«AMALIY MATEMATIKA VA AXBOROT TEXNOLOGIYALARINING ZAMONAVIY MUAMMOLARI»
XALQARO ILMIY-AMALIY ANJUMAN

АМАЛИЙ МАТЕМАТИКА ВА АХБОРОТ ТЕХНОЛОГИЯЛАРИНИНГ ЗАМОНАВИЙ МУАММОЛАРИ

ХАЛҚАРО ИЛМИЙ-АМАЛИЙ АНЖУМАН

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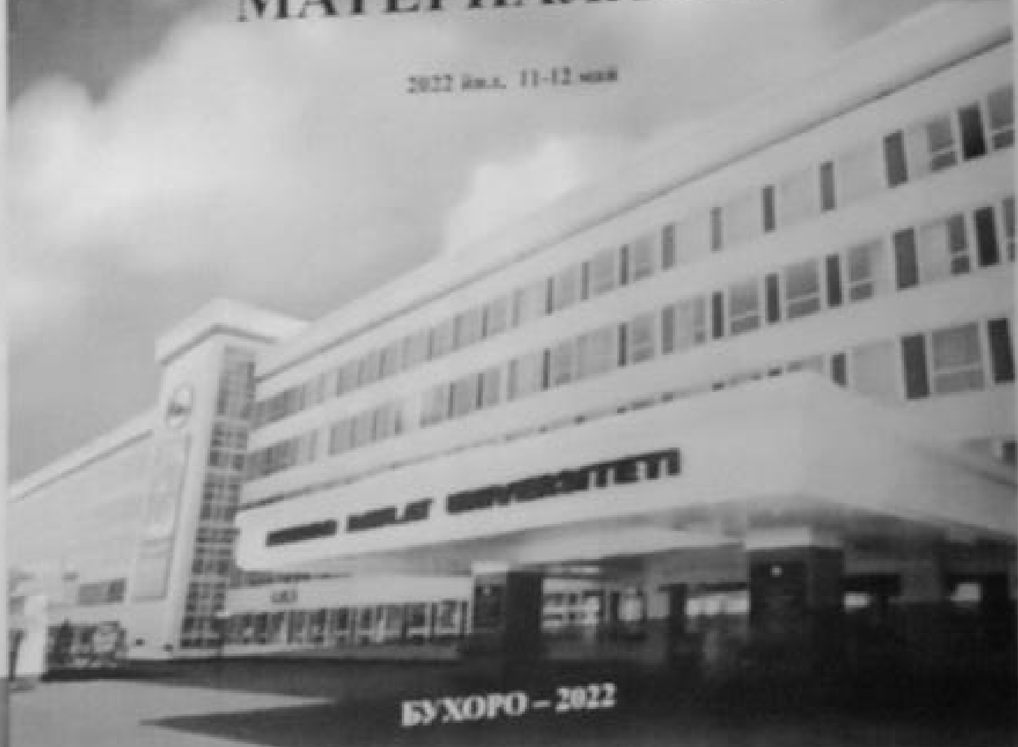

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$$\mu_0 = \left(\max_{x \in T} \int_T \frac{v^2(t)dt}{u(x,t) - m} \right)^{-1}$$

We introduce two bounded and self-adjoint operators $H_\mu^{(1)}$ and $H_\lambda^{(2)}$ (so-called channel operators). They act in $L_2(T^2)$ by

$$H_\mu^{(1)} = H_0 - \mu V_1, \quad H_\lambda^{(2)} = H_0 - \lambda V_2$$

Set

$$E_{\mu,\lambda} = \min\{\xi : \xi \in \sigma_{ess}(H_{\mu,\lambda})\}$$

Then $E_{\mu,\lambda} \in \sigma_{ess}(H_{\mu,\lambda})$ is called the lower bound of the essential spectrum of $H_{\mu,\lambda}$.

The main result of the present note is the following theorem.

Theorem 1. *For the essential spectrum of $H_{\mu,\lambda}$ we have*

$$\sigma_{ess}(H_{\mu,\lambda}) = \sigma(H_\mu^{(1)}) \cup \sigma(H_\lambda^{(2)}).$$

For the lower bound $E_{\mu,\lambda}$ the following assertions hold

(i) If $v(x') \neq 0$ for some $x' \in \Lambda$, then for all $\mu, \lambda > 0$ we have $E_{\mu,\lambda} < m$;

(ii) Let $v(x') = 0$ for all $x' \in \Lambda$.

(ii$_1$) For any $\mu > \mu_0$ and $\lambda > 0$ we have $E_{\mu,\lambda} < m$;

(ii$_2$) For any $\mu \leq \mu_0$ and $\lambda > 0$ we have $E_{\mu,\lambda} = \min \sigma(H_\lambda^{(2)})$.

Moreover, $\max(\sigma(H_{\mu,\lambda})) = M$ for any $\mu, \lambda > 0$.

This result plays a key role in the analysis of the discrete spectrum of $H_{\mu,\lambda}$. In [1] the discrete spectrum of $H_{\mu,0}$ was discussed.

