Mathematical modeling of the combined heat supply system of the solar house

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Abstract. Today, improving energy efficiency in residential heat supply systems, saving fuel and energy resources, and increasing the efficiency of using devices based on renewable energy sources is an urgent issue. The aim of the article is to develop a mathematical model of the heat balance of one-story country houses based on the use of solar energy in a non-stationary mode and conduct theoretical research. The heat balance scheme of the solar house with autonomous heat supply and the electric-heat scheme of the physical model are proposed. Based on the proposed schemes, a mathematical model and calculation algorithm of the heat balance of a one-story rural house of the research object in non-stationary mode were developed, based on the mathematical modeling, the influence of the heat capacity of the wall structure on the temperature regime of the building was studied. The main temperature characteristics, which analyzed the change of the indoor air temperature of the building depending on the ambient temperature, were built on the basis of the MATLAB/SIMULINK program. The mathematical model of the building's heat balance in dynamic mode and the obtained calculation results are recommended for use in the development of energy-efficient solar houses.

1 Introduction

Currently, a number of reforms are being carried out in Uzbekistan regarding the rational use of natural fuel energy resources, the application of energy-saving technologies to economic sectors, and the wide introduction of modern technologies through the radical modernization of production. According to the content of the current laws and decisions, reducing the consumption of energy and resources in heat supply systems, widely introducing energy-saving technologies in production sectors, expanding the use of renewable energy sources, and improving energy efficiency in economic sectors are indicated as priority tasks [1-4].

Currently, in our Republic, the development of innovative technologies based on the use of renewable energy sources, the implementation of scientific and technical developments, the improvement of the energy efficiency of renewable energy sources, the promotion of the expansion and localization of their production are carried out at the level of state policy [1]. It is important to conduct research based on the modeling of the heat balance of

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buildings in order to evaluate the possibilities of using solar energy in the heat supply of residential buildings, to develop and implement solar energy-based heat supply systems.

Scientific and research work on increasing the efficiency of using solar energy in the heat supply of buildings is being carried out by specialist scientists in the world [5-11]. Modeling of solar collector heat supply systems and evaluation of solar collector application efficiency in residential heat supply, optimization and control of parameters of solar heat supply systems have been thoroughly studied [12-16]. Even in the climatic conditions of the city of Karshi, scientific research was conducted on the use of solar energy in various technological processes. But the analysis of the conducted scientific studies shows that the creation and implementation of combined systems of solar and traditional heat supply of rural houses is not sufficiently studied.

The issue of heat balance modeling of an experimental solar house based on a combined heat supply system was considered in the article. An overview of the experimental solar house is shown in Figure 1 and the heat balance diagram is shown in Figure 2.

A dynamic model of the heat balance equation of a solar house with combined autonomous heat supply was developed, and a mathematical model of the process was created according to the block diagram developed on this basis.

Fig. 1. General view of the experimental solar house (Karshi city).

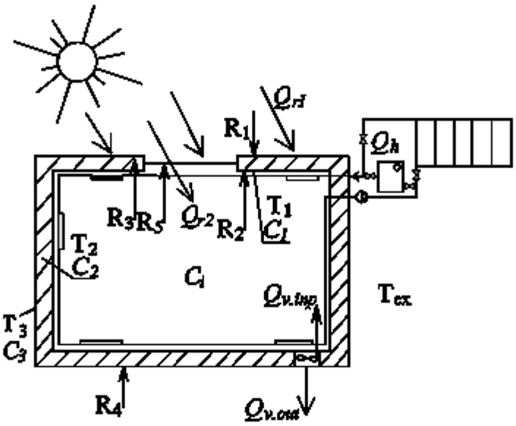


Fig. 2. Heat balance scheme of a solar house with autonomous heat supply.

