

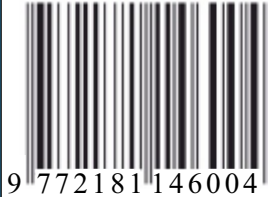
BUXORO DAVLAT UNIVERSITETI ILMIY AXBOROTI



Научный вестник Бухарского государственного университета
Scientific reports of Bukhara State University

9/2024

E-ISSN 2181-1466



9 772181 146004

ISSN 2181-6875



9 772181 687004



@buxdu_uz



@buxdu1



@buxdu1



www.buxdu.uz

9/2024

BUXORO DAVLAT UNIVERSITETI ILMIY AXBOROTI
SCIENTIFIC REPORTS OF BUKHARA STATE UNIVERSITY
НАУЧНЫЙ ВЕСТНИК БУХАРСКОГО ГОСУДАРСТВЕННОГО УНИВЕРСИТЕТА

Ilmiy-nazariy jurnal

2024, № 9, sentabr

Jurnal 2003-yildan boshlab **filologiya** fanlari bo'yicha, 2015-yildan boshlab **fizika-matematika** fanlari bo'yicha, 2018-yildan boshlab **siyosiy** fanlar bo'yicha, **tarix** fanlari bo'yicha 2023 yil 29 avgustdan boshlab O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasining dissertatsiya ishlari natijalari yuzasidan ilmiy maqolalar chop etilishi lozim bo'lgan zaruriy nashrlar ro'yxatiga kiritilgan.

Jurnal 2000-yilda tashkil etilgan.

Jurnal 1 yilda 12 marta chiqadi.

Jurnal O'zbekiston matbuot va axborot agentligi Buxoro viloyat matbuot va axborot boshqarmasi tomonidan 2020-yil 24-avgust № 1103-sonli guvohnoma bilan ro'yxatga olingan.

Muassis: Buxoro davlat universiteti

Tahririyat manzili: 200117, O'zbekiston Respublikasi, Buxoro shahri Muhammad Iqbol ko'chasi, 11-uy.

Elektron manzil: nashriyot_buxdu@buxdu.uz

TAHRIR HAY'ATI:

Bosh muharrir: Xamidov Obidjon Xafizovich, iqtisodiyot fanlari doktori, professor

Bosh muharrir o'rinbosari: Rasulov To'liqin Husenovich, fizika-matematika fanlari doktori (DSc), professor

Mas'ul kotib: Shirinova Mexrigiyo Shokirovna, filologiya fanlari bo'yicha falsafa doktori (PhD)

Kuzmichev Nikolay Dmitriyevich, fizika-matematika fanlari doktori (DSc), professor (N.P. Ogaryov nomidagi Mordova milliy tadqiqot davlat universiteti, Rossiya)

Danova M., filologiya fanlari doktori, professor (Bolgariya)

Margianti S.E., iqtisodiyot fanlari doktori, professor (Indoneziya)

Minin V.V., kimyo fanlari doktori (Rossiya)

Tashqarayev R.A., texnika fanlari doktori (Qozog'iston)

Mo'minov M.E., fizika-matematika fanlari nomzodi (Malayziya)

Mengliyev Baxtiyor Rajabovich, filologiya fanlari doktori, professor

Adizov Baxtiyor Rahmonovich, pedagogika fanlari doktori, professor

Abuzalova Mexriniso Kadirovna, filologiya fanlari doktori, professor

Amonov Muxtor Raxmatovich, texnika fanlari doktori, professor

Barotov Sharif Ramazonovich, psixologiya fanlari doktori, professor, xalqaro psixologiya fanlari akademiyasining haqiqiy a'zosi (akademigi)