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CUBIC NUMERICAL RANGE OF 3 × 3 BLOCK OPERATOR MATRICES

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Block operator matrices are matrices the entries of which are linear operators between Banach or Hilbert spaces. They arise in various areas of mathematics and its applications. Let $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ be complex Hilbert spaces, and consider $\mathcal{H} := \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3$. With respect to this decomposition, every bounded linear operator $\mathcal{A} \in L(\mathcal{H})$ has a 3 × 3 block operator matrix representation

$$\mathcal{A} := \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{12}^* & A_{22} & A_{23} \\ A_{13}^* & A_{23}^* & A_{33} \end{pmatrix} \tag{1}$$

with bounded linear entries $A_{ij} \in L(\mathcal{H}_j, \mathcal{H}_i)$, $i, j = 1,2,3$ such that $A_{ii}^* = A_{ii}, i = 1,2,3$. In the following we denote by

$$S_{\mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3} := S_{\mathcal{H}_1} \times S_{\mathcal{H}_2} \times S_{\mathcal{H}_3} = \{(f_1 f_2 f_3)^t \in \mathcal{H} : \|f_i\| = 1, i = 1,2,3\}$$

the product of the unit spheres $S_{\mathcal{H}_i}$ in $\mathcal{H}_i$; we also write S^3 or $S_\mathcal{H}$ instead of $S_{\mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3}$ if the decomposition $H = \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3$ is clear (note the slight difference in notation between $S_{\mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3}$ and the unit sphere $S_{\mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3}$ in $H_1 \oplus H_2 \oplus H_3$). In this case $S^3 = \{f = (f_1 f_2 f_3)^t \in H : \|f_i\| = 1, i = 1,2,3\}$.

Definition 1. For $f = (f_1 f_2 f_3)^t \in S_{\mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3}$ we introduce the 3 × 3 matrix

$$\mathcal{A}_f := \begin{pmatrix} (A_{11}f_1, f_1) & (A_{12}f_2, f_1) & (A_{13}f_3, f_1) \\ (A_{12}^*f_1, f_2) & (A_{22}f_2, f_2) & (A_{23}f_3, f_2) \\ (A_{13}^*f_1, f_3) & (A_{23}^*f_2, f_3) & (A_{33}f_3, f_3) \end{pmatrix} \in M_3(C).$$

Then the set

$$W_{\mathcal{H}_1\oplus\mathcal{H}_2\oplus\mathcal{H}_3}(\mathcal{A}) = \bigcup_{f\in S^3} \sigma_p(\mathcal{A}_f)$$

is called cubic numerical range of $\mathcal{A}$ (with respect to the block operator matrix representation (1)). For a fixed decomposition of $\mathcal{H}$, we also write

$$W^3(\mathcal{A}) = W_{\mathcal{H}_1\oplus\mathcal{H}_2\oplus\mathcal{H}_3}(\mathcal{A}).$$

Let $a_{ij}(f) = (A_{ij}f_j, f_i)$ for $i,j = 1,2,3$ and

$$E_k(f) = \frac{1}{3}(a_{11}(f) + a_{22}(f) + a_{33}(f)), \ \text{if} \ a_{11}(f) = a_{22}(f) = a_{33}(f), \quad a_{12}(f) = a_{23}(f) = a_{13}(f) = 0;$$

$$E_k(f) = \frac{1}{3}(a_{11}(f) + a_{22}(f) + a_{33}(f)) + 2\sqrt{-\frac{P(f)}{3}}\cos\frac{\Phi(f)+2\pi k}{3} \ \text{otherwise},$$

where

$$P(f) := -\frac{1}{6}((a_{11}(f) - a_{22}(f))^2 + (a_{11}(f) - a_{33}(f))^2 + (a_{22}(f) - a_{33}(f))^2 - |a_{12}(f)|^2 - |a_{23}(f)|^2 - |a_{13}(f)|^2;$$

$$Q(f) := -\frac{2}{27}(a_{11}(f) + a_{22}(f) + a_{33}(f))^3 + \frac{1}{3}(a_{11}(f) + a_{22}(f) + a_{33}(f))(a_{11}(f)a_{22}(f) + a_{22}(f)a_{33}(f) + a_{11}(f)a_{33}(f) - |a_{12}(f)|^2 - |a_{23}(f)|^2 - |a_{13}(f)|^2) + a_{11}(f)a_{22}(f)a_{33}(f) + 2Re\left(a_{12}(f)a_{23}(f)\overline{a_{13}(f)}\right) + |a_{12}(f)|^2 a_{33}(f) + |a_{23}(f)|^2 a_{11}(f) + |a_{13}(f)|^2 a_{22}(f),$$

$$\Phi(f) = \arccos\left(-\frac{3Q(f)}{2P(f)}\sqrt{-\frac{3}{P(f)}}\right).$$

The main result of this note is the following theorem.

Theorem 1 For the cubic numerical range of $\mathcal{A}$ we have

$$W^3(\mathcal{A}) = \bigcup_{k=1}^{3}\bigcup_{f\in S^3}\{E_k(f)\}.$$

This theorem plays a key role in the estimate of the bounds of $\mathcal{A}$ in terms of cubic numerical range.

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A SET OF FIXED POINTS OF A COVID-19 SPREADING MODEL WITH VACCINATED CASE

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In the work [1] authors proposed SIARD epidemic model and based on this model we consider the following discrete-time SAIRVD model:

$$W = \begin{cases} S^{(1)} = S(1-v_s) - (\beta_i\alpha I + \beta_a\alpha^2 A)S + \omega R + \rho V \\ A^{(1)} = A(1-\mu-\gamma_a-d_a-v_a) + \beta_a\alpha^2 AS \\ I^{(1)} = I(1-\gamma_i-d_i) + \beta_i\alpha IS + \mu A \\ R^{(1)} = R(1-\omega-v_R) + \gamma_a A + \gamma_i I + \gamma_v V \\ V^{(1)} = V(1-\rho-d_v-\gamma_v) + v_a A + v_s S + v_R R \\ D^{(1)} = D + d_a A + d_i I + d_v V \end{cases} \quad (1)$$

This model describes the interaction of susceptible population S, asymptomatic (unreported) infected population A, symptomatic (reported) infected population I, recovered population R, vaccinated population V and death population D. This system also closed system as many other systems, so for simplicity if we assume $S + A + I + R + V + D = 1$ then $S' + A' + I' + R' + V' + D' = 1$