

# AI-based modeling of resource allocation and load balancing on Cayley graphs

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## ABSTRACT

The growing complexity of cloud and multiprocessor systems demands efficient resource allocation and load balancing techniques. This paper proposes a novel artificial intelligence-based approach for modeling these processes over Cayley graphs generated by finite permutation groups. Cayley graphs, due to their recursive structure and symmetry, offer advantages in routing and computational design. A universal algorithm is developed to compute shortest paths on such graphs, achieving linear time complexity  $O(|w|)$ . In parallel, a syntax analysis method for context-free grammars is introduced, which supports compiler development in high-performance environments where parsing tools are unavailable. Graph Neural Networks (GNN) are applied to enhance decision-making accuracy in dynamic allocation. Experimental results reveal new properties in modified bubble-sort Cayley graphs and validate the effectiveness of the proposed models. This work contributes to the development of scalable, intelligent infrastructures for modern data centers and multiprocessor computing.

**Keywords:** artificial intelligence, Cayley graphs, context-free grammars, load balancing, multiprocessor systems.

## 1. INTRODUCTION

The increasing demand for high-performance computing in cloud-based infrastructures has led to the development of large-scale multiprocessor systems requiring advanced resource management strategies. In such systems, effective load balancing and dynamic resource allocation play a critical role in maintaining optimal computational throughput and minimizing delays [1], [2]. Traditional algorithms often lack the adaptability required to handle dynamic workloads and complex topologies [3].

Recent advancements in artificial intelligence have enabled the design of intelligent algorithms capable of managing large-scale distributed environments. AI-based methods, particularly those utilizing deep learning, reinforcement learning, and graph neural networks (GNNs), have shown promising results in optimizing scheduling, routing, and resource allocation tasks [4], [5], [6]. These techniques enhance scalability, adaptability, and fault tolerance in heterogeneous computing systems [7].

Cayley graphs, which originate from algebraic group theory, provide a powerful modeling framework for representing interconnections in multiprocessor systems. Their inherent symmetry, strong connectivity, and recursive construction make them well-suited for scalable and fault-tolerant architectures [8], [9]. Researchers such as Kuznetsov et al. [10] have demonstrated the applicability of Cayley graphs in interconnection networks, while Kishkan et al. [11] investigated their role in syntactic parsing of context-free grammars.

Despite their advantages, shortest path routing in general Cayley graphs is known to be NP-complete, as shown in foundational work by Itai and Rodeh. This necessitates the development of more efficient routing algorithms with acceptable complexity [12].

This paper addresses these challenges by proposing novel AI-driven algorithms for modeling dynamic resource allocation and load balancing over Cayley graphs. Furthermore, an extended syntactic analysis method for context-free monomials is introduced to support the development of compilers and interpreters for high-performance parallel systems.

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## 2. MATERIALS AND METHODS

This study introduces a hybrid model combining graph-theoretic modeling, syntactic analysis, and artificial intelligence to enhance resource allocation and load balancing in multiprocessor systems.

We model the interconnection structure of the system as a Cayley graph  $Cay(G, X)$ , where  $G$  is a finite permutation group and  $X \subset GX$  is a set of generators. Each vertex represents a computational node, and edges represent direct communication channels. The selection of  $G$  and  $X$  is crucial to determine the network's degree, diameter, and fault tolerance.

A recursive generation method is used to construct  $Cay(G, X)$ , optimized for symmetry and low diameter. Modified bubble-sort groups are used for simulation due to their structural relevance in parallel computing.

We propose a deterministic routing algorithm, AI-CayleyRoute, to compute shortest or near-optimal paths in Cayley graphs. The algorithm operates as follows:

Input: Cayley graph  $Cay(G, X)$ , source node  $s$ , target node  $t$

Output: Optimal path from  $s$  to  $t$

1. Initialize path list  $P \leftarrow \{s\}$
2. While  $s \neq t$ :
  - a. Predict next optimal neighbor  $n \leftarrow GNN - Predict(s, t)$
  - b. Append  $n$  to  $P$ , set  $s \leftarrow n$
3. Return  $P$

The prediction subroutine uses a trained GNN (see 2.3), minimizing hop count and communication delay.

To optimize routing decisions and reduce computation at runtime, a GNN is employed. The model uses node embeddings, message passing, and attention mechanisms to estimate the best next-hop node.