Baqoyeva Muhabbat Qayumovna, filologiya fanlari doktori, professor

Bo'riyev Sulaymon Bo'riyevich, biologiya fanlari doktori, professor

Jumayev Rustam G'aniyevich, siyosiy fanlar nomzodi, dotsent

Djurayev Davron Raxmonovich, fizika-matematika fanlari doktori, professor

Durdiyev Durdimurod Qalandarovich, fizika-matematika fanlari doktori, professor

Olimov Shirinboy Sharofovich, pedagogika fanlari doktori, professor

Qahhorov Siddiq Qahhorovich, pedagogika fanlari doktori, professor

Umarov Baqo Bafoyevich, kimyo fanlari doktori, professor

Murodov G'ayrat Nekovich, filologiya fanlari doktori, professor

O'rayeva Darmonoy Saidjonovna, filologiya fanlari doktori, professor

Navro'z-zoda Baxtiyor Nigmatovich, iqtisodiyot fanlari doktori, professor

Hayitov Shodmon Ahmadovich, tarix fanlari doktori, professor

To'rayev Halim Hojiyevich, tarix fanlari doktori, professor

Rasulov Baxtiyor Mamajonovich, tarix fanlari doktori, professor

Eshtayev Alisher Abdug'aniyevich, iqtisodiyot fanlari doktori, professor

Quvvatova Dilrabo Habibovna, filologiya fanlari doktori, professor

Axmedova Shoir Nematovna, filologiya fanlari doktori, professor

Bekova Nazora Jo'rayevna, filologiya fanlari doktori (DSc), professor

Amonova Zilola Qodirovna, filologiya fanlari doktori (DSc), dotsent

Hamroyeva Shahlo Mirjonovna, filologiya fanlari doktori (DSc), dotsent

Nigmatova Lola Xamidovna, filologiya fanlari doktori (DSc), dotsent

Jamilova Bashorat Sattorovna, filologiya fanlari doktori, professor

Boboyev Feruz Sayfullayevich, tarix fanlari doktori

Jo'rayev Narzulla Qosimovich, siyosiy fanlar doktori, professor

Xolliyev Askar Ergashovich, biologiya fanlari doktori, professor

Artikova Hafiza To'ymurodovna, biologiya fanlari doktori, professor

Norboyeva Umida Toshtemirovna, biologiya fanlari doktori, professor

Hayitov Shavkat Ahmadovich, filologiya fanlari doktori, professor

Qurbonova Gulnoz Negmatovna, pedagogika fanlari doktori (DSc), professor

Ixtiyarova Gulnora Akmalovna, kimyo fanlari doktori, professor

Rasulov Zubaydullo Izomovich, filologiya fanlari doktori (DSc), professor

Mirzayev Shavkat Mustaqimovich, texnika fanlari doktori, professor

Samiyev Kamoliddin A'zamovich, texnika fanlari doktori, professor

Esanov Husniddin Qurbonovich, biologiya fanlari doktori, dotsent

Raupov Soyib Saidovich, tarix fanlari nomzodi, professor

Zaripov Gulmurot Toxirovich, texnika fanlari nomzodi, professor

Jumayev Jura, fizika-matematika fanlari nomzodi, dotsent

Klichev Oybek Abdurasulovich, tarix fanlari doktori, dotsent

G'aybulayeva Nafisa Izattullayevna, filologiya fanlari doktori (DSc), dotsent

MUNDARIJA * СОДЕРЖАНИЕ *** CONTENTS**

МАТЕМАТИКА * MATHEMATICS *** МАТЕМАТИКА**

Отакулов С., Рахимов Б.Ш.	О локально-относительной управляемости одного класса дифференциальных включений	3
Ergashova Sh.H.	Ikki bozonli sistemaga mos Shryodinger operatori xos qiymatlarining cheksizligi	11
Жураев Ф.М.	Задача с условием Геллерстедта на не параллельных характеристиках для вырождающегося нагруженного уравнения параболо-гиперболического типа	17
Muxammadjonov A.A., Abduraxmonova R.A., Vasiyeva G.A.	Cho‘zilgan uchburchak gumbaz qirralari bo‘ylab uch quvlovchi va bir qochuvchining tutish differensial o‘yini	29
Imomova Sh.M., Mardonova M.A.	Koshi masalasini taqribiy yechish	39
Durdiev U.D.	Initial-boundary value problem for the integro-differential equation of beam vibration	47
Akhadkulov H.A.	Exploring rigidity for circle diffeomorphisms with break type of singularitie	52
Tosheva N.A., Ravshanova M.I.	Silvestr alomati va uning tatbiqlari	56
Муминов М.Э., Раджабов Т.А.	Условия существования решений уравнений в частных производных с кусочно-постоянными аргументами	63