In the mathematical modeling of the research object, a heat-electric scheme was first built, taking into account the physical aspects of the model. For this, the internal environment and the components of the heat capacity indicators in it were determined. The amount of heat supplied or consumed according to the specified quantities, its effect on the change of the temperature of the internal environment, the thermal resistance of the heat-receiving layer and other factors leading to the change of the total heat capacity were expressed mathematically on the basis of the theory of electro-thermal similarity. The electrical-thermal scheme of the mathematical model is depicted in Figure 3.

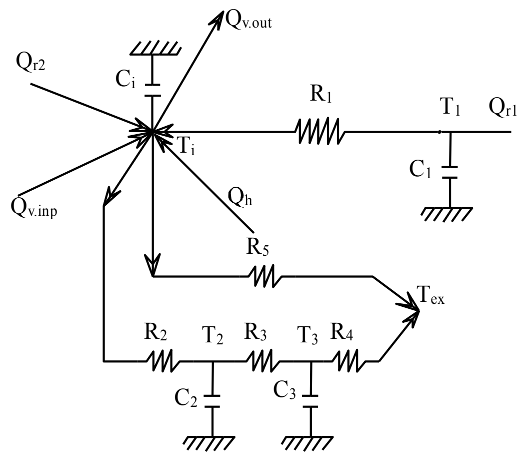


Fig. 3. Electric-heat scheme of the model constructed for the experimental solar house.

Here: Q_{p1} – radiation (heat) falling directly on the front of the research object, W; Q_{p2} – solar radiation (heat) falling directly into the internal environment of the research object, W; $Q_{ven.l}$ – heat that leaves the research facility through ventilation, W; $Q_{ven.input}$ – heat input due to ventilation, W; Q_{heat} – heat supplied from an external source to heat the object, W; T_1 – the temperature of the internal wall of the object, °C; T_2 – the internal temperature of the facility structure, °C; T_3 – the external temperature of the facility structure, °C; $T_т$ – outdoor temperature, °C; C_i – heat capacity of the indoor air of the research object, $\frac{J}{kg{\cdot}К}$; C_1 – heat capacity of the front wall of the object, $\frac{J}{kg{\cdot}К}$ C_2 – internal heat capacity of object construction, $\frac{J}{kg{\cdot}К}$$C_3$ – heat capacity of the structure, $\frac{J}{kg{\cdot}К}$. R_1 – thermal resistance of the front part of the research object, $\frac{m^2{\cdot}К}{W}$. R_2 – thermal resistance of the internal side of the object structure, $\frac{m^2{\cdot}К}{W}$; R_3 – internal thermal resistance of the object structure, $\frac{m^2{\cdot}К}{W}$; R_4 – thermal resistance of the outer part of the structure, $\frac{m^2{\cdot}К}{W}$; R_5 – total thermal resistance of the glass wall section, $\frac{m^2{\cdot}К}{W}$.

2 Materials and methods

The dynamic operating mode of the research facility can be modeled using a system of linear differential equations. It will be possible to first express these equations in the form of matrices, and then transfer them to the form of a dynamic model through Matlab/Simulink software. The heat balance equation for the dynamic state of the heat supply system of the solar house will have the following form:

$$\begin{cases} \mathrm{c}_1 m_1 \dfrac{dt_1}{d\tau} = \alpha_{и} \cdot F \cdot (t_{и} - t_1) - \dfrac{F}{R_1} \cdot (t_1 - t_2) \\ \mathrm{c}_2 m_2 \dfrac{dt_2}{d\tau} = \dfrac{F}{R_1} \cdot (t_1 - t_2) - \dfrac{F}{R_2} \cdot (t_2 - t_3) \\ \mathrm{c}_3 m_3 \dfrac{dt_3}{d\tau} = \dfrac{F}{R_2} \cdot (t_2 - t_3) - \dfrac{F}{R_3} \cdot (t_3 - t_4) - \alpha_T . F \cdot (t_4 - t_T) + q_{light \cdot} \cdot F \cdot k_{fl} \cdot \alpha_{fl} \\ \mathrm{c}_{in} m_{in} \dfrac{dt_{и}}{d\tau} = G_c \cdot \mathrm{c}_c \cdot (t_{к} - t_{ч}) + q_{light \cdot} \cdot F_{win} \cdot k_{win} \cdot \alpha_{win} - G_{air} \cdot \mathrm{c}_{air} (t_{in} - t_{out.}) - \\ -kF \cdot (t_{in} - t_{out}) - \dfrac{F_{win}}{R_{win}} \cdot (t_{in} - t_{out}) \end{cases} \quad (1)$$

