t})\overline{\boldsymbol{g_1}(\boldsymbol{t})}\,\boldsymbol{dt}$$

tenglik yordamida aniqlanadi. $\boldsymbol{f}=(\boldsymbol{f_0},\boldsymbol{f_1})$ ning normasi esa

$$\|\boldsymbol{f}\|=\sqrt{|\boldsymbol{f_0}|^2+\int\limits_{\mathbb{T}^{\boldsymbol{d}}}|\boldsymbol{f_1}(\boldsymbol{t})|^2\,\boldsymbol{dt}}$$

kabi aniqlanadi.

Mazkur ishda $\mathbb{C}\oplus\boldsymbol{L_2}\left(\mathbb{T}^{\boldsymbol{d}}\right)$ Gilbert fazosidagi quyidagi ikkinchi tartibli operatorli matritsani qaraymiz.

$$\boldsymbol{h_{\mu,\lambda}}(\boldsymbol{\gamma}):=\begin{pmatrix}\boldsymbol{h_{00}}(\boldsymbol{x}) & \dfrac{\boldsymbol{\lambda}}{\sqrt{\boldsymbol{2}}}\boldsymbol{h_{01}}\\ \dfrac{\boldsymbol{\lambda}}{\sqrt{\boldsymbol{2}}}\boldsymbol{h_{01}^*} & \boldsymbol{h_{11}^0}(\boldsymbol{\gamma},\boldsymbol{x})-\boldsymbol{\mu V}\end{pmatrix}$$

Bu yerda matritsaviy elementlar

$$h_{00}(x)f_0=u(x)f_0,\qquad h_{01}f_1=\int\limits_{\mathbb{T}^d}v(t)f_1(t)dt,$$

$$(h_{01}^*f_0)(y)=v(y)f_0,\qquad \left(h_{11}{}^0(\gamma;x)f_1\right)(y)=w_\gamma(x,y)f_1(y),$$

$$(vf_1)(y)=\int\limits_{\mathbb{T}^d}f_1(t)dt$$

qoidalar yordamida ta’sir qiladi. $u_1(\cdot),v(\cdot)$ funksiyalar $\mathbb{T}^d$ da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar.

Ikkinchi tartibli matritsaviy model operatorning muhim spektri tushunchasiga to’xtalib o’tamiz. Matritsaviy operatorni quyidagicha ko’rinishda ifodalaymiz:

$$\boldsymbol{h_{\mu,\lambda}}(\gamma;x):=h_{00}(\gamma)+v_{\mu,\lambda}.$$

Bu yerda, $h_{00}(\gamma;x):=\begin{pmatrix}h_{00}(x) & 0\\ 0 & h_{11}^0(\gamma,x)\end{pmatrix}$, $v_{\mu,\lambda}:=\begin{pmatrix}0 & \frac{\lambda}{\sqrt{2}}h_{01}\\ \frac{\lambda}{\sqrt{2}}h_{01}^* & -\mu v\end{pmatrix}$.

Bizga yaxshi ma'lumki, chekli o'lchamli qo'zg'alishlarda muhim spektrning o'zgarmasligi to'g'risidagi mashhur Veyl teoremasiga ko'ra $\boldsymbol{h}_{\mu,\lambda}(\gamma;x)$ va $h_{00}(\gamma;x)$ operatorli matritsalarning muhim spektrlari ustma-ust tushadi.

Demak, $\boldsymbol{h}_{\mu,\lambda}(\gamma;x)$ operatorli matritsaning aniqlanishiga ko'ra, uning muhim spektri uchun

$$\sigma_{ess}\left(\boldsymbol{h}_{\mu,\lambda}(\gamma;x)\right)=\sigma_{ess}\left(h_{00}(x)\right)$$

tenglik o'rinli bo'lib, operatorli matritsaning muhim spektri $\sigma_{ess}\left(h_{00}(x)\right)=\overline{Imw_\gamma(x;\cdot)}=\left[m_\gamma(x);\ M_\gamma(x)\right]$ kesmadan iborat. Bu yerda,

$$m_\gamma(x)=min_{y\in\mathbb{T}^d}w_\gamma(x,y),\ M_\gamma(x)=max_{y\in\mathbb{T}^d}w_\gamma(x,y).$$

Operatorli matritsaning diskret spektrini aniqlashda muhim bo'lgan hamda $\mathbb{C}\setminus\left[m_\gamma(x);\ M_\gamma(x)\right]$ sohada regulyar bo'lgan $\Delta_{\mu,\lambda}(\gamma,x;z)$ funksiyani qaraymiz:

$$\Delta_{\mu,\lambda}(\gamma,x;z)=\left(u(x)-z-\frac{\lambda^2}{2}\int\limits_{\mathbb{T}^d}\frac{v^2(t)dt}{w_\gamma(x,t)-z}\right)\left(1-\int\limits_{\mathbb{T}^d}\frac{dt}{w_\gamma(x,t)-z}\right)$$
$$-\frac{\mu\lambda}{2}\left(\int\limits_{\mathbb{T}^d}\frac{v(t)dt}{w_\gamma(x,t)-z}\right)^2$$

Quyidagi lemma $\boldsymbol{h}_{\mu,\lambda}(x)$ operatorli matritsaning xos qiymatlari va $\Delta_{\mu,\lambda}(\gamma,x;\cdot)$ funksiya nollari orasidagi munosabatni ifodalaydi.

1-Lemma. $z\in\mathbb{C}\setminus[m(x);M(x)]$ soni $\boldsymbol{h}_{\mu,\lambda}(\gamma;x)$ operatorning xos qiymati bo'lishi uchun $\Delta_{\mu,\lambda}(\gamma,x;z)=0$ bo'lishi zarur va yetarli.

Isbot. $z\in\mathbb{C}\setminus[m(x);M(x)]$ soni $\boldsymbol{h}_{\mu,\lambda}(\gamma;x)$ operatorning xos qiymati bo'lsin. U holda

$$\boldsymbol{h}_{\mu,\lambda}(\gamma;x)f=zf,\ \ f=\begin{pmatrix}f_0\\f_1\end{pmatrix}.$$

tenglamani qaraymiz. 1-lemmadan $\boldsymbol{h}_{\mu,\lambda}(\gamma;x)$ operatorning diskret spektri uchun

$$\sigma_{disc}\left(\boldsymbol{h}_{\mu,\lambda}(\gamma;x)\right)=\left\{z\in\mathbb{C}\setminus[m(x),M(x)]:\Delta_{\mu,\lambda}(\gamma,x;z)=0\right\}$$

tenglik kelib chiqadi.

Ishning asosiy natijasi quyidagi teoremada o'z aksini topgan.

Teorema. $\boldsymbol{h}_{\mu,\lambda}(\gamma;x)$ ooperator ko'pi bilan 3 ta xos qiymatga ega. Ulardan ikkitasi muhim spektrdan chapda, bittasi esa muhim spektrdan o'ngda joylashgan bo'ladi.

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