FIZIKA * PHYSICS *** ФИЗИКА**

Ergashev S.Sh., Sanaqulov Sh.M., Boymurodova R.A.	Maydanak observatoriyasida ekzosayyoralar kuzatuvi	70
Тешаев М.Х., Ҳомидов Ф.Ф., Жалолов Ф.Б., Нарзуллоев М.А.	Йиғилган массали қовушқоқ-эластик пластинканинг хос тебранишлари	75
Саипназаров Ж. М.	Распространение волн в двухслойной упругой среде	82
Химматов И.Ф., Улин С.Е.	Моделирование нейтронозахватной терапии с использованием GEANT4	91
Xidirov B.G‘., Maxmanov U.K.	РЗНТ miqdorining qiymati organik quyosh elementlarning optik va strukturaviy xossalariga ta’siri	95
Халилов Ш.Ф.	Қовушқоқ эластик муҳит билан алоқада бўлган цилиндрик қобикқа хос тўлқинларни тарқалиши	102
Shermatov B.N., Murodov S.N., Shermatov A.B., Nishonov I.E., Xoldorov O.N., Mirov M.F.	Kiselev qora tuynukning xususiyatlarini o‘rganishda kvazi davriy tebranishlar va ularning umumiy xususiyatlari	106
Sharopov U.B., Akbarova F.Dj.	Elektron nurlanishda rux oksid kristallari sirtida zaryadlarning hosil bo‘lishi	110

INITIAL-BOUNDARY VALUE PROBLEM FOR THE INTEGRO-DIFFERENTIAL EQUATION OF BEAM VIBRATION

Durdiev Umidjon Durdimuratovich,

Head of the department Differential equations, Bukhara State University,
Senior Research Fellow at the Institute of Mathematics named after V.I.Romanovskiy
at the Academy of sciences of the Republic of Uzbekistan
umidjan93@list.ru, u.d.durdiev@buxdu.uz

Abstract. This paper considers the initial boundary value problem for the integro-differential equation of vibration of a finite-length beam. From a physical standpoint, the boundary conditions stipulate that the beam is fixed at one end and free at the other. Subsequently, the vibration operator of a homogeneous beam is treated using the method of separation of variables for eigenvalues and eigenfunctions, which reduces the problem to a Fredholm integral equation of the second kind. The method of successive approximations is employed to derive a solution to these equations, and the existence theorem is subsequently demonstrated.

Keywords: integro-differential, kernel, beam vibration equation, Fredholm integral equation, eigenvalues and eigenfunctions.

НАЧАЛЬНО-КРАЕВАЯ ЗАДАЧА ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ КОЛЕБАНИЙ БАЛКИ

Аннотация. В данной работе рассматривается начальная краевая задача для интегро-дифференциального уравнения колебаний балки конечной длины. С физической точки зрения граничные условия описывают, что балка закреплена с одной стороны и свободна с другой. Затем, используя метод разделения переменных для оператора вибрации однородной балки, с помощью собственных значений и собственных функций задача сводится к интегральным уравнениям Фредгольма второго рода. К этим уравнениям применяется метод последовательных приближений и доказывается теорема существования решения.

Ключевые слова: интегро-дифференциальное, ядро, уравнение колебаний балки, интегральное уравнение Фредгольма, собственные значения и собственные функции.

INTEGRO-DIFFERENTIAL BALKA TEBRANISH MASALASI UCHUN BOSHLANG'ICH-CHEGARAVIY MASALA

Annotatsiya. Ushbu ishda chekli uzunlikdagi balka tebranishlarining integro-differensial tenglamasi uchun boshlang'ich chegaraviy masala ko'rib chiqiladi. Fizik nuqtayi nazardan, chegara shartlari balkaning bir tomonida mustahkamlangan, ikkinchisi esa erkin ekanligini tavsiflaydi. So'ngra, bir xil balkaning tebranish operatori uchun o'zgaruvchilarni ajratish usulidan foydalanib, masala xos qiymatlari va xos funksiyalaridan foydalangan holda ikkinchi turdagi Fredgolm integral tenglamalariga keltiriladi. Bu tenglamalarga ketma-ket yaqinlashish usuli qo'llaniladi va yechimning mavjudligi teoremasi isbotlanadi.

Kalit so'zlar: integro-differensial, yadro, balka tebranish tenglamasi, Fredgolm integral tenglamasi, xos qiymatlar va xos funksiyalar.