The heat transfer coefficient and the thermal resistance of the layers were found from the formula (2)[5-12]:

$$k = \frac{1}{\frac{1}{\alpha_{in}} + \frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3} + \frac{1}{\alpha_{т}}} \quad (2)$$

By simplifying the right and left sides of equation (1), we get the following equations:

$$\frac{dt_1}{d\tau} = \frac{(-\alpha_{и} . F - \frac{F}{R_1})}{m_1 c_1} \cdot t_1 + \frac{F}{R_1 \cdot m_1 c_1} \cdot t_2 + \frac{\alpha_1 . F \cdot t_{in}}{m_1 c_1} \quad (3)$$

$$\frac{dt_2}{d\tau} = \frac{F}{R \cdot m_2 c_2} \cdot t_1 + \frac{-\frac{F}{R_1} - \frac{F}{R_2}}{m_2 c_2} \cdot t_2 + \frac{F}{R_2 \cdot m_2 c_2} \cdot t_3 \quad (4)$$

$$\frac{dt_3}{d\tau} = \frac{F}{R_2 \cdot m_2 c_3} \cdot t_2 + \frac{-\frac{F}{R_2} - \frac{F}{R_3}}{m_3 c_3} \cdot t_3 + \frac{-\frac{F}{R_3} - \alpha_T . F}{m_3 c_3} \cdot t_4 + \frac{\alpha_T . F}{m_3 c_3} t_T + \frac{F \cdot k_{fl} \cdot \alpha_{fl}}{m_3 c_3} \cdot q_{light \cdot} \quad (5)$$

$$\frac{dt_{in}}{d\tau} = \left(\frac{-G_{air} \cdot \mathrm{c}_{air} - kF - \frac{F_{win}}{R_{win}}}{m_{in} c_{in}} \right) \cdot t_{in} + \left(\frac{G_{air} \cdot \mathrm{c}_{air} + kF + \frac{F_{win}}{R_{win}}}{m_{in} c_{in}} \right) \cdot t_T + \left(\frac{1}{m_{in} in} \right) \cdot Q_{heat} + \\ + \left(\frac{k_{win} \cdot \alpha_{win} \cdot F_{win}}{m_{in} c_{in}} \right) \cdot q_{light} \quad (6)$$

Equations (5) – (6) can be expressed in matrix form:

$$\dot{x} = Ax + Bu \quad (7)$$

$$y = Cx + Du \quad (8)$$

(7) and (8) are vector exponents of Eqs:

$$x = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_{in} \end{bmatrix}; \quad \dot{x} = \begin{bmatrix} \grave{t_1} \\ \grave{t_2} \\ \grave{t_3} \\ \grave{t_{in}} \end{bmatrix}; \quad y = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_{in} \end{bmatrix}; \quad and = \begin{bmatrix} t_4 \\ t_t \\ Q_{heat} \\ q_{rad} \end{bmatrix};$$

$$C = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \end{bmatrix}; \qquad D = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \end{bmatrix};$$

$$A = \begin{bmatrix} \frac{\left(-\alpha_1.F - \frac{F}{R_1}\right)}{m_1 c_1}; & \frac{F}{R_1 \cdot m_1 c_1}; & 0; & \frac{\alpha F}{m_1 c_1}; \\ \frac{F}{R_1 \cdot m_2 c_2}; & \frac{-\frac{F}{R_1} - \frac{F}{R_2}}{m_2 c_2}; & \frac{F}{R_2 \cdot m_2 c_2}; & 0; \\ 0; & \frac{F}{R_3 \cdot m_3 c_3}; & \frac{-\frac{F}{R_2} - \frac{F}{R_3}}{m_3 c_3}; & 0; \\ 0; & 0; & 0; & \frac{-G_{ven} \cdot c_{in} - \text{к}F - \frac{F_{ml}}{R_{ml}}}{m_{in} c_{in}}; \end{bmatrix}$$