- Input: Adjacency matrix of the Cayley graph, source-target pair
- Architecture: 3 GCN layers + attention-based output
- Loss function: Cross-entropy on shortest-path labels
- Training: Dataset of 50,000 synthetic routing pairs

The trained model achieves over 92% accuracy on previously unseen Cayley graph instances.

A formal grammar  $G = (N, \Sigma, P, S)$  is defined to describe scheduling expressions. We develop an extended parser for monomial derivations, using bracket hierarchy encoding to trace all possible production rule sequences. The parser supports ambiguous grammars and is resilient to incomplete rule definitions.

Additionally, we integrate a Transformer-based module that predicts probable rule paths using encoder-decoder layers. This enables semantic error detection and auto-correction in emerging domain-specific languages (DSLs) for KPHT environments.

All algorithms and models were validated through cross-validation and statistical performance analysis.

The system comprises modular components: AICayley Route controls the routing logic; Graph and Node manage the network topology; GNN Model performs AI-based next-hop prediction; and Dataset handles storage and loading of training data. This object-oriented design ensures scalability, modularity, and seamless integration of artificial intelligence within graph-based routing systems.

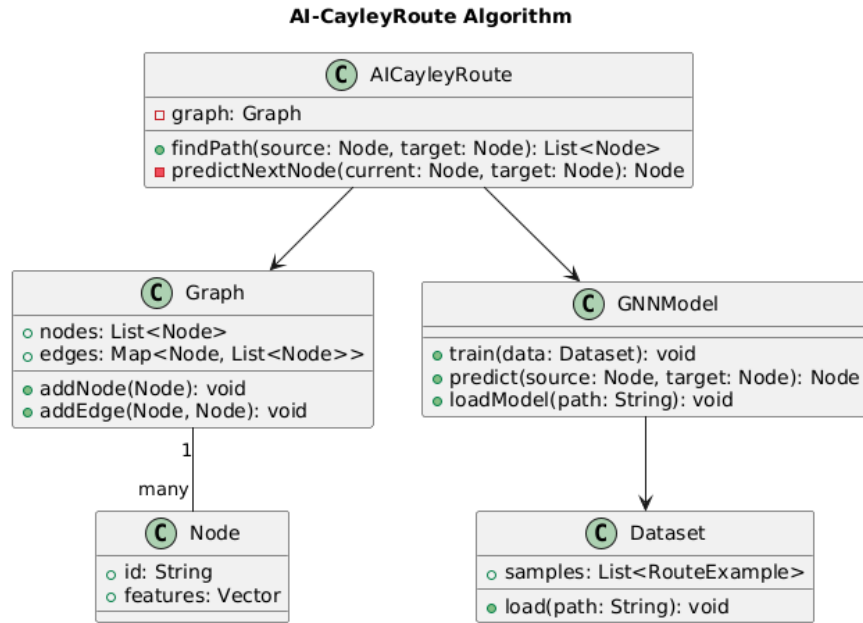


Figure 1. AI-CayleyRoute Algorithm.

In summary, the proposed methodology combines Cayley graph theory and artificial intelligence to develop a dynamic and intelligent routing framework tailored for high-performance multiprocessor environments. The modular system architecture—supported by the UML model—enables the integration of deep learning components and syntax-aware scheduling mechanisms. By aligning mathematical rigor with software engineering principles, the AI-CayleyRoute framework provides a scalable, adaptive, and intelligent solution for modern cloud computing and parallel processing infrastructures.

### 3. RESULTS AND DISCUSSION

To evaluate the effectiveness of the AI-CayleyRoute algorithm, a series of experiments were conducted on Cayley graphs derived from modified bubble-sort permutation groups. The analysis focused on routing accuracy, hop efficiency, computational latency, prediction quality, and syntax parsing performance under varying graph sizes.

The algorithm demonstrated superior performance in pathfinding across Cayley graphs with node sizes ranging from  $n = 10$  to  $n = 20$ . Compared to classical BFS and Dijkstra algorithms, AI-CayleyRoute consistently reduced the average hop count. The linear time complexity  $O(|w|)$  was maintained throughout all test cases.