Introduction and statement of the problem. A beam is a linear structural element with a length that is significantly greater than its width and height. It is employed in load-bearing structures with diverse bearing conditions, primarily for its ability to withstand bending forces. They are manufactured with a variety of cross-sections, including round, square (beam), box beam, T-beam, I-beam, channel, and others.

In practice, a horizontally positioned beam typically bears the vertical transverse weight load. However, in some cases, the effect of probable horizontal transverse forces must also be considered, such as in the context of wind load or an earthquake. The loaded beam, in turn, exerts a force on the supports, which may be columns, hangers, walls, or other beams (bars). The load is then transmitted further and ultimately, in the majority of cases, is taken up by the structural elements that are working in compression – the supports.

The case of truss construction, in which the rods rest on a horizontal beam, can be distinguished from the aforementioned examples and examined separately.

The fourth-order partial differential equation is derived from numerous problems associated with rod, beam, and plate vibrations, which have significant applications in the field of structural mechanics [1, 282 p], [2, 298 p.], [3, 143 p.].

In recent years, there has been a notable increase in interest in the study of direct and inverse problems associated with the equation of beam vibrations. This is evidenced by a growing body of literature on the subject, as evidenced by references [4–16]. In [11], the inverse problem of determining the time-dependent coefficient in the transverse vibration equation of a beam is considered. From a physical perspective, this coefficient represents the stiffness of the beam. In [15], we address a direct problem pertaining to the vibration equation of an infinite beam, specifically the Cauchy problem. Additionally, we examine an inverse problem, namely determining the time-dependent coefficient associated with the lower-order term in the aforementioned equation. The direct problem of nonlocal in time and inverse problems with integral overriding conditions for the beam vibration equation are addressed in [16]. In this paper we consider the initial boundary value problem for the integro-differential equation of vibration of a finite-length beam.

Let us consider the integro-differential equation of beam vibration

$$w_{tt} + a^2 w_{xxxx} = \int_0^t k(t)w(x, t - \tau)d\tau, \#(1)$$

in the domain

$$G = \{(x, t): 0 < x < l, 0 < t \leq T\}, \#(2)$$

with initial

$$w(x, 0) = \varphi(x), \quad w_t(x, 0) = \psi(x), \quad x \in [0, l], \#(3)$$

and boundary conditions

$$w(0, t) = w_x(0, t) = w_{xx}(l, t) = w_{xxx}(l, t) = 0, \quad t \in [0, T], \#(4)$$

where l is the length of the beam, T is the time interval.

Problem. It is required to determine the function $w(x, t) \in C_{x,t}^{4,2}(G)$, satisfying the equalities (1) – (4) given numbers a, l, T and sufficiently smooth functions $k(t), \varphi(x), \psi(x)$.

Solution of the initial-boundary value problem. Let us introduce in equation (1) the notation $F(x, t) = \int_0^t k(t)w(x, t - \tau)d\tau$. Then equation (1), get the following form:

$$w_{tt} + a^2 w_{xxxx} = F(x, t). \#(5)$$

To solve the equation (5) with initial (2) and boundary conditions (3), we use the method of separation of variables by representing $w(x, t) = X(x)T(t)$ and obtain the spectral problem with respect to $X(x)$. This problem is fully investigated in [17].

$$w(x, t) = \sum_{n=1}^{\infty} w_n(t)Y_n(x) + \sum_{n=1}^{\infty} v_n(t)Y_n(x). \#(6)$$

$\lambda_n = -d_n^4$ is eigenvalues of the spectral problem, where

$$d_n = \frac{\pi}{k} \left(n - \frac{1}{2} + (1)^n \theta_n \right), \quad \theta_n = O\left(\frac{1}{n^2}\right),$$

$$X_n(x) = \begin{cases} a_n \operatorname{ch} d_n \left(x - \frac{l}{2} \right) + b_n \sin d_n \left(x - \frac{l}{2} \right), & n = 2k - 1, \\ c_n \operatorname{sh} d_n \left(x - \frac{l}{2} \right) + f_n \cos d_n \left(x - \frac{l}{2} \right), & n = 2k, \end{cases} \#(7)$$

is eigenfunction system, where

$$a_n = \operatorname{sh}^{-1} \left(\frac{d_n l}{2} \right), \quad b_n = \cos^{-1} \left(\frac{d_n l}{2} \right), \quad c_n = -\operatorname{ch}^{-1} \left(\frac{d_n l}{2} \right), \quad f_n = \sin^{-1} \left(\frac{d_n l}{2} \right).$$

Normalizing the system of functions (7), we gets

$$Y_n(x) = \frac{X(x)}{\|X(x)\|}, \quad \|X(x)\| = \begin{cases} \sqrt{l} \operatorname{ctg} \left(\frac{d_n l}{2} \right), & n = 2k - 1, \\ \sqrt{l} \operatorname{tg} \left(\frac{d_n l}{2} \right), & n = 2k. \end{cases} \#(8)$$

Note that system (9) is orthonormal and full in the space $L_2[0, l]$ and forms an orthonormal basis therein.