$$B = \begin{bmatrix} 0\,; & 0\,; & 0; & 0; \\ 0; & 0; & 0; & 0; \\ \frac{\left(-\frac{F}{R_3} - \alpha_{\text{т}}.F\right)}{m_3 c_3}; & 0; & 0; & \frac{F \cdot k_p \cdot \alpha_p}{m_3 c_3}; \\ 0; & \frac{G_{in} \cdot \text{c}_{in} + kF + \frac{F_{ml}}{R_{ml}}}{m_{in} c_{in}}; & \frac{1}{m_{in} \cdot c_{in}}; & \frac{k_{f.l} \cdot \alpha_{f.l} \cdot F_{ml}}{m_{in} c_{in}} \end{bmatrix}$$

3 Results and Discussions

According to the results of mathematical modeling, the analysis of the thermal regime of the research object is carried out by entering the dynamic model into the Matlab/Simulink program. The modular diagram of the dynamic model written in the Matlab/Simulink programming language is presented in Figure 4.

The structure of the object, that is, the solar house, affects its internal air temperature (Figure 5). If we take into account that the solar house heating season for the southern regions of our Republic consists of November-March, it is possible to analyze the change in the indoor air temperature of the house during a month depending on the outdoor air temperature.

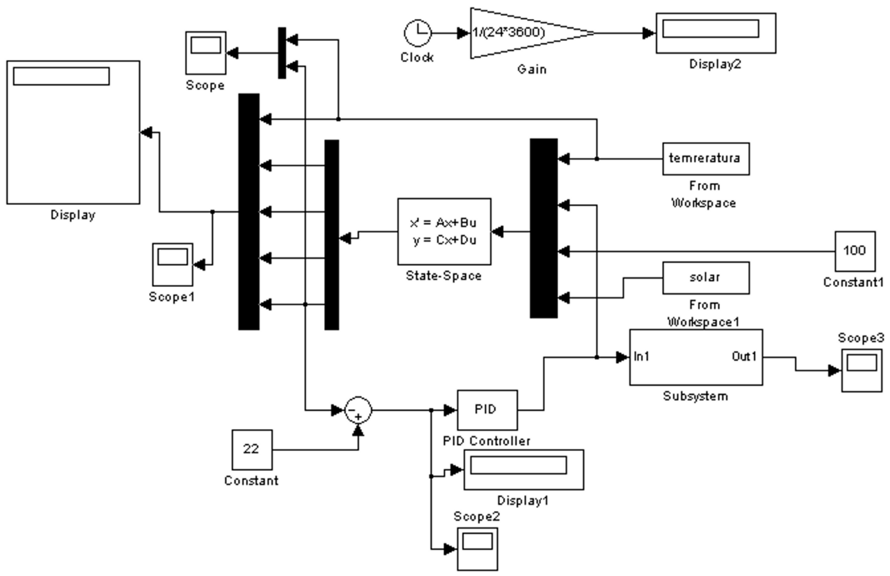
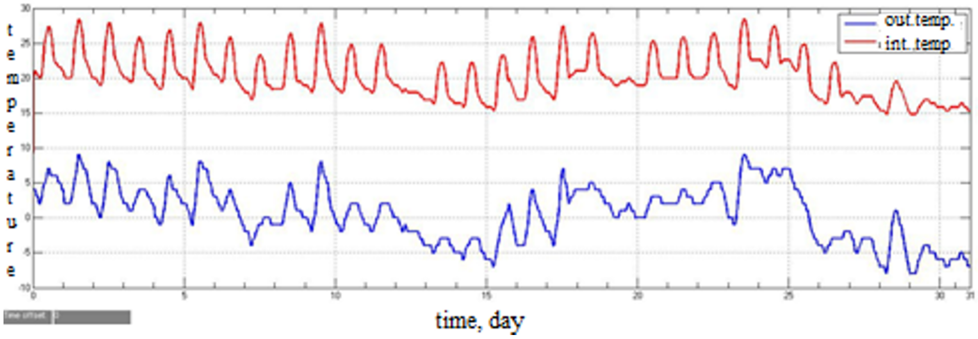
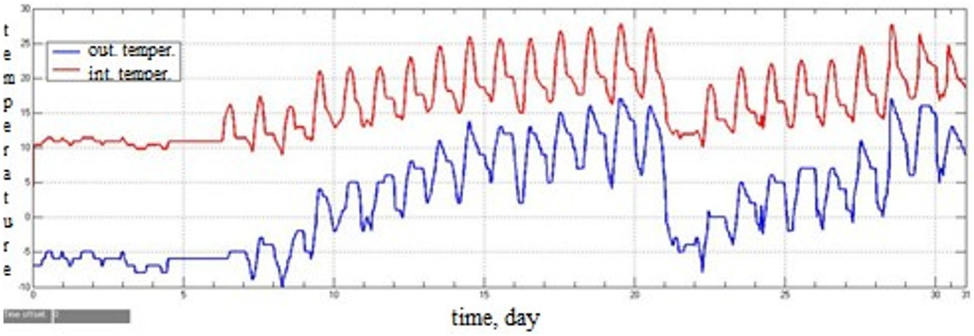