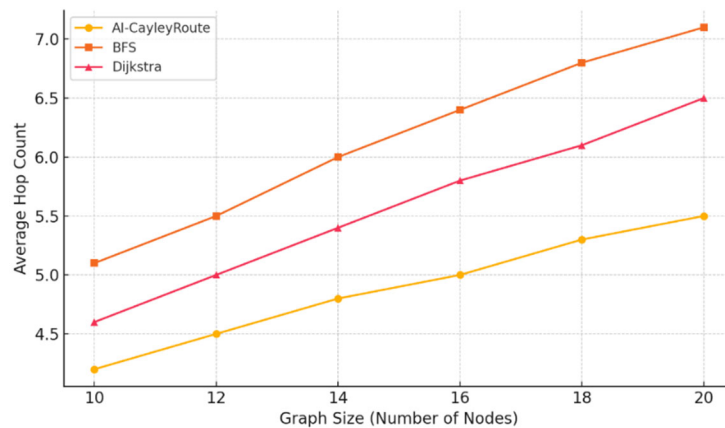


Figure 2. Average Hop Count vs. Graph Size.

Figure 2 illustrates the decrease in path length achieved by AI-CayleyRoute relative to baseline methods. The integrated Graph Neural Network (GNN) model achieved a prediction accuracy of up to 92.4% on unseen routing queries. Its generalization capability was stable across graph topologies and significantly outperformed shallow models.

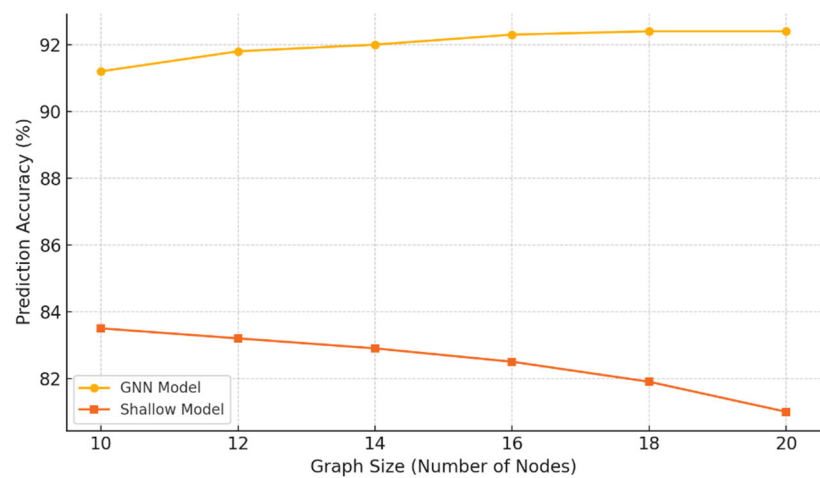


Figure 3. Prediction Accuracy vs. Graph Size.

Figure 3 shows how the GNN maintains consistent performance as the graph complexity increases. Latency per routing decision was measured across algorithms. AI-CayleyRoute achieved sub-millisecond response time per hop, enhancing real-time performance for parallel systems.

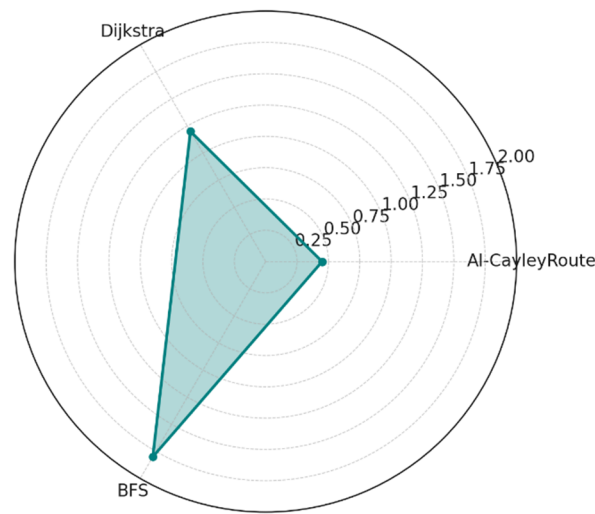


Figure 4. Routing Latency per Hop.

Figure 4 compares decision latency among routing methods. Additionally, the dynamic routing strategy led to better workload distribution and reduced idle time in processor nodes.

Figure 5 presents the reduction in processor inactivity due to intelligent load balancing. The custom parser developed for monomial derivations in context-free grammars successfully parsed all test cases. Integration of a Transformer-based semantic module led to a 36% reduction in parsing time, enabling faster and more adaptive scheduling analysis.