Then the solution of the stated problem (1) – (4) will be sought in the form of the sum of series (6),

$$w_n(t) = \varphi_n \cos ad_n^2 t + \frac{\psi_n}{a d_n^2} \sin ad_n^2 t,$$

where

$$\begin{aligned} \varphi_n &= \int_0^t \varphi(x) Y_n(x) dx, \quad \psi_n = \int_0^l \psi(x) Y_n(x) dx, \quad v_n(t) = \frac{1}{ad_n^2} \int_0^l F_n(s) \sin a d_n^2 (t-s) ds, \\ F_n(t) &= \int_0^l F(x, t) Y_n(x) dx, \end{aligned}$$

and $Y_n(x)$ is determining by the formula (8).

Upon inserting $F(x, t)$ into the solution, we obtain:

$$\begin{aligned} w(x, t) &= \sum_{n=1}^{\infty} \left(\varphi_n \cos ad_n^2 t + \frac{\psi_n}{a d_n^2} \sin ad_n^2 t \right) Y_n(x) + \\ &+ \sum_{n=1}^{\infty} \frac{Y_n(x)}{ad_n^2} \int_0^l \sin ad_n^2 (t-s) \int_0^l \int_0^s k(s) u(\xi, s-\tau) d\tau d\xi ds. \quad \#(9) \end{aligned}$$

For the sake of convenience, we introduce the following notation

$$\begin{aligned} G_0(x, t) &= \sum_{n=1}^{\infty} \left(\varphi_n \cos ad_n^2 t + \frac{\psi_n}{a d_n^2} \sin ad_n^2 t \right) Y_n(x), \\ K(x, t-s) &= \sum_{n=1}^{\infty} \frac{Y_n(x)}{ad_n^2} k(s) \sin ad_n^2 (t-s). \end{aligned}$$

Subsequently, our solution (9) assumes the following form:

$$w(x, t) = G_0(x, t) + \int_0^l \int_0^l \int_0^s K(x, t-s) w(\xi, s-\tau) d\tau d\xi ds.$$

The Fredholm integral equation of the second kind has been obtained. The equation is to be solved using the method of successive approximations for the Fredholm integral equation of the second kind, with the solution presented in the following form:

$$w(x, t) = \sum_{k=0}^{\infty} w_k(x, t), \quad \#(10)$$

where

$$w_0(x, t) = G_0(x, t), \quad w_k = \int_0^l \int_0^l \int_0^s K(x, t-s) w_{k-1}(\xi, s-\tau) d\tau d\xi ds.$$

In estimating the value of u_n in the domain G , we have

$$\begin{aligned} |w_0| &\leq g_0, \\ |w_1| &\leq \left| \int_0^l \int_0^l \int_0^s K(x, t-s) w_0(\xi, s-\tau) d\tau d\xi ds \right| \leq K_0 g_0 \frac{l^3}{2}, \\ |w_2| &\leq \left| \int_0^l \int_0^l \int_0^s K(x, t-s) w_1(\xi, s-\tau) d\tau d\xi ds \right| \leq K_0^2 g_0 \left(\frac{l^3}{2} \right)^2, \\ &\dots \\ |w_k| &\leq \left| \int_0^l \int_0^l \int_0^s K(x, t-s) w_{k-1}(\xi, s-\tau) d\tau d\xi ds \right| \leq K_0^k g_0 \left(\frac{l^3}{2} \right)^k, \end{aligned}$$