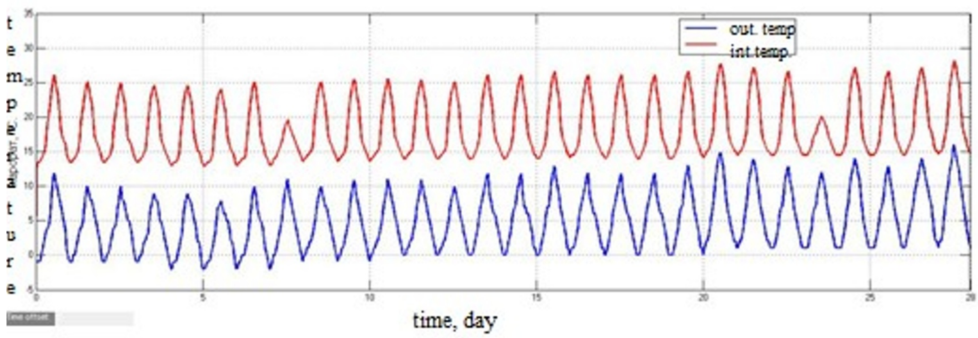
Fig. 4. Modular scheme of the dynamic model of the research object.



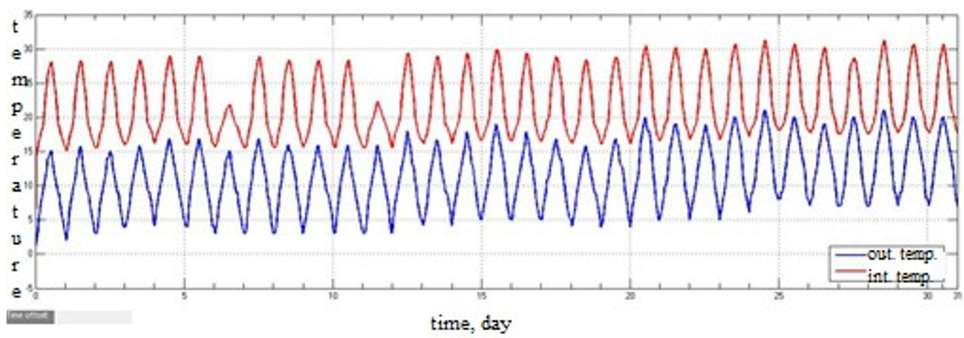
a) for December;



b) for January;



c) for February;