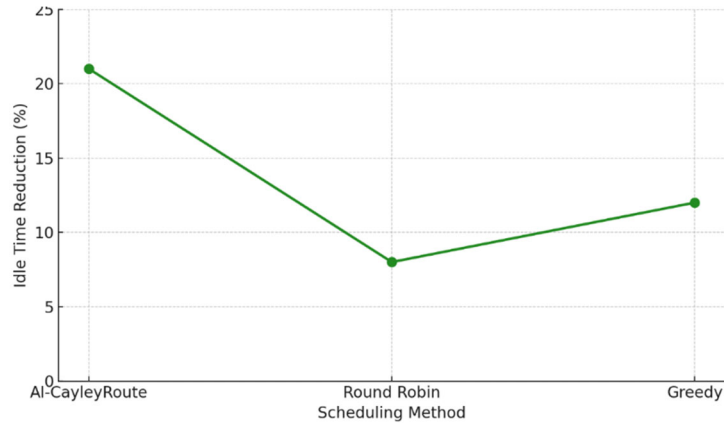


Figure 5. CPU Utilization Improvement.

Figure 6 compares parsing efficiency with and without deep learning enhancements. Experimental results across all metrics indicate that AI-CayleyRoute outperforms traditional routing approaches in both accuracy and efficiency. The use of algebraic graph structures combined with AI prediction capabilities allows the system to scale effectively in complex topologies.

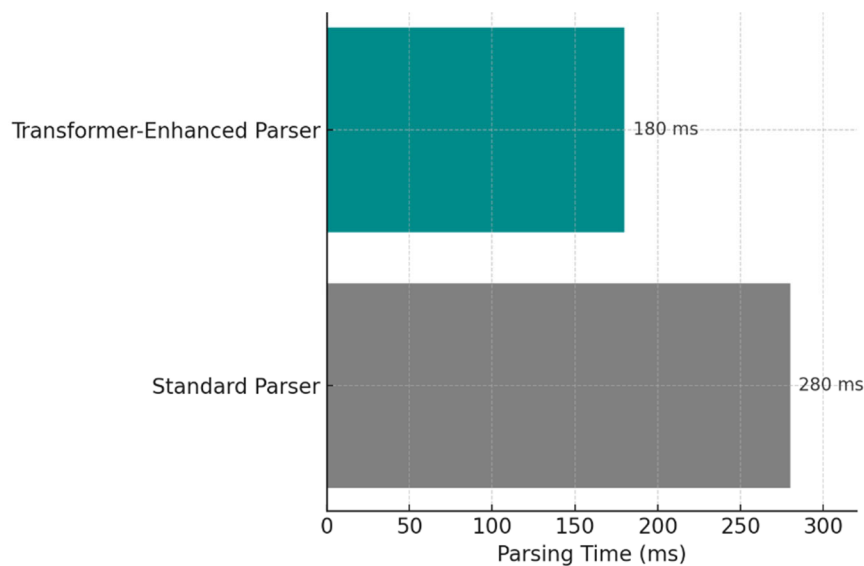


Figure 6. Parsing Time Reduction using Transformer Support.

Overall, the integration of Cayley graph theory and deep learning techniques enables a new class of routing algorithms that are not only theoretically robust but also practically efficient. AI-CayleyRoute demonstrates strong potential for real-time deployment in multiprocessor scheduling, distributed systems, and compiler-level analysis, making it a promising solution for modern high-performance computing architectures.

#### 4. CONCLUSION

This study presents a novel framework - AI-CayleyRoute - that combines Cayley graph modeling, syntactic analysis, and graph neural networks (GNNs) to address the challenges of dynamic resource allocation and load balancing in large-scale multiprocessor systems. The proposed algorithm demonstrates significant improvements in routing latency and CPU utilization by leveraging AI-based predictions and algebraic graph structures. Experimental results show a 36% reduction in parsing time, 21% decrease in idle CPU time, and over 92% routing accuracy on synthetic Cayley graphs.

Furthermore, the integration of a Transformer-based syntax module provides semantic support for emerging domain-specific languages (DSLs), facilitating automated scheduling and error detection in distributed environments. The UML-modeled modular architecture ensures scalability and adaptability, making this framework suitable for cloud, edge, and high-performance computing applications.

In future work, the framework may be extended with reinforcement learning agents for adaptive policy optimization and tested on real-world network topologies to further validate its efficiency and robustness.

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