where

$$\begin{aligned} K_0 &= \max_{x \in [0, l], s \in [0, T]} |K(x, t-s)| = \max_{x \in [0, l], s \in [0, T]} \left| \sum_{n=1}^{\infty} \frac{Y_n(x)}{ad_n^2} k(s) \sin ad_n^2 (t-s) \right| \leq \\ &\leq \frac{C_0 l^2 k_0}{a \pi^2 \sqrt{l}} \sum_{n=1}^{\infty} \frac{1}{n^2} \leq \frac{2 C_0 l^2 k_0}{a \pi^2}, \end{aligned}$$

$$C_0 = \max\{C_1, 6\}, \quad C_1 = \frac{4}{(1 - e^{-d_1 l})^2}, \quad k_0 = \max_{s \in [0, T]} |k(s)|, \quad g_0 = \max_{x \in [0, l], t \in [0, T]} |G_0(x, t)|.$$

Additionally, there are C_i positive constants that are dependent on both l and T .

In order for the series to converge, it is necessary that $K_0 \frac{l^3}{2} < 1$. This leads to the conclusion that the condition for l is as follows:

$$l < \left(\frac{a\pi^2}{C_0 k_0} \right)^{\frac{2}{9}}.$$

Subsequently, the series (10) is demonstrated to satisfy the following estimate:

$$|w(x, t)| \leq g_0 \sum_{k=1}^{\infty} \left(K_0 \frac{l^3}{2} \right)^k. \quad \#(11)$$

This proves the following:

Lemma 1. For any $(x, t) \in G$ and for all l that satisfy the following condition:

$$l < \left(\frac{a\pi^2}{C_0 k_0} \right)^{\frac{2}{9}},$$

estimate (11) is true.

The formal term-by-term differentiation of the integral equation (9) yields the following result:

$$w_{tt}(x, t) = \sum_{n=1}^{\infty} -(\varphi_n a^2 d_n^4 \cos ad_n^2 t + \psi_n ad_n^2 \sin ad_n^2 t) Y_n(x) + \\ - \sum_{n=1}^{\infty} Y_n(x) ad_n^2 \int_0^l \sin ad_n^2 (t-s) \int_0^l \int_0^s k(s) u(\xi, s-\tau) d\tau d\xi ds, \quad \#(12)$$

$$w_{xxxx}(x, t) = \sum_{n=1}^{\infty} d_n^4 \left(\varphi_n \cos ad_n^2 t + \frac{\psi_n}{a d_n^2} \sin ad_n^2 t \right) Y_n(x) + \\ + \sum_{n=1}^{\infty} \frac{d_n^2 Y_n(x)}{a} \int_0^l \sin ad_n^2 (t-s) \int_0^l \int_0^l k(s) u(\xi, s-\tau) d\tau d\xi ds. \quad \#(13)$$

Lemma 2. If the functions $\varphi(x)$, $\psi(x)$ satisfy the specified conditions,

$$\varphi(x) \in C^5[0, l], \quad \varphi^{VI}(x) \in L_2[0, l], \quad \varphi(0) = \varphi'(0) = \varphi''(l) = \varphi'''(l) = \varphi^{IV}(0) = \varphi^V(0) = 0,$$

$$\varphi(x) \in C^3[0, l], \quad \varphi''''(x) \in L_2[0, l], \quad \psi(0) = \psi'(0) = \psi''(l) = \psi'''(l),$$

then the following representations hold:

$$\varphi_n = \frac{\varphi_n^{VI}}{d_n^6}, \quad \psi_n = \frac{\psi_n''''}{d_n^4}, \quad \#(14)$$

where

$$\varphi_n^{VI} = \begin{cases} \frac{1}{||X_n||} \int_0^l \varphi^{VI}(x) \left(a_n \operatorname{ch} d_n \left(x - \frac{l}{2} \right) + b_n \sin d_n \left(x - \frac{l}{2} \right) \right) dx, & n = 2k - 1, \\ \frac{1}{||X_n||} \int_0^l \varphi^{VI}(x) \left(c_n \operatorname{sh} d_n \left(x - \frac{l}{2} \right) - f_n \cos d_n \left(x - \frac{l}{2} \right) \right) dx, & n = 2k, \end{cases}$$

$$\psi_n'''' = \int_0^l \psi_n''''(x) Y_n(x) dx.$$

By integrating by parts, the integrals over φ_n six times and over ψ_n four times, while taking into account the conditions set forth in Lemma 2, we arrive at the result (14). In accordance with the conclusions of Lemma 2, the series (9), (12), and (13) are majorized by the convergent numerical series

$$C^* \sum_{n=1}^{\infty} \frac{1}{n^2} (|\varphi_n^{VI}| + |\psi_n''''|),$$

it has been demonstrated that the series (9) converges uniformly. Therefore, the sum of the series satisfies the conditions stated in Problems (1) – (4).