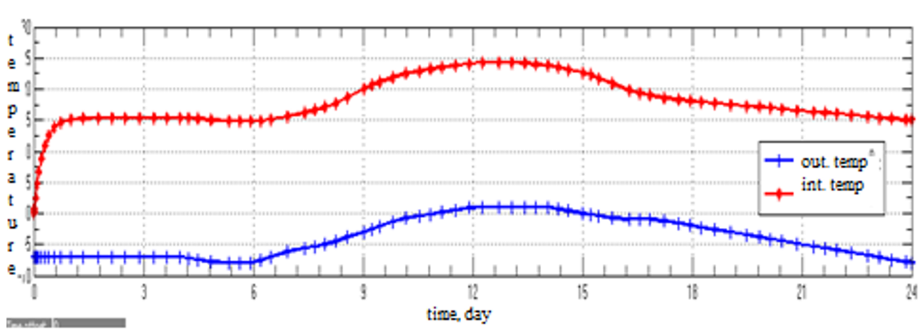


g) for March.

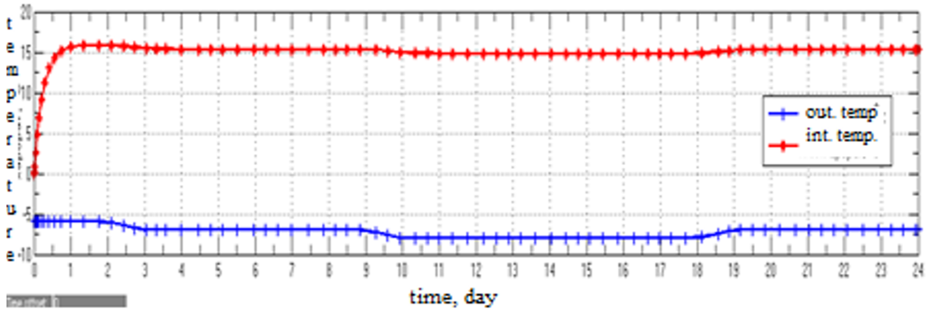
Fig. 5. Temperature characteristics of the solar house (obtained using Matlab/Simulink software).

In pictures 5a and b, it can be seen that the air temperature has cooled down to -5÷-10 degrees from December 26 to January 7, 2020. This situation was repeated on January 20-22 (Figure 5b). Due to cloudy weather conditions at this time, the indoor air temperature of the research facility was maintained at 22-24 ^{0}C using a water heating boiler. In Figures 5, b, v and g, organic fuel savings have been achieved as a result of not using a water heating boiler for heating purposes, as the average temperature of the outside air during the day is around 20 ^{0}C.

Figure 6 shows the temperature characteristics for the typical days of the study (December 29, 2020 and January 4, 2021).



a) December 29, 2020



b) January 4, 2021

Fig. 6. Temperature characteristics of the solar house for typical days.

As a result of modeling and calculating the thermal regime of the research object, it was possible to bring the internal temperature environment of the object to the required value within the specified time interval and the possibility of establishing the necessary temperature regime in the solar house without increasing the thickness of the thermal insulation layers, that is, without allowing the material and resource costs to increase.

4 Conclusions

The following conclusions and recommendations were developed based on the results of experiments and calculations based on the heat balance scheme of the proposed solar house with autonomous heat supply and the electric-heat scheme of the physical model for the experimental solar house:

- A mathematical model of the heat balance of a one-story rural house in non-stationary mode was developed and a calculation algorithm was proposed.
- Based on mathematical modeling, the influence of the heat capacity of the wall structure on the temperature regime of the building was studied.
- The proposed model allows to analyze the temperature change of the indoor air of the building depending on the ambient temperature.
- The mathematical model of the building's heat balance in the dynamic mode and the obtained calculation results can be used in the development of energy-efficient solar houses.

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