The following theorem is thus derived:

Theorem 1. *If the functions φ_n , ψ_n satisfy the conditions set forth in Lemma 2 and $k(t) \in C[0, T]$, then there exists a unique solution to the problem (1) – (4). This solution is determined by the sum of the series (9).*

REFERENCES:

1. Strutt, J. Baron Rayleigh, *The Theory of Sound*, London: Macmillan, 1877. Translated under the title *Teoriya zvuka*, Moscow: Gosudarstv. Izdat. Tekhn. Teor. Lit., 1955, vol. 1.
2. Krylov, A.N. *Vibratsiya sudov (Ship Oscillations)*, Moscow, 2012.
3. Tikhonov A.N. and Samarskii A.A. *Uraveniya matematicheskoi fiziki (Equations of Mathematical Physics)*, Moscow: Nauka, 1966.
4. Ascanelli, A., Cicognani, M., and Colombini, F. The global Cauchy problem for a vibrating beam equation // *J. Differ. Equat.*, 2009, vol. 47, pp. 1440–1451.
5. Sabitov, K.B. A remark on the theory of initial–boundary value problems for the equation of rods and beams // *Differ. Equations*, 2017, vol. 53, № 1, pp. 86–98.
6. Sabitov, K.B. Cauchy problem for the beam vibration equation // *Differ. Equations*, 2017, vol. 53, № 5, pp. 658–664.
7. Vatul'yan, A.O. and Vasil'ev, L.V. On determining the parameters of fastening an inhomogeneous beam in the presence of damping // *Izv. Saratov. Univ. Nov. Ser. Mat. Mekh. Inf.*, 2016, vol. 16, № 4, pp. 449–456.
8. Akhtyamov A.M. and Il'gamov M.A. Flexural model for a notched beam: Direct and inverse problems // *J. Appl. Mech. Tech. Phys.*, 2013, vol. 54, № 1, pp. 132–141.
9. Vatulyan A.M., Bur'yan A.Yu., and Osipov A.V. On the identification of variable stiffness in the analysis of transverse vibrations of a beam // *Vestn. Donsk. Gos. Tekh. Univ.*, 2010, vol. 10, № 6, pp. 825–833.
10. Loktionov A.P. Inverse Cauchy problem for beams in building structures // *Build. Reconstr.*, 2022, vol. 2, pp. 13–25.
11. Durdiev U.D. Inverse problem of determining an unknown coefficient in the beam vibration equation // *Differential Equations*, 2022, vol. 58, № 1, pp. 36–43.
12. Sabitov K.B. Initial–boundary value problems for the beam vibration equation with allowance for its rotational motion under bending // *Differential Equations*, 2021, vol. 57, № 3, pp. 342–352.
13. Sabitov K.B. and Fadeeva O.V. Initial–boundary value problem for the equation of forced vibrations of a cantilever beam // *Vestn. Samarsk. Gos. Tekh. Univ. Ser. Fiz.-Mat. Nauki*, 2021, vol. 25, № 1, pp. 51–66.
14. Durdiev U.D. Problem of determining the reaction coefficient in a fractional diffusion equation // *Differential Equations*, 57(9):1195–1204, 2021.
15. Durdiev U.D. Inverse Problem of Determining the Unknown Coefficient in the Beam Vibration Equation in an Infinite Domain // *Differential Equations*, 2023, vol. 59, № 4, 462-472. DOI: 10.1134/S0012266123040031.
16. Durdiev U.D. A Time-Nonlocal Inverse Problem for the Beam Vibration Equation with an Integral Condition // *Differential Equations*, 2023, vol. 59, № 3, 359-370. DOI: 10.1134/S0012266123030060.
17. Sabitov K.B., Fadeeva O.V. Initial-boundary value problem for the equation of forced vibrations of a cantilever beam // *Vestn. Samarsk. Gos. Tekh. Univ. Ser. Fiz.-Mat. Nauki*, 2021, vol. 25, № 1, pp. 51-66 <https://doi.org/10.14498/vsgtu1